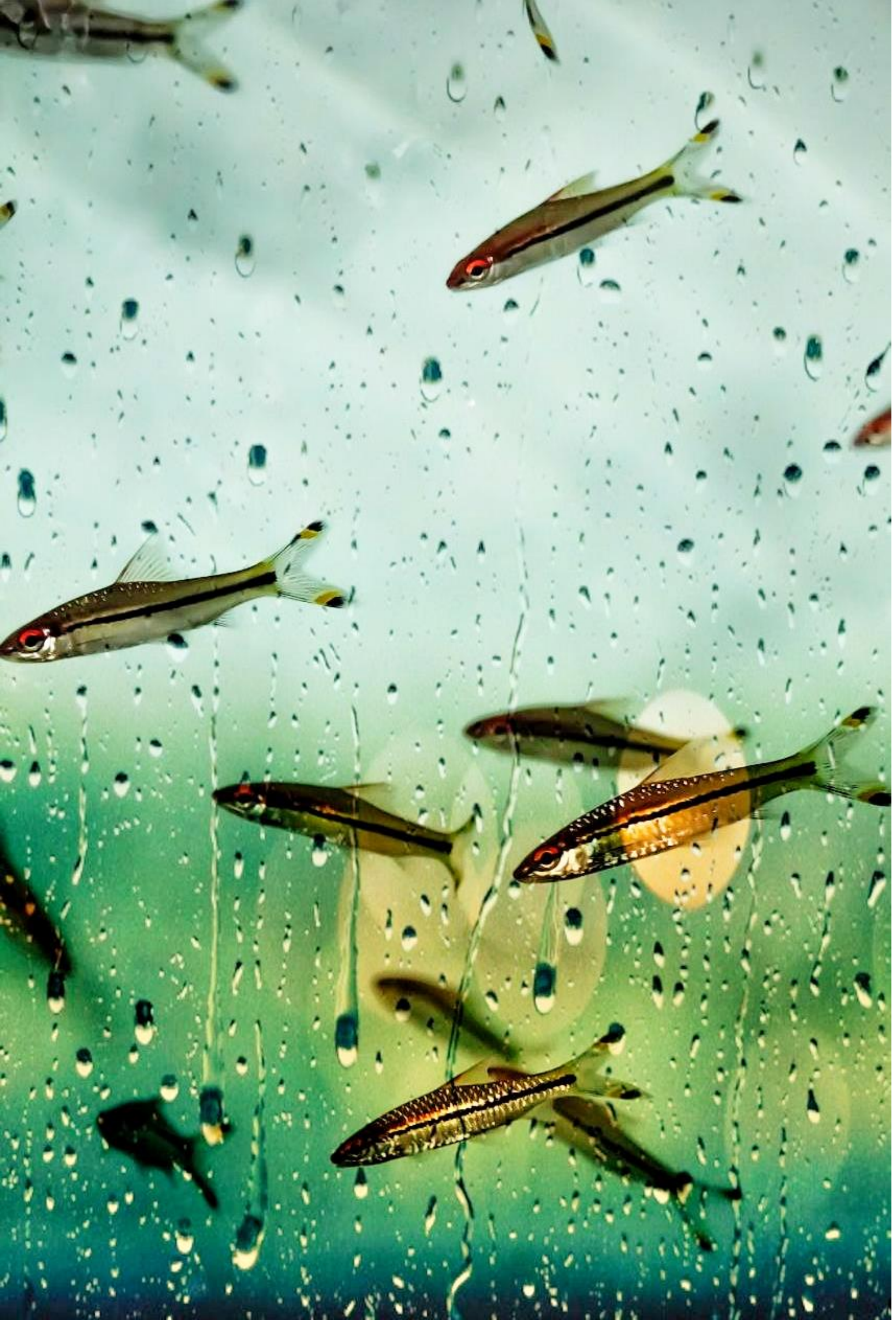

The role of intrinsic noise in biological systems: from animal groups to populations to evolution

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Bengaluru, Karnataka, India

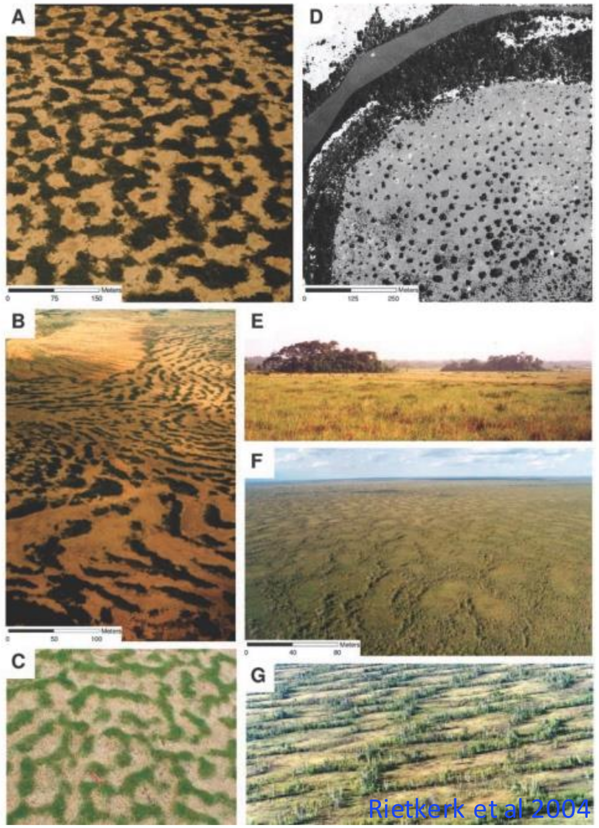
Behaviour Emergence and Evolution Workshop,
IMSc Chennai, 15th June 2026.

Students/postdocs: Jahanvi Tiwari, Jitesh Jhawar, Harikrishnan Rajendran, Ashwin Karichannavar, Arshed Nabeel, Vivek Jadhav, Cassandre Aimon, Shikhara Bhat

Collaborators (most recent): Guy Theraulaz, Clement Sire, Ramon Escubedo, Tim Rogers, Richard Morris, Danny Raj.

Funding: IISc, MoE, DST, DBT, CEFIPRA, SERB/ANFR, STC-ISRO.

Theoretical Ecology *and* Evolution Lab



Self-Organisation in Ecology

- Predictive ecology; Nonequilibrium Physics Approaches
- Mechanisms and Functions
- Theory, Simulations, Experiments, and Field Data

Outline for today's talk: Noise to Order!

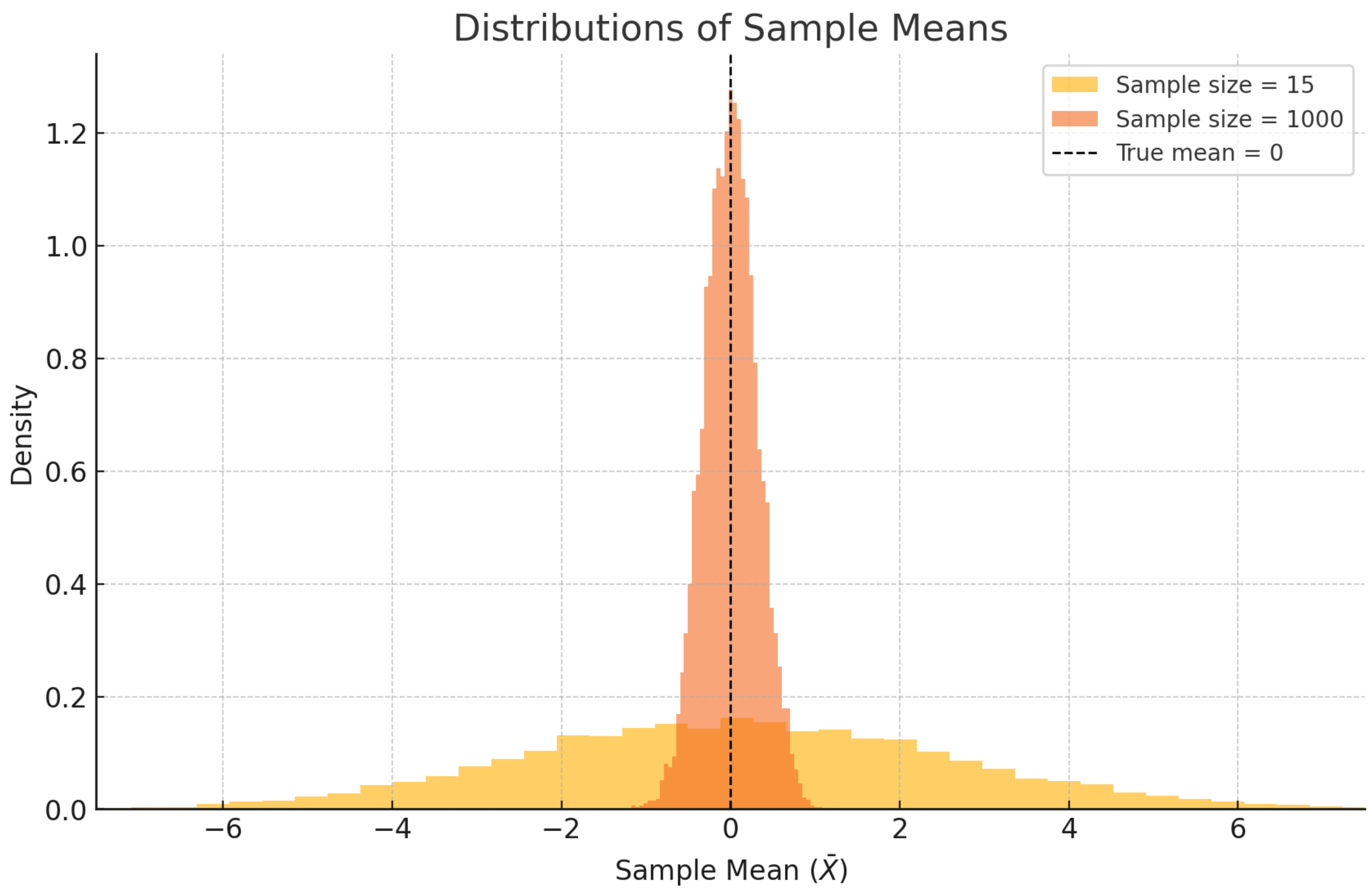
- Theme 1: Deciphering dynamics of fish schools
    - Noise helps characterize the nature of synchrony.
    - Experiments of fish schools

- A novel method: Data-driven equation discovery

- Theme 2: Evolution and Noise

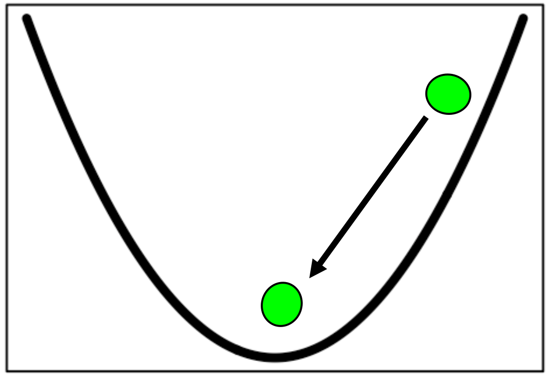
Noise and sample mean

- Take N random numbers: X_1 to X_N
- Estimate the sample mean!

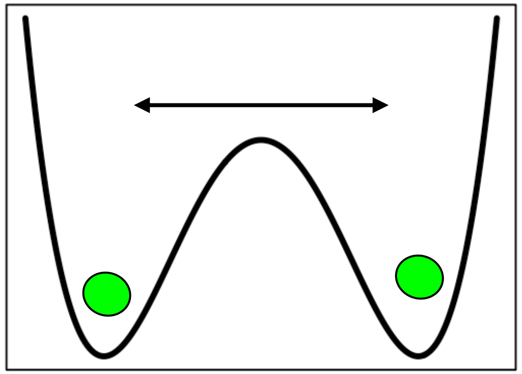


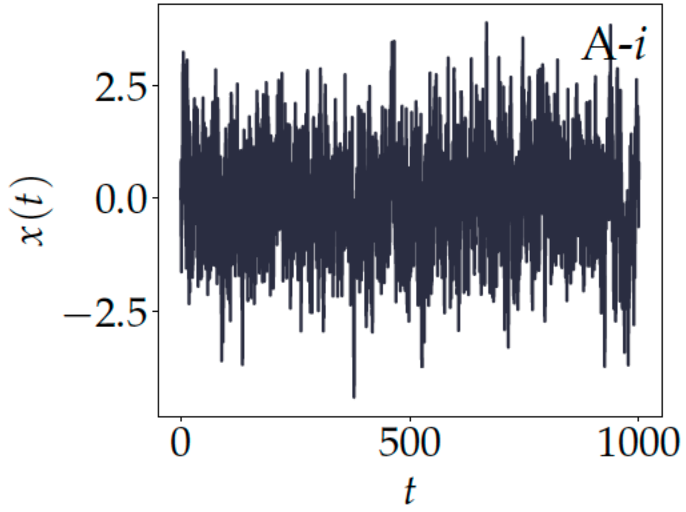
Some cartoon diagrams: potential landscapes

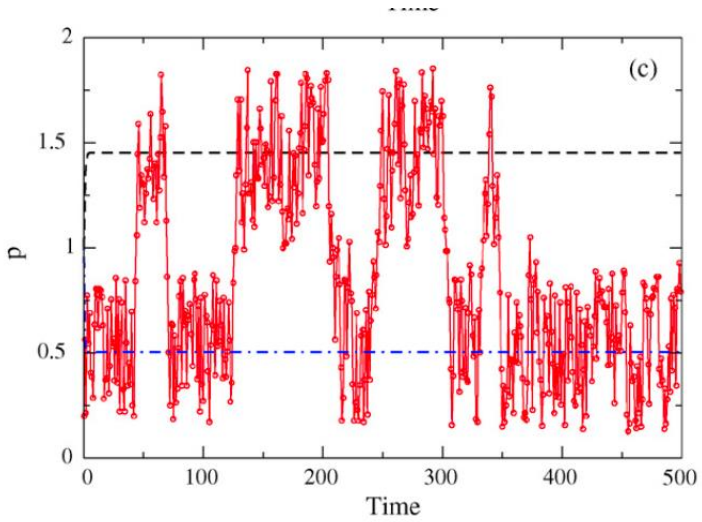
Unistable System + noise



Bistable System + noise

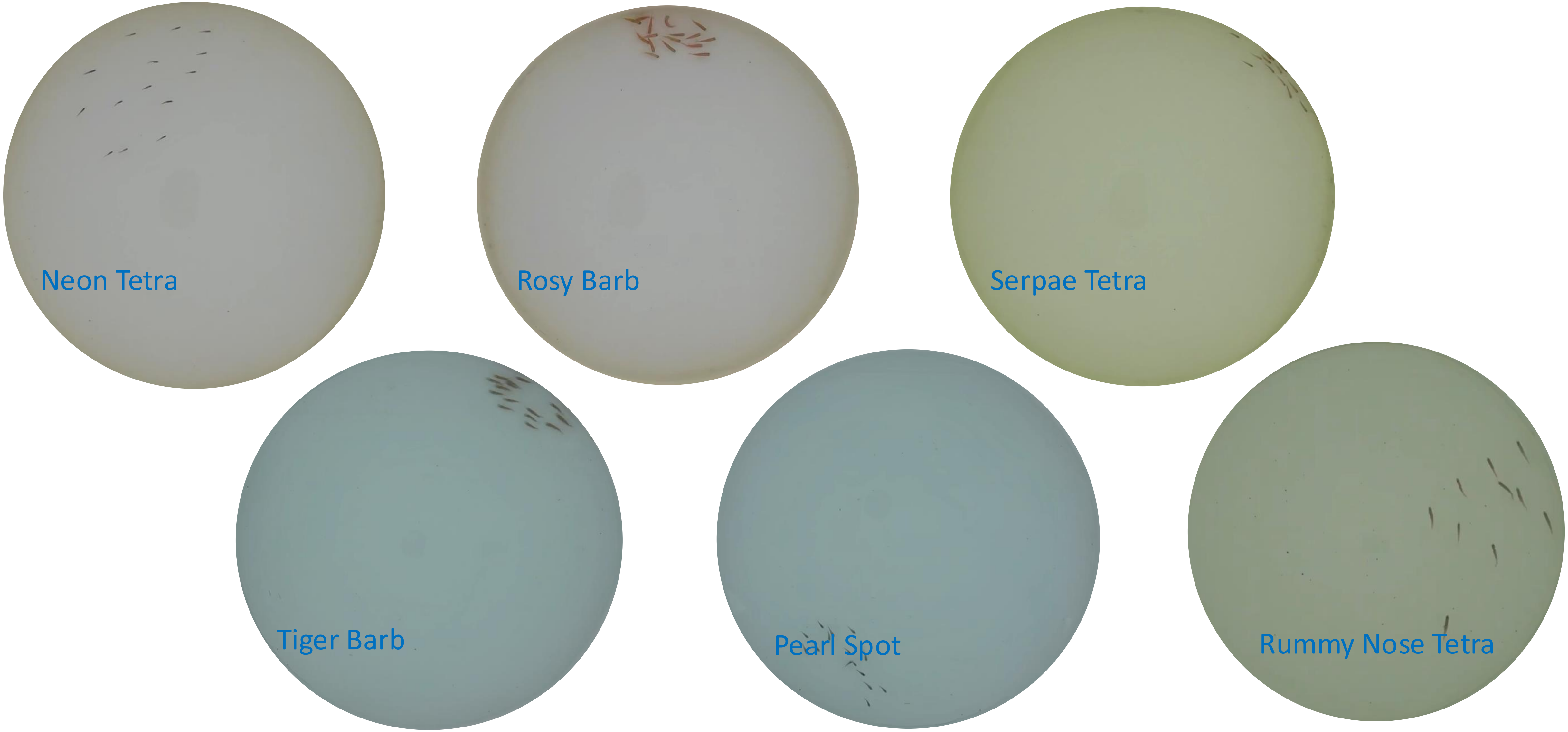






Guttal and Jayaprakash, 2007

Theme 1: The role of noise in collective synchrony!

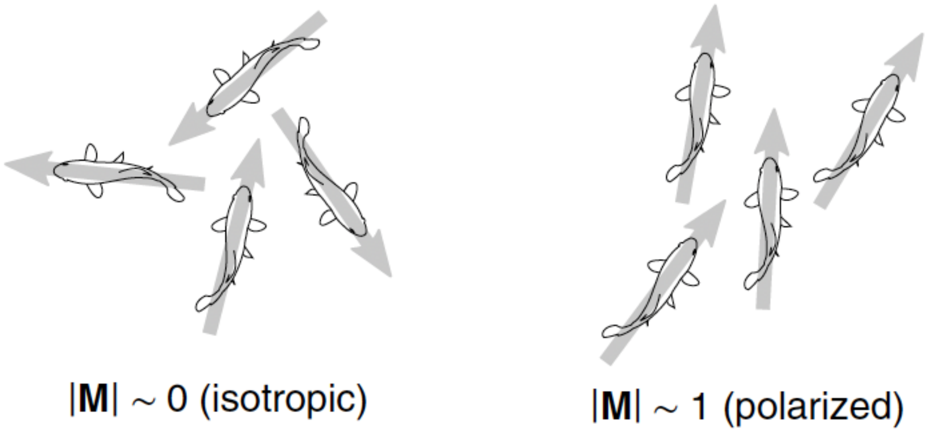


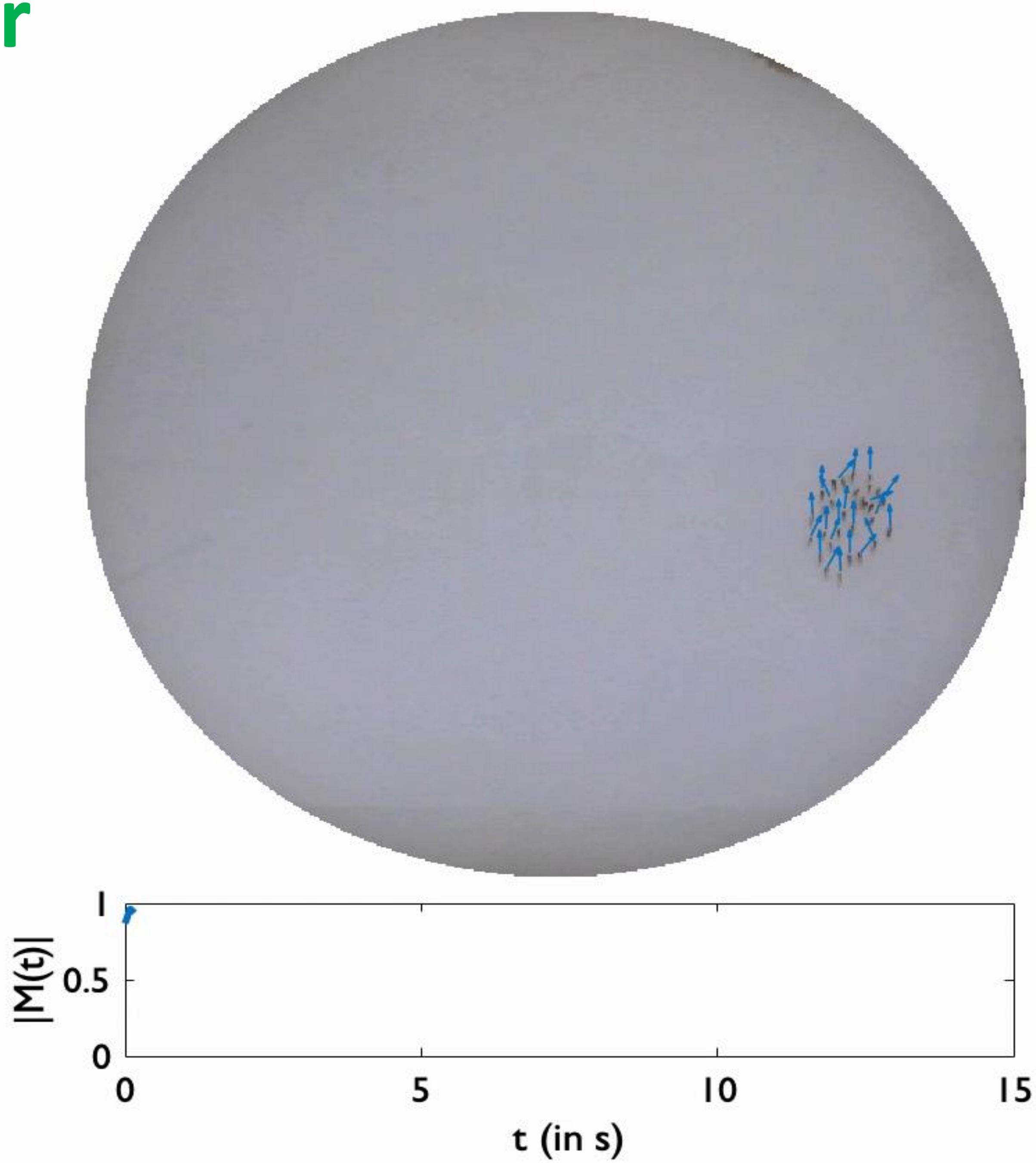
Jitesh Jhawar, Amith-Kumar, Viraj Torsekar, Jahanvi Tiwari, Ashrit Mangalwedhekar, Cassandre Aimon, Arshed Nabeel, Guy Theraulaz, etc

Synchrony Order Parameter

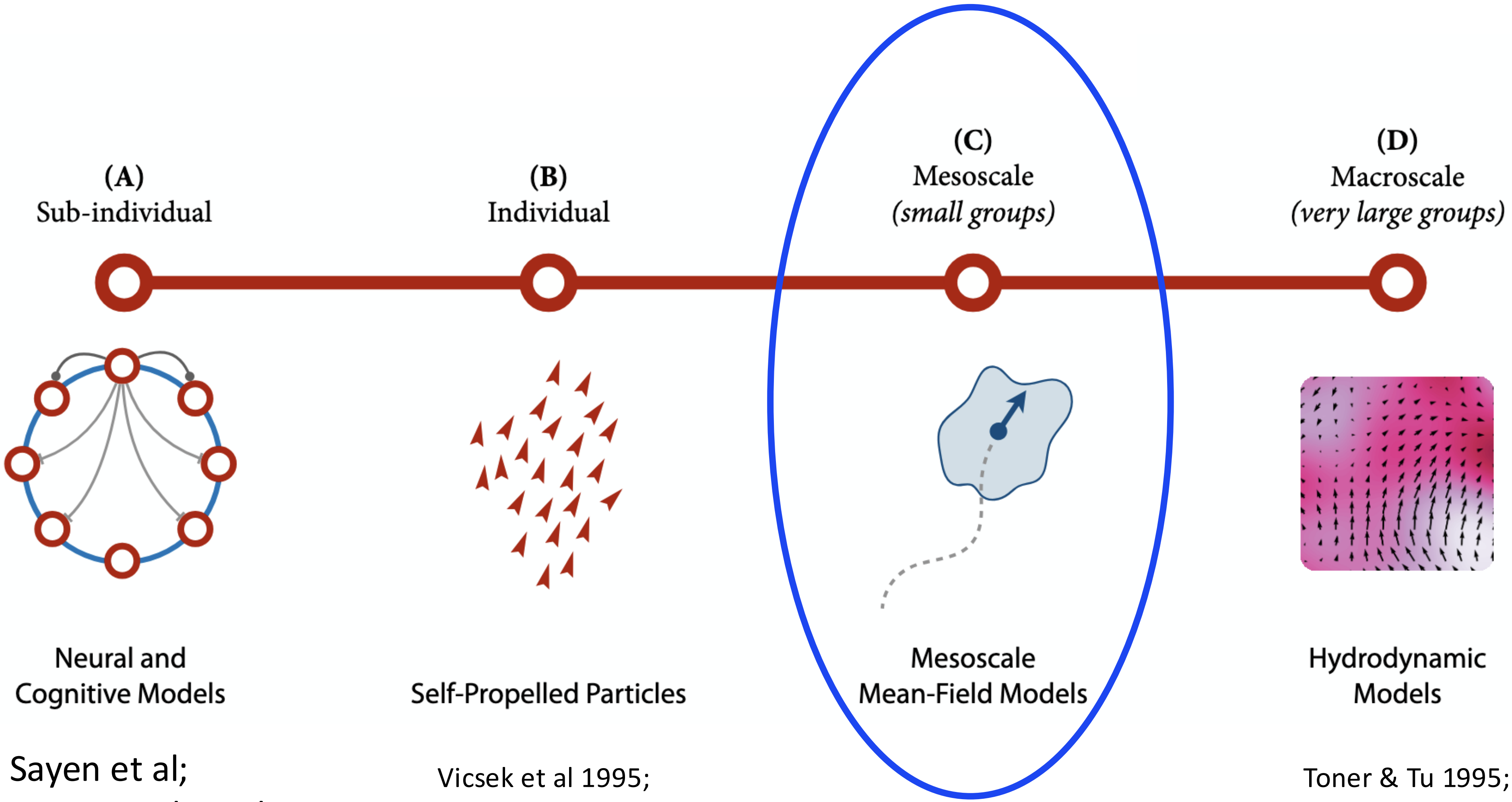
Group Polarisation

$$\boldsymbol{M}(t_n) = \frac{1}{N}\sum_{i=1}^{N} \hat{\boldsymbol{v}}_i(t_n).$$





Various approaches to modelling collective dynamics



Sayen et al;
Romanczuk et al

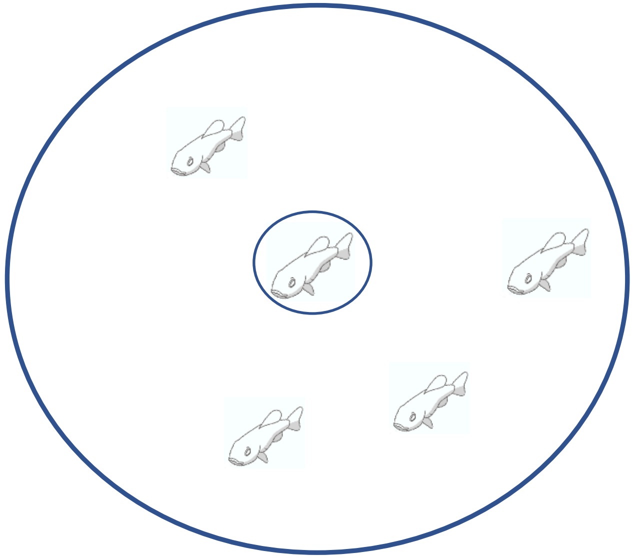
Vicsek et al 1995;
Couzin et al; 2002

Biancaloni et al; Jhawar et al

Toner & Tu 1995; Ramaswamy, 2010;
Marchetti et al Rev Mod Phys 2014

Agent-based models of flocking systems (Vicsek et al 1995; Couzin et al; 2002)

- A key assumption: Individuals move in **an average direction of neighbours**, with a noise



noise term

$$\theta_i(t+1) = \langle\theta\rangle_r + \eta(t)$$

Coarse-grained/hydrodynamic models

- Typically, in the thermodynamic limit (N large):

Flocking order-parameter: $\mathbf{m} = \frac{1}{N}\sum_i \mathbf{v}_i(t)$

drift term — "extrinsic"-noise term — spatial coupling

$$\frac{\partial \mathbf{m}}{\partial t} = (\alpha - \beta|\mathbf{m}|^2)\mathbf{m} + \eta(t) + \nabla \text{ terms}(c, \mathbf{m})$$

See Toner Tu 1995; Ramaswamy, 2010; Marchetti et al Rev Mod Phys 2014

Our approach:

1. **Data-driven** characterization of ***stochastic and nonlinear dynamics of synchrony*** (via Ito-SDEs)

$$\frac{d\mathbf{m}}{dt} = \mathbf{f}(\mathbf{m}) + \mathbf{g}(\mathbf{m}) \cdot \eta(t)$$

Drift
Deterministic part

Diffusion
Stochastic part

2. **Theory-Derived SDEs and Compare with Data-Derived SDEs**

Examples of SDE models in Ecology

- ## Stochastic Population Models:

Deterministic term: Could be logistic model term.

Noise term: Demographic noise term or Environmental noise term

- ## Evolutionary models:

Deterministic term: Selection

Noise term: Random genetic drift

$$\frac{d\mathbf{m}}{dt} = \mathbf{f}(\mathbf{m}) + \mathbf{g}(\mathbf{m}) \cdot \eta(t)$$

Drift (f(m)): Deterministic part

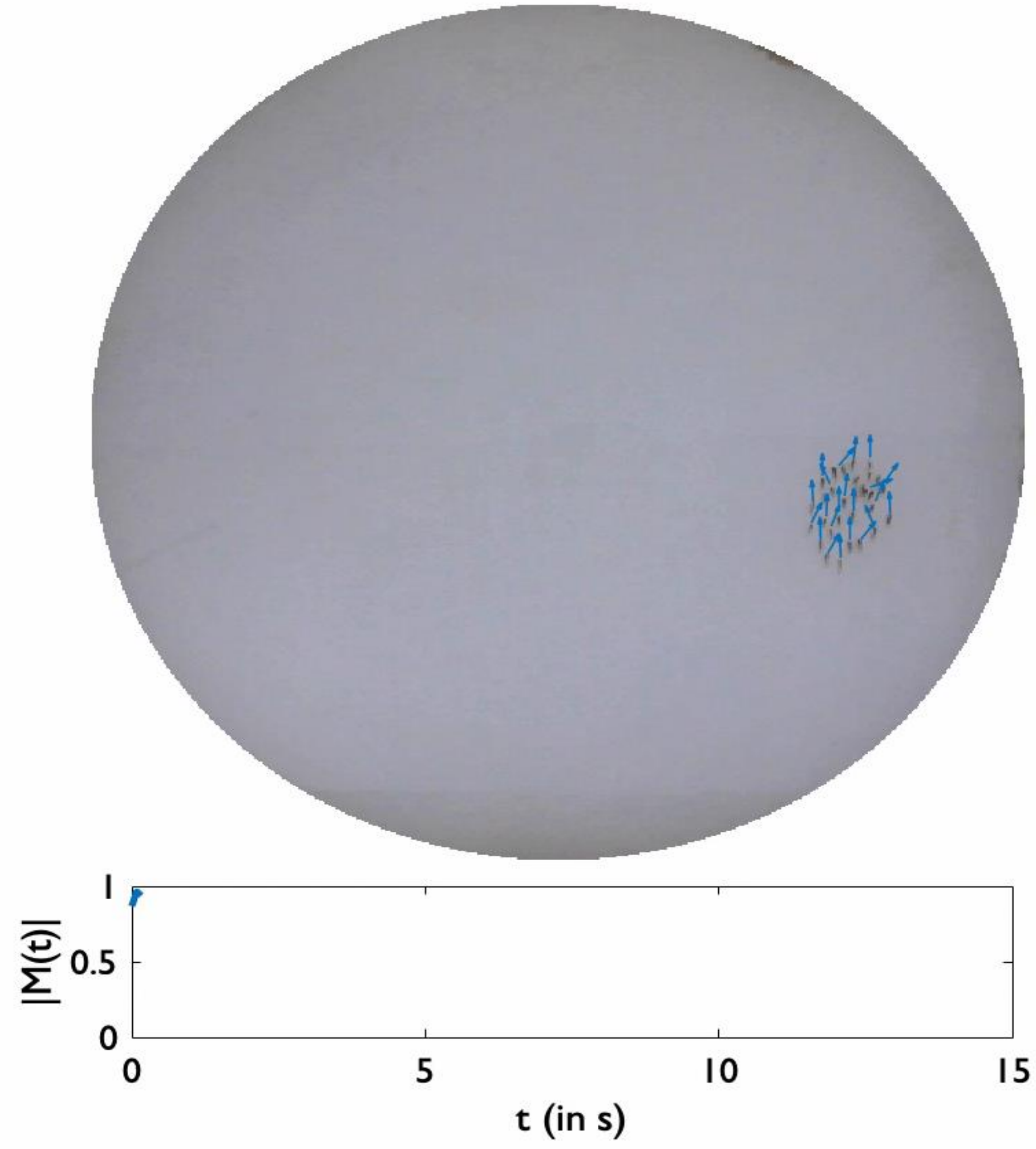
Diffusion (g(m)): Stochastic part

Synchrony Order Parameter

Group Polarisation

$$\boldsymbol{M}(t_n) = \frac{1}{N}\sum_{i=1}^{N} \hat{\boldsymbol{v}}_i(t_n).$$

$$\dot{m} = f(m) + g(m)\eta(t),$$



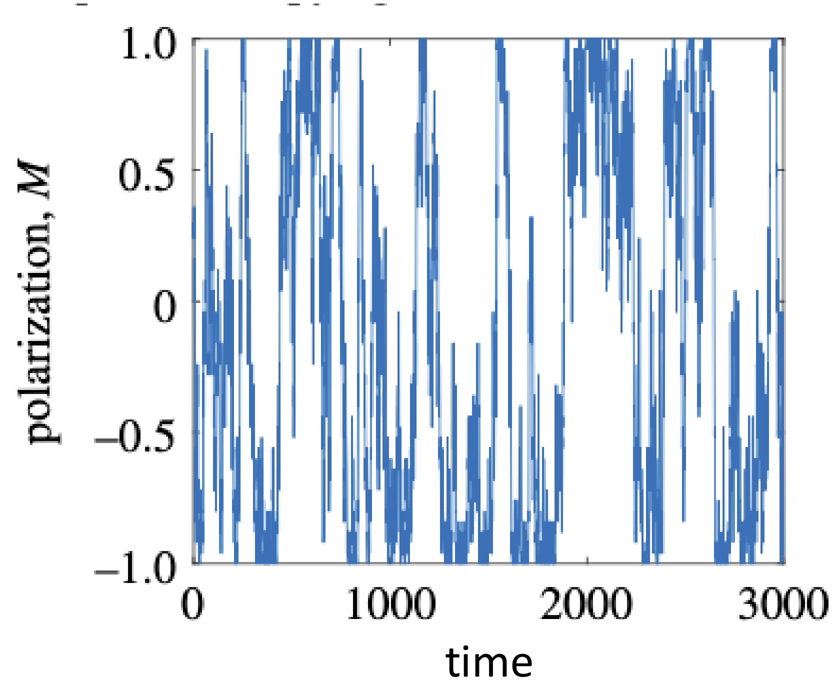
Inverse problem:

Data to equations!

Methods: Noise helps characterize the underlying dynamics!

$$\dot{m} = f(m) + g(m)\eta(t),$$

PyDaddy Data Driven Dynamics



Jhawar and Guttal, 2020; Nabeel, et al 2025.

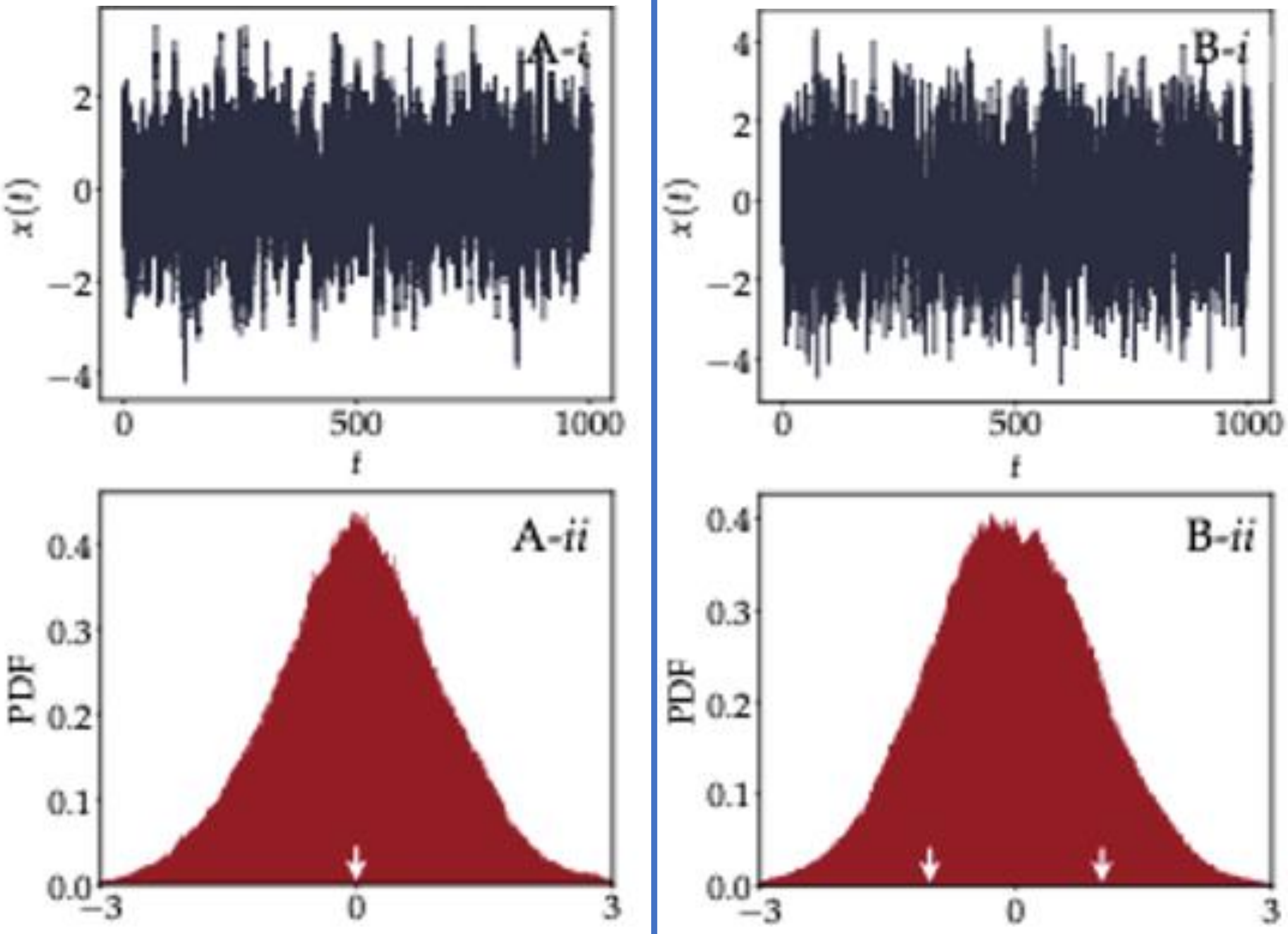
See: **PyDaDDy:** https://github.com/tee-lab/PyDaddy

$$\dot{x} = -x + \sqrt{2} \cdot \eta(t),$$

$$\dot{x} = x - x^3 + 2\left(\sqrt{1 + x^2}\right) \cdot \eta(t).$$

Time series

Distribution

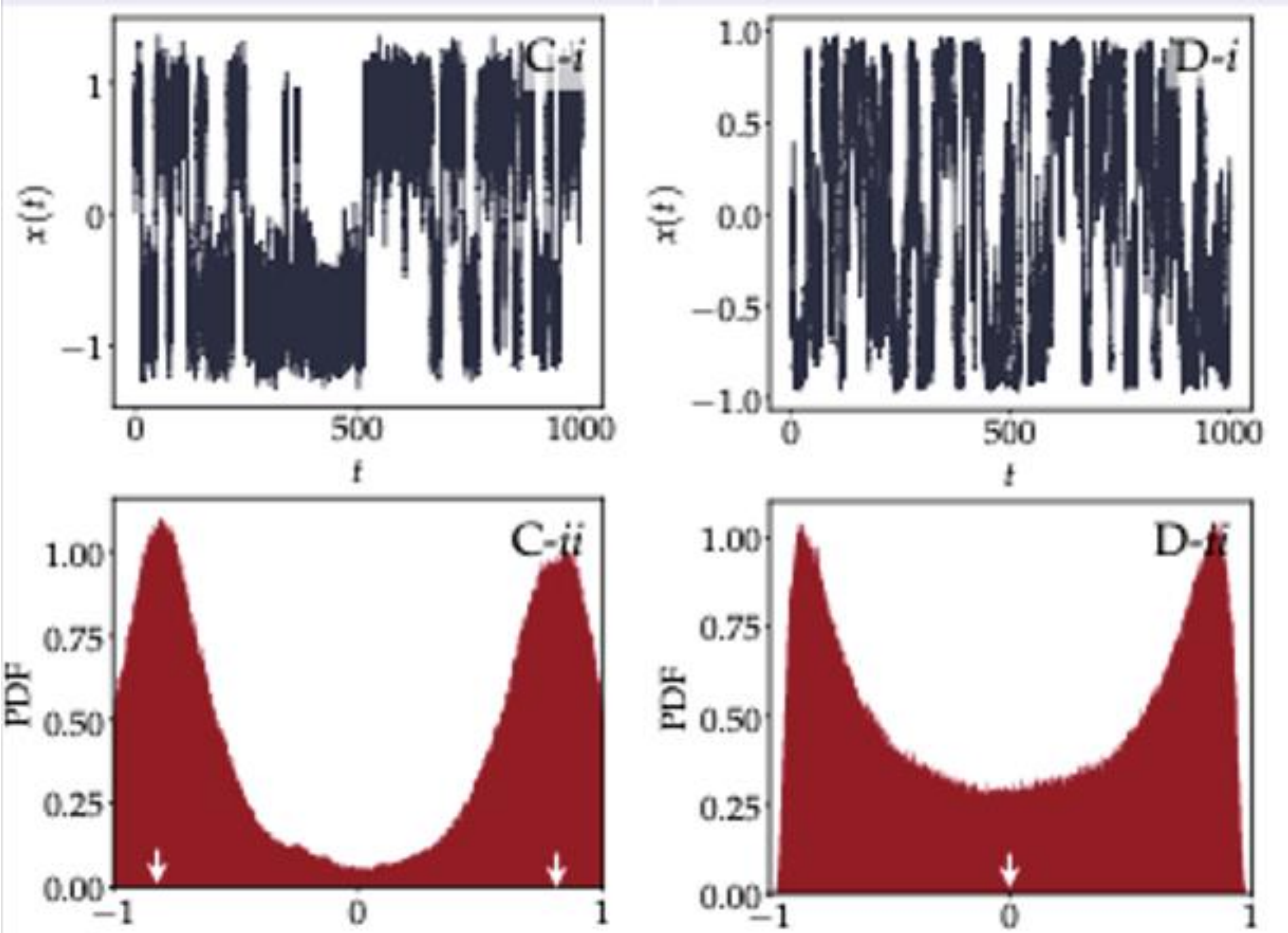


Possible to recover and distinguish above models, even if we had only time series data!

$$\dot{x} = 2x - 3x^3 + \frac{1}{2} \cdot \eta(t), \qquad \dot{x} = -x + \sqrt{2}(1 - x^2) \cdot \eta(t).$$

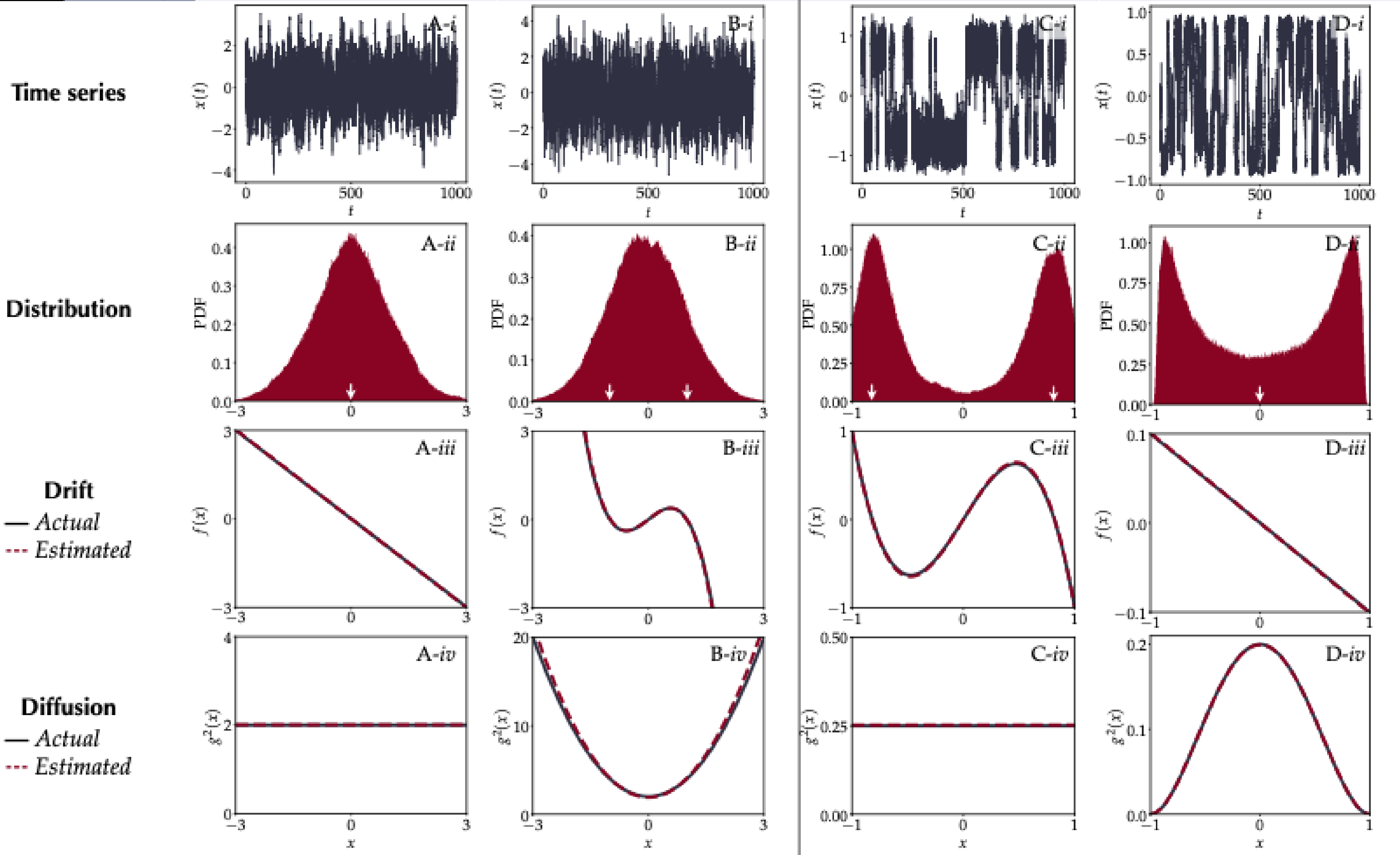
Time series

Distribution



Possible to recover and distinguish above models, even if we had only time series data!

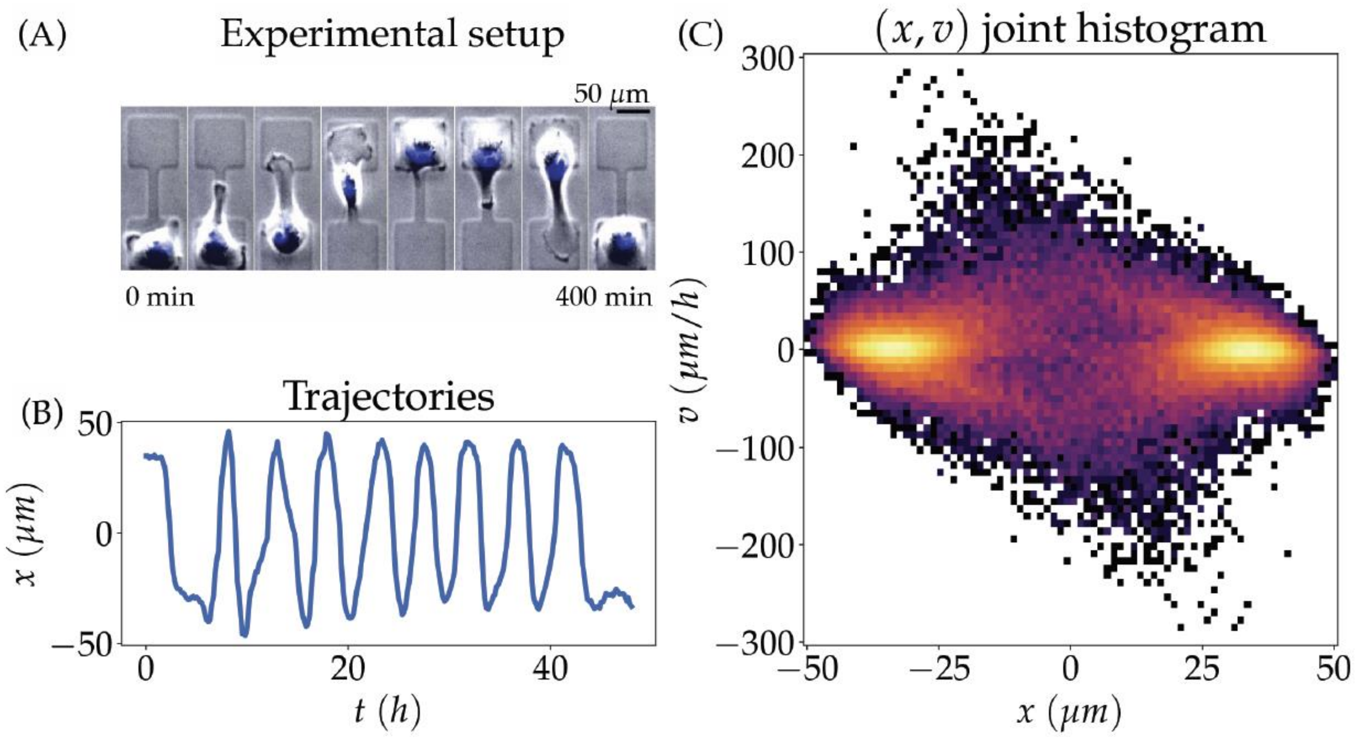
		Unimodal distributions		Bimodal distributions	
Actual	Drift (f)	$-x$	$x - x^3$	$2x - 3x^3$	$-x$
	Diffusion (g^2)	2	$2(1 + x^2)$	0.25	$2(1 - x^2)^2 = 2x^4 - 4x^2 + 2$



Estimated	Drift (f)	$-(0.995 \pm .022)\,x$	$(0.902 \pm .099)\,x - (0.967 \pm .025)\,x^3$	$(1.924 \pm .019)\,x - (-2.879 \pm .024)\,x^3$	$-(0.098 \pm .014)\,x$
	Diffusion	$(2.003 \pm .004)$	$(1.957 \pm .009) + (2.063 \pm .009)\,x^2$	$(0.254 \pm .001)$	$(0.198 \pm .002)\,x^4 - (0.398 \pm .002)\,x + (0.200 \pm .000)$

Single cell oscillations => SDEs

- Van-der pol like oscillator + Noise.



$$\dot{x} = v,$$

$$\dot{v} = f(x, v) + g(x, v) \cdot \eta(t),$$

Nabeel, et al, The American Naturalist, 2025

PyDaddy: https://github.com/tee-lab/PyDaddy

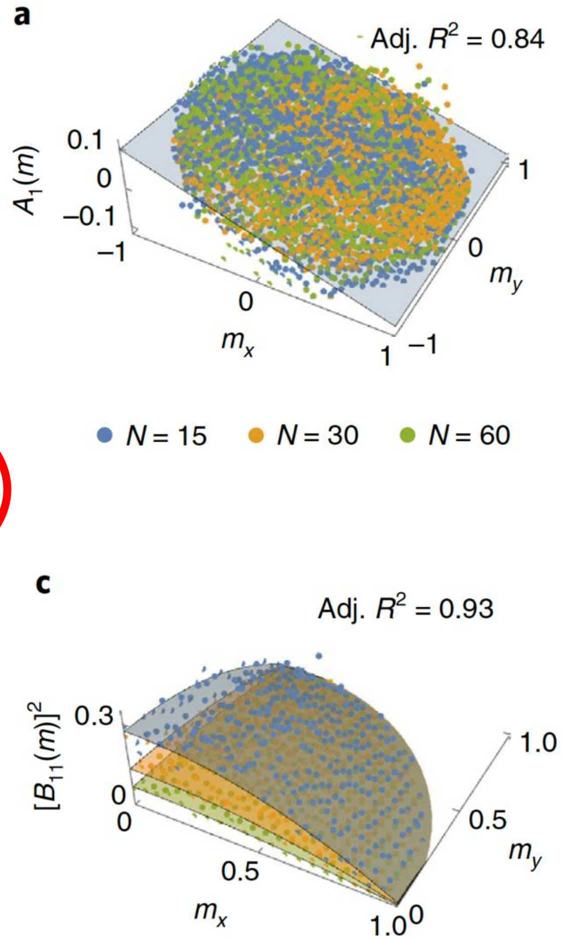
Data-derived dynamical equation for fish schooling:

Fish species: Etroplus suratensis (Karimeen)

Linear Drift with **Quadratic Diffusion**

Noise-induced synchrony

$$\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t} = -\alpha\ \mathbf{m} + \left[\frac{\beta\left(1-|\mathbf{m}|^2\right)+\alpha}{N}\right]^{1/2} \mathbb{1}\cdot\boldsymbol{\eta}(t)$$

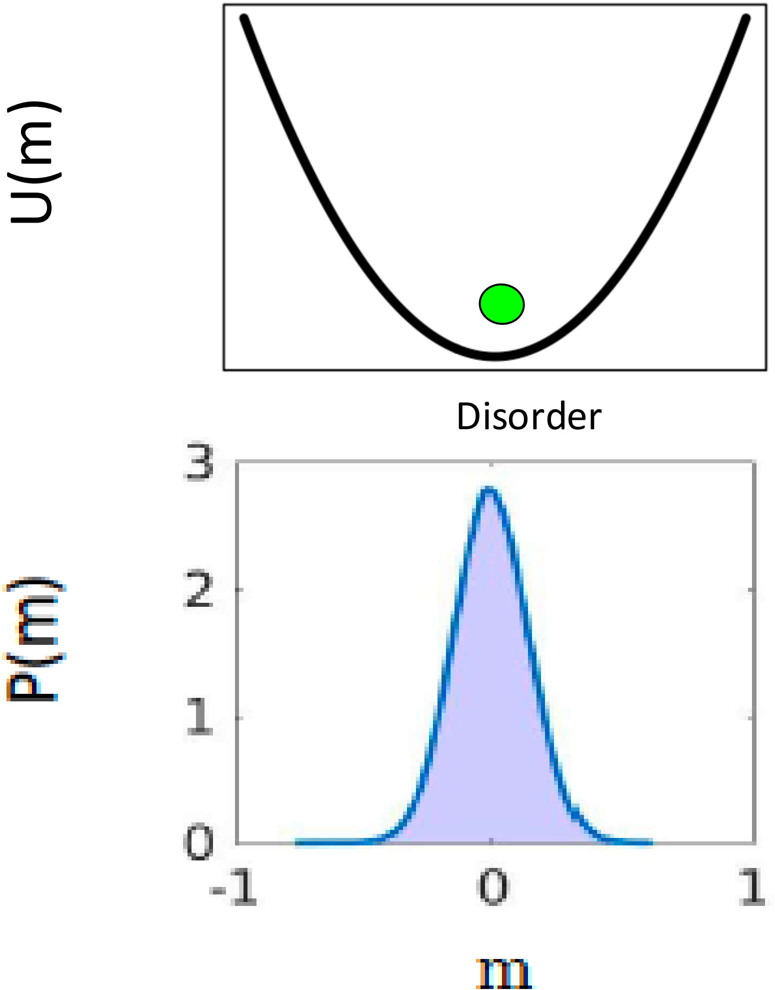


Noise-induced schooling;
Stochastic Pairwise model
Jhawar et al, 2020, Nature physics.

$$\frac{d\mathbf{m}}{dt} = -a\,\mathbf{m} + \sqrt{\frac{a + c\left(1 - |\mathbf{m}|^2\right)}{N}}\,\mathbb{1} \cdot \boldsymbol{\eta}(t)$$

Our naïve expectation of such an equation

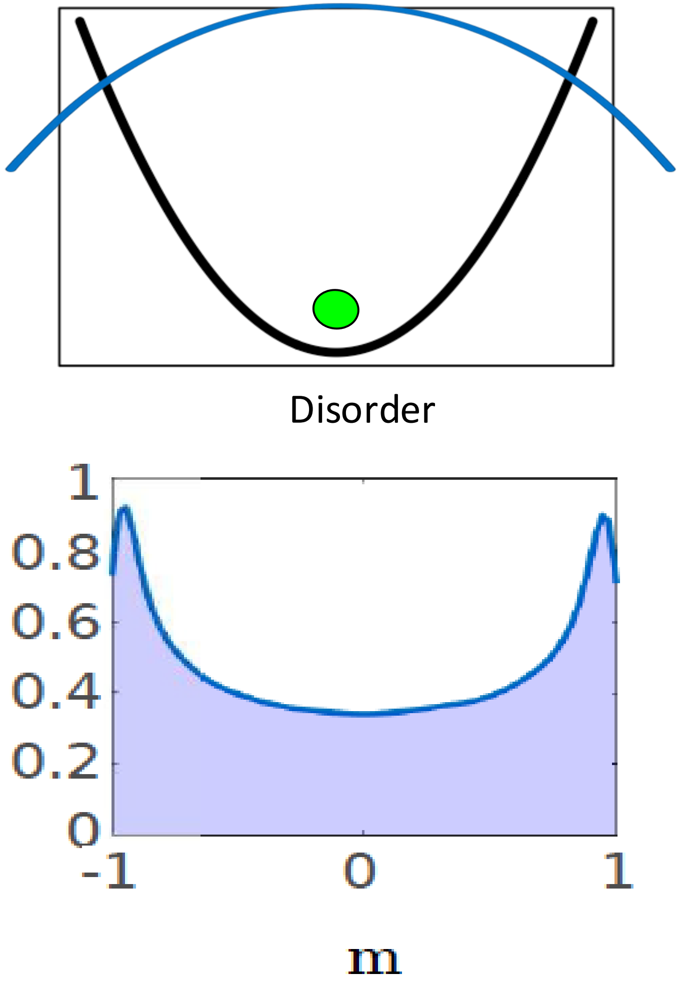
Disorder is the attractor



What's actually happening:

Small flocks: High Intrinsic Noise

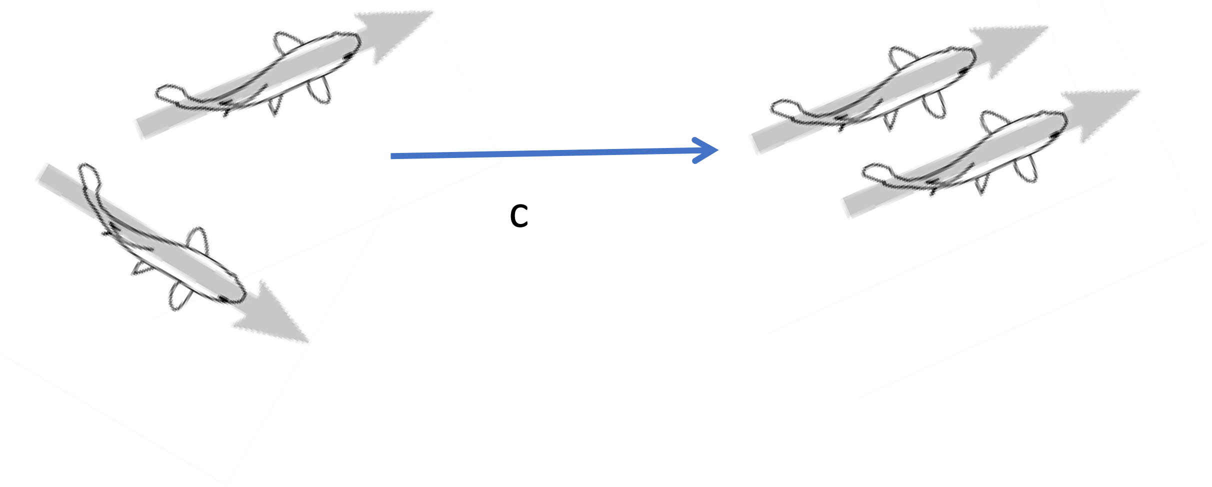
Ordered state is most likely



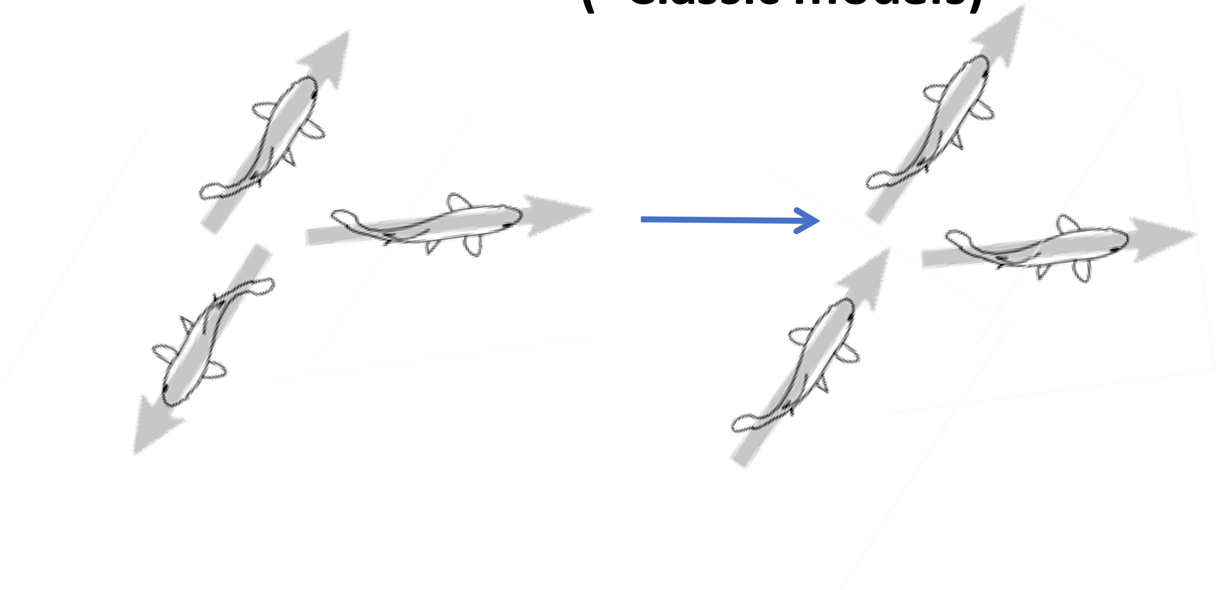
Is there a microscopic model that can explain these results?

A ‘spin’ model : Stochastic social interaction model

Pairwise copying interaction



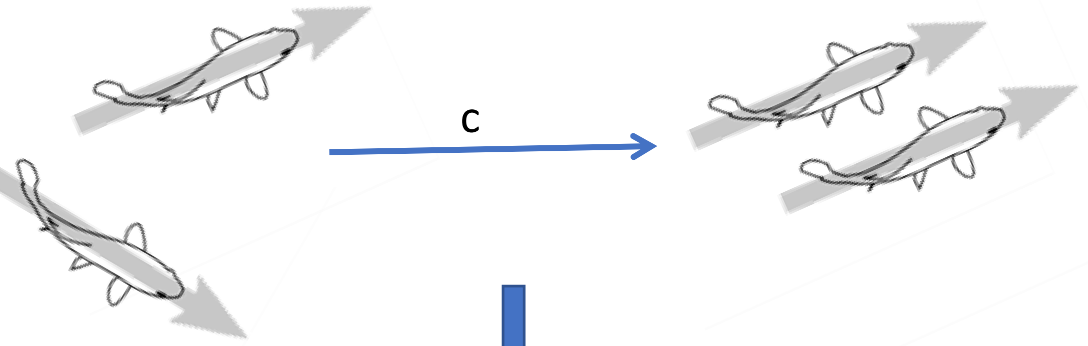
Ternary or average interaction
(~Classic models)



Group polarization: $$\mathbf{m} = \frac{1}{N}\sum_i \mathbf{v}_i(t)$$

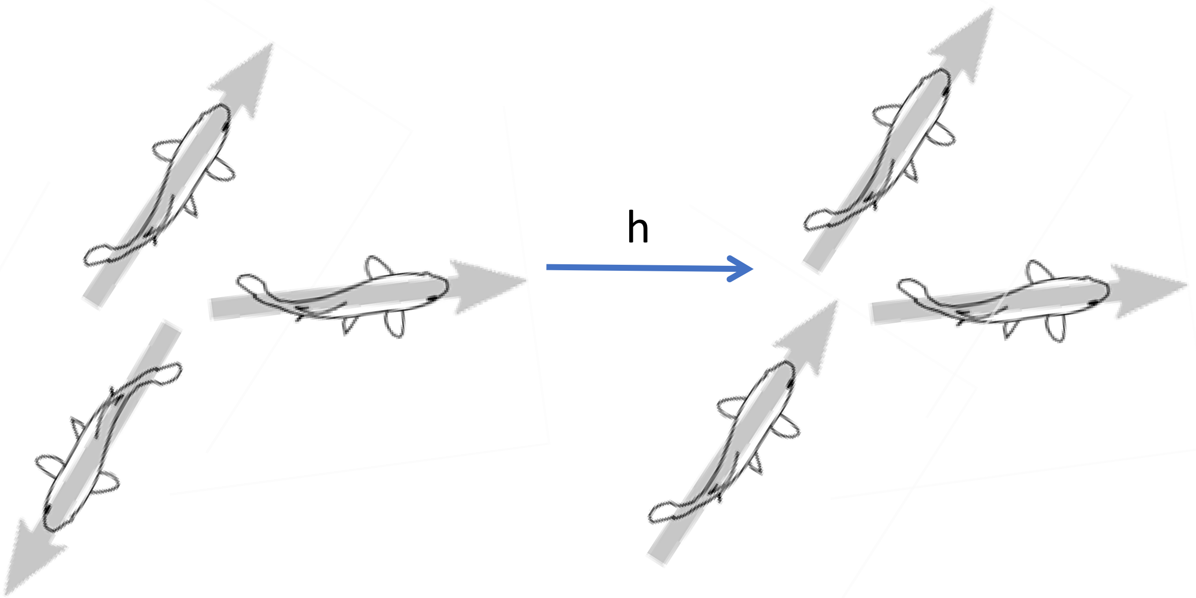
Mesoscopic Theory of Polarisation Dynamics

Only pairwise copying alignment interaction



$$\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t} = -a\,\mathbf{m} + \sqrt{\frac{a + c\left(1 - |\mathbf{m}|^2\right)}{N}}\,\mathbb{1}\cdot\boldsymbol{\eta}(t)$$

Ternary or average alignment interaction (Classic models)



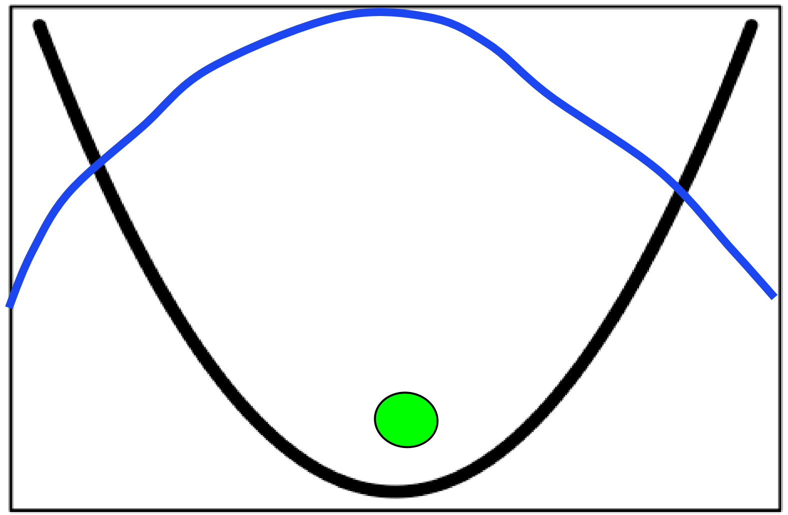
$$\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t} = -a\,\mathbf{m} + h\,\mathbf{m}\left(1 - |\mathbf{m}|^2\right) + \sqrt{\frac{a + (c+h)\left(1 - |\mathbf{m}|^2\right)}{N}}\,\mathbb{1}\cdot\boldsymbol{\eta}(t)$$

For the math: see: Bincaloni et al, 2014; Jhawar et al 2019; Jhawar et al 2020, Nature Physics

Pairwise copying alignment interaction

$$\frac{d\mathbf{m}}{dt} = -a\,\mathbf{m} + \sqrt{\frac{a + c\left(1 - |\mathbf{m}|^2\right)}{N}}\;\mathbb{1}\cdot\boldsymbol{\eta}(t)$$

Disorder is the attractor



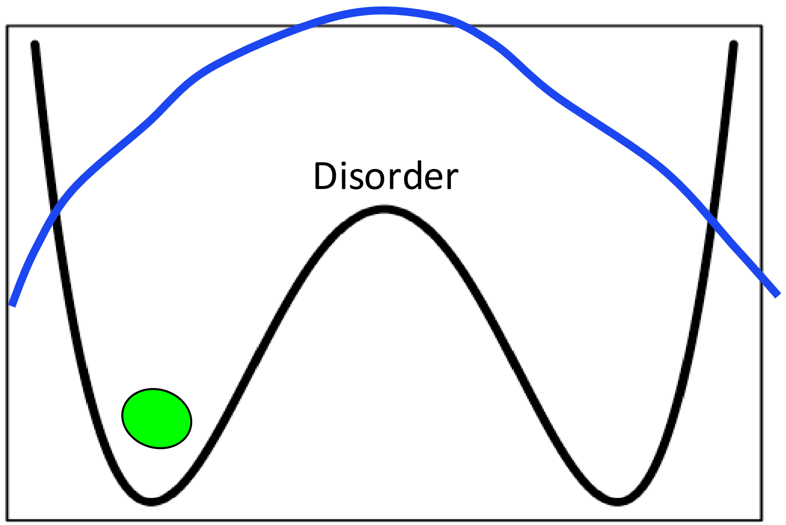
Disorder

Noise-induced synchrony for small flocks

Ternary or average alignment interaction (Classic models)

$$\frac{d\mathbf{m}}{dt} = -a\,\mathbf{m} + h\,\mathbf{m}\left(1 - |\mathbf{m}|^2\right) + \sqrt{\frac{a + (c+h)\left(1 - |\mathbf{m}|^2\right)}{N}}\;\mathbb{1}\cdot\boldsymbol{\eta}(t)$$

Order is the attractor



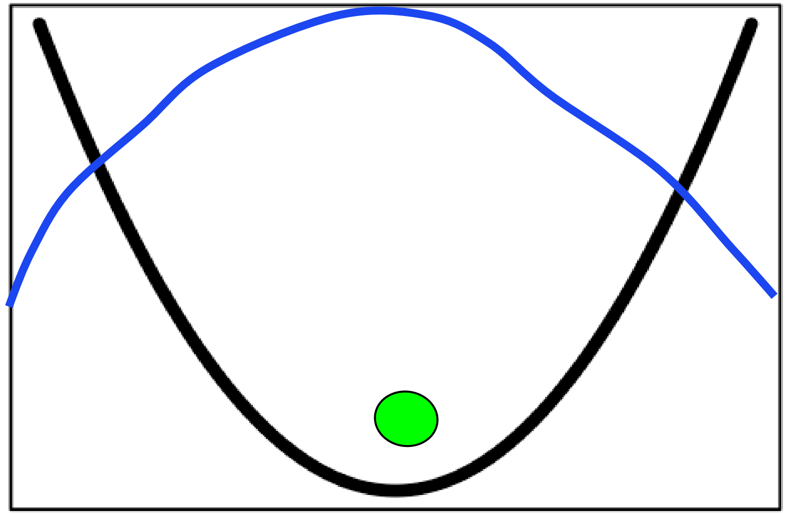
Order Order

Deterministic order

Pairwise copying alignment interaction

$$\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t} = -a\,\mathbf{m} + \sqrt{\frac{a + c\left(1 - |\mathbf{m}|^2\right)}{N}}\,\mathbb{1} \cdot \boldsymbol{\eta}(t)$$

Disorder is the attractor

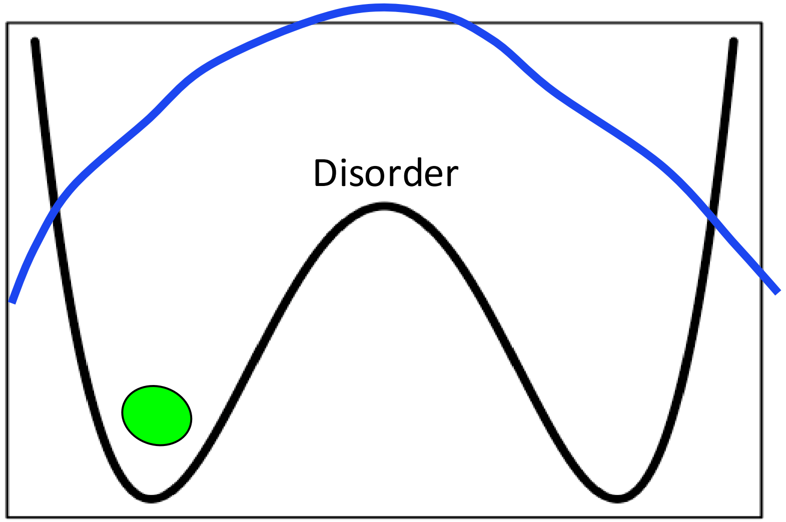


Noise-induced synchrony in Etroplus Suratensis

Ternary or average alignment interaction (Classic models)

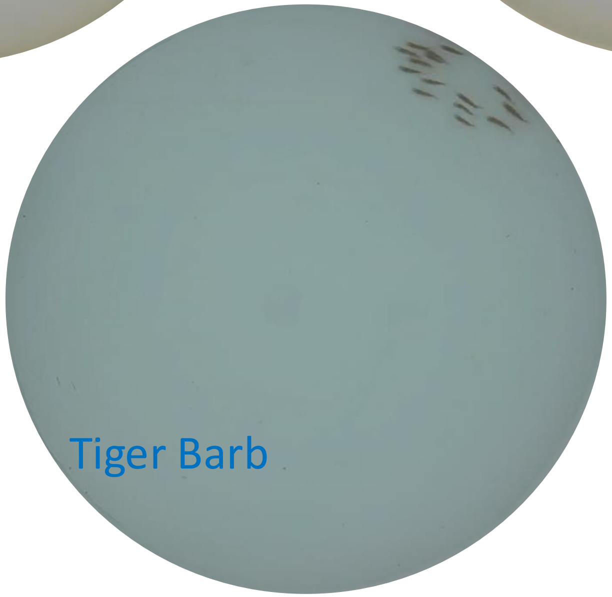
$$\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t} = -a\,\mathbf{m} + h\,\mathbf{m}\left(1 - |\mathbf{m}|^2\right) + \sqrt{\frac{a + (c+h)\left(1 - |\mathbf{m}|^2\right)}{N}}\,\mathbb{1} \cdot \boldsymbol{\eta}(t)$$

Order is the attractor

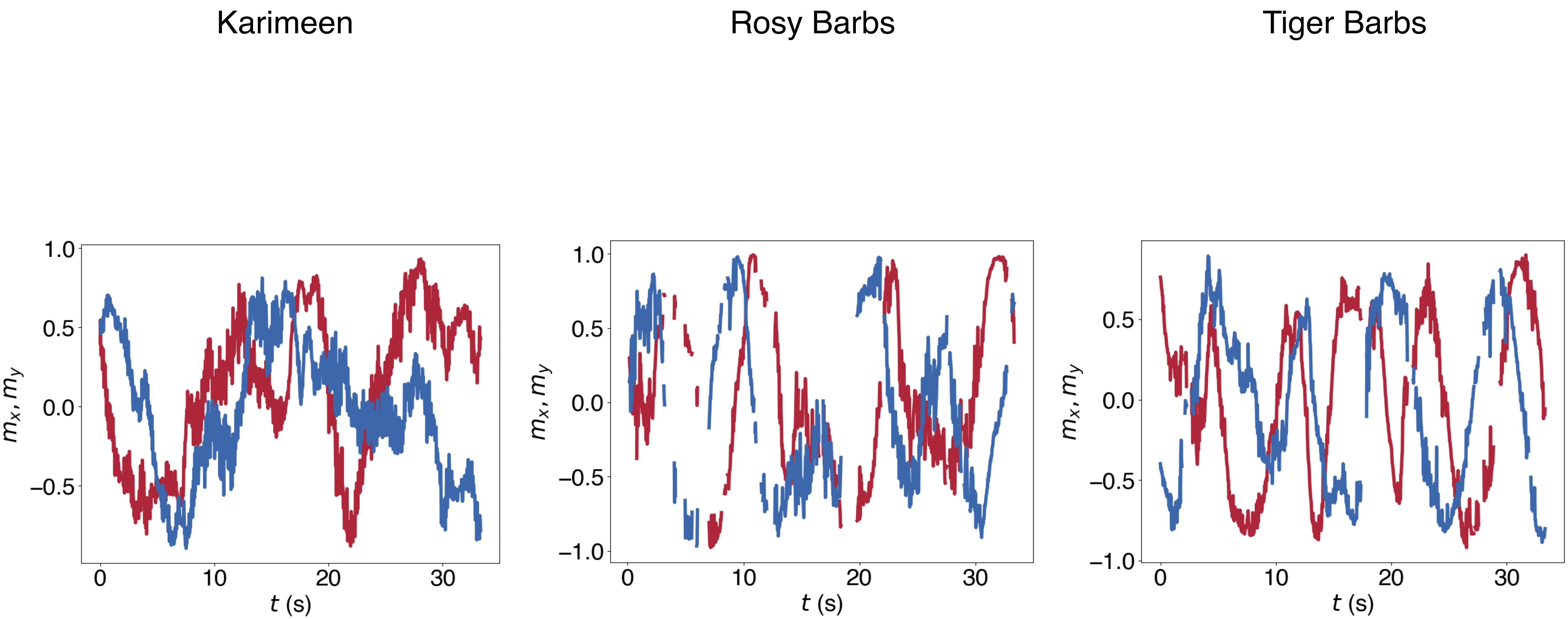


Deterministic order

Do other fish species show similar dynamical structure?

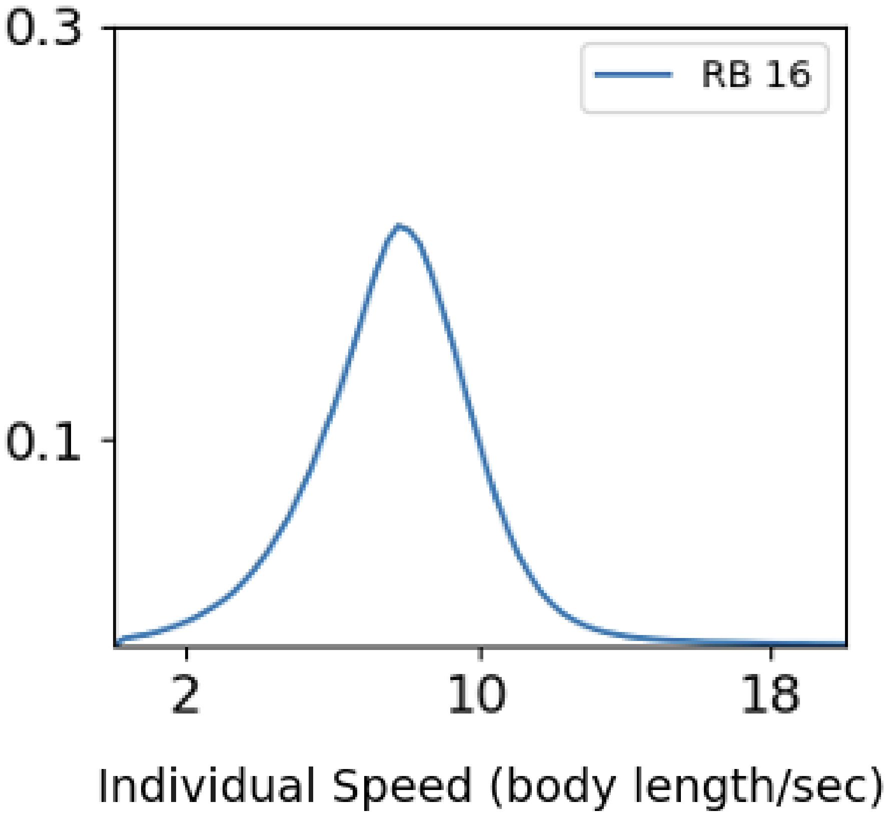


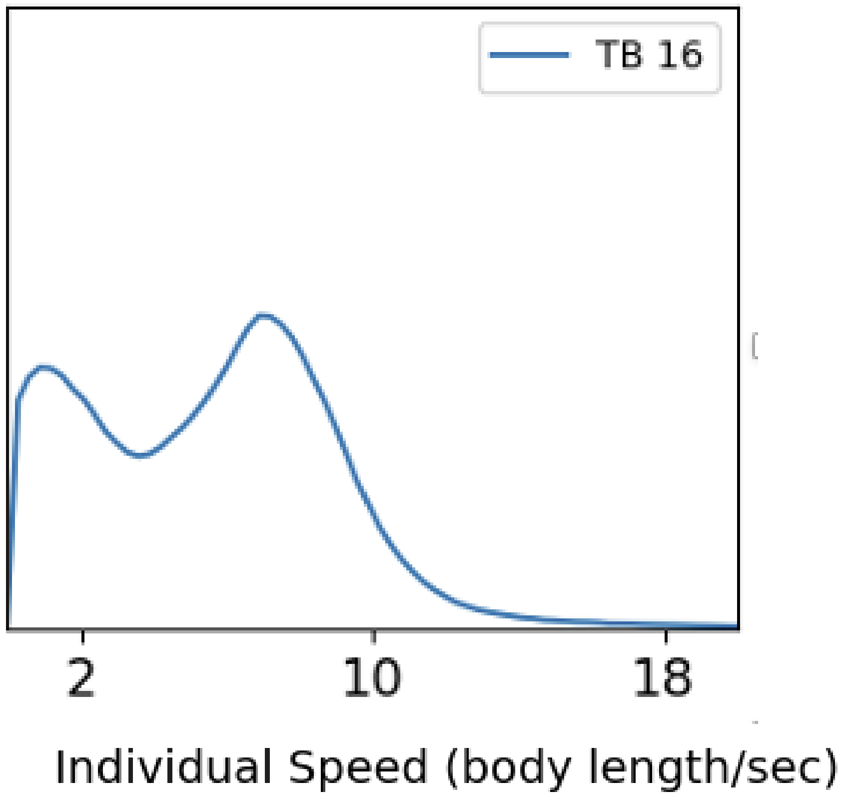
Ongoing Work: Arshed Nabeel, Viraj Torsekar, Jahanvi Tiwari, Ashrit Mangalwedhekar, Cassandre Aimon, Guy Theraulaz, etc



Karimeen
Rosy Barbs
Tiger Barbs
m_x, m_y
t (s)
1.0
0.5
0.0
−0.5
−1.0
0
10
20
30

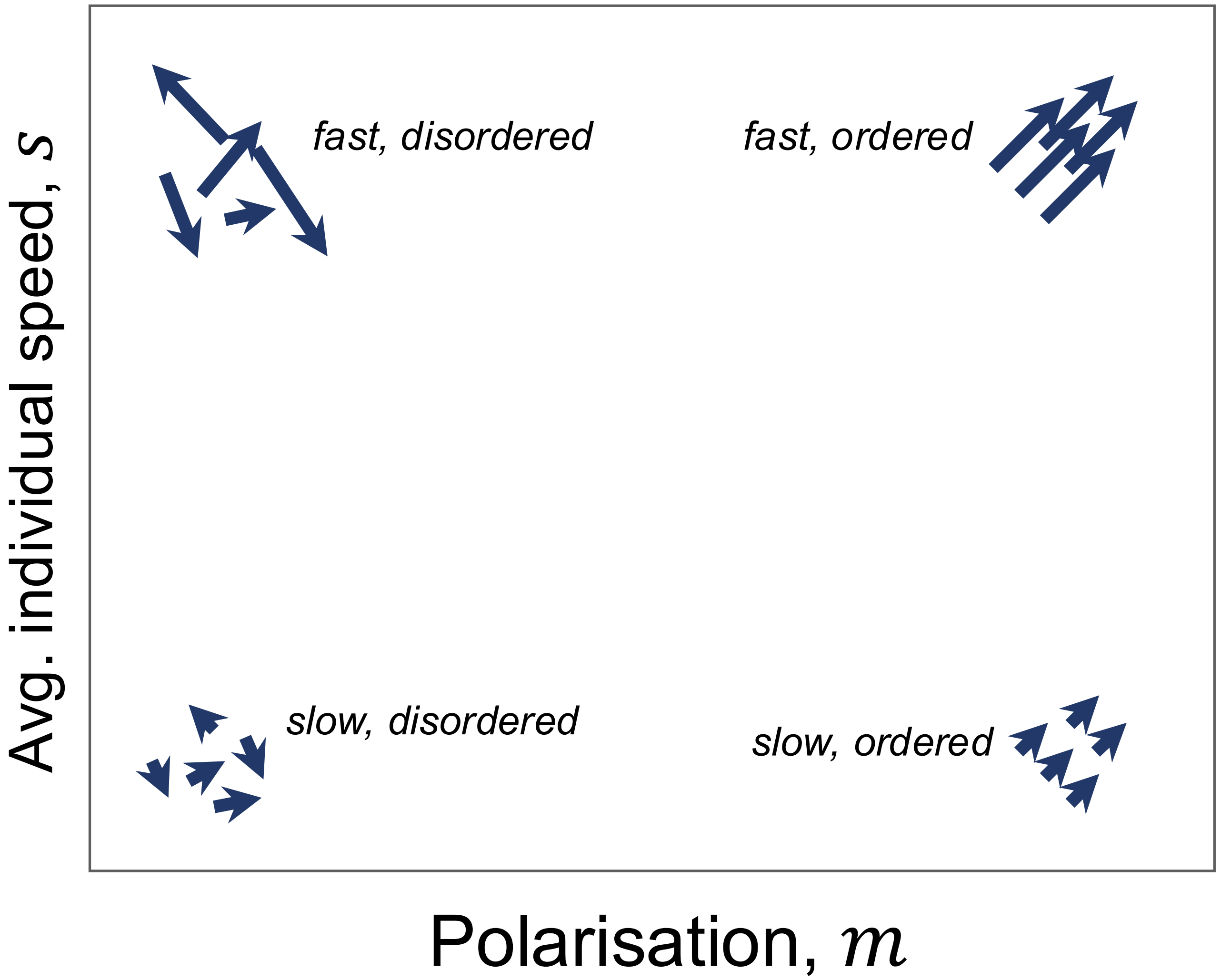
Speed variations can be very different across species



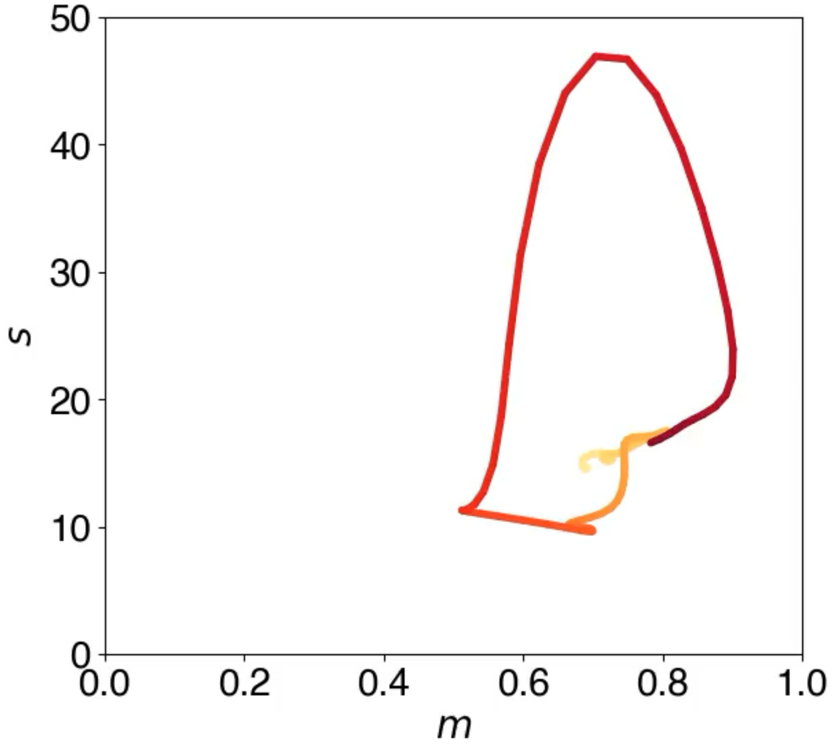


Ongoing work of Jahanvi Tiwari, Arshed Nabeel, Juee Dhar, Viraj Torsekar

The m-s space (Synchrony – Speed phase-plane)



The m-s space (synchrony – speed phase-plane)



As the school swims in the arena, it is also *swimming* in the $m - s$ space.

How does the school *swim* in the $m - s$ space?

Modelling the dynamics

strength of the fluctuations

$$\frac{d}{dt}\begin{bmatrix}\mathbf{m}\\ s\end{bmatrix} = \mathbf{f}(m, s) + G(m, s) \cdot \eta(t)$$

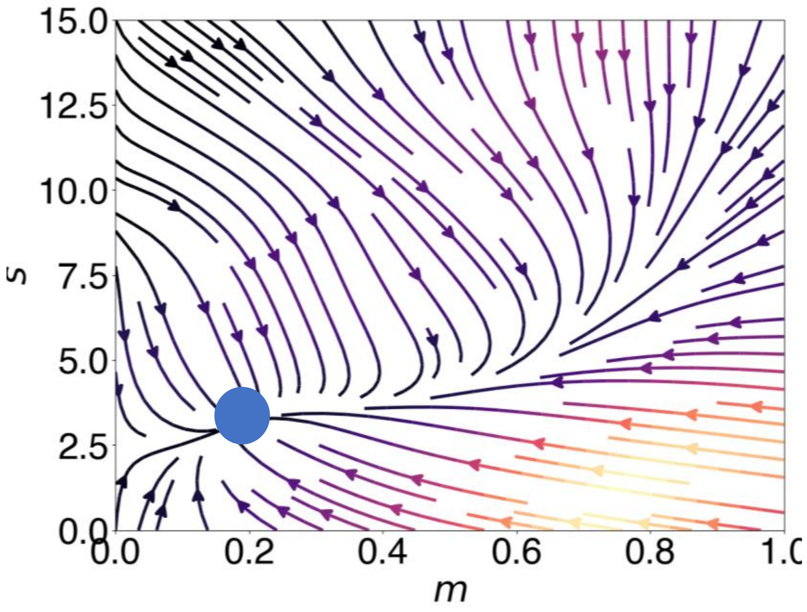
The dynamics of the school in the m-s space

the average rate of change of m and s

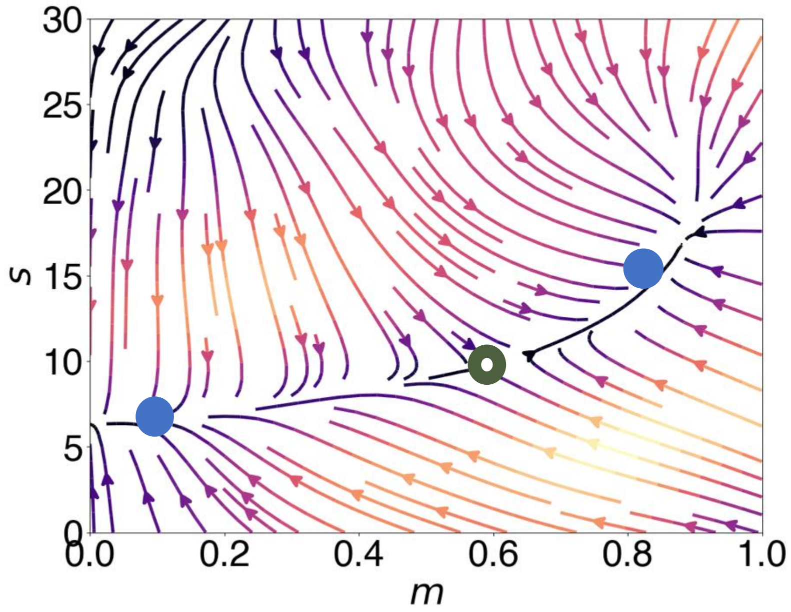
stochastic fluctuations about the average change

Data-Derived Flow Diagrams/Equations: fixed points, null clines and stability

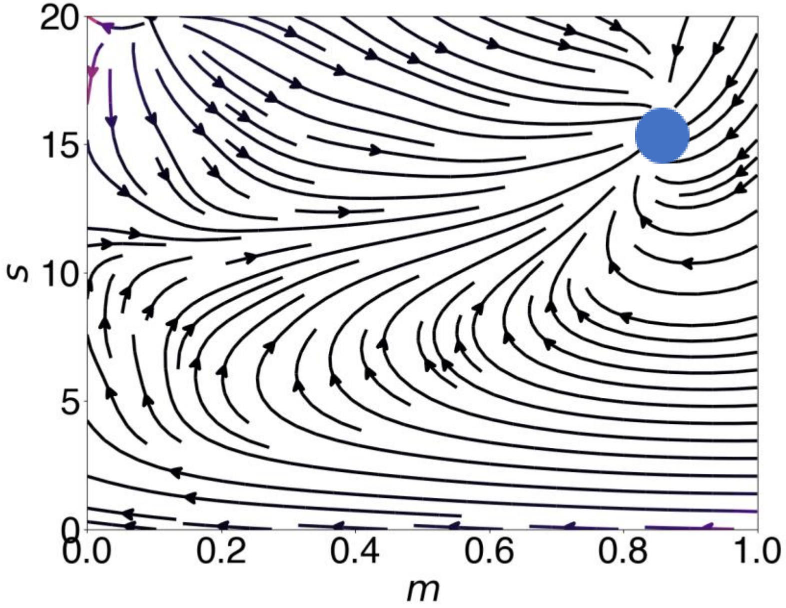
Class I: Intrinsic Noise-induced order!



Class II: Bistable Deterministic Order!



Class III: Deterministic order!



Ongoing work of Arshed Nabeel (with many others)

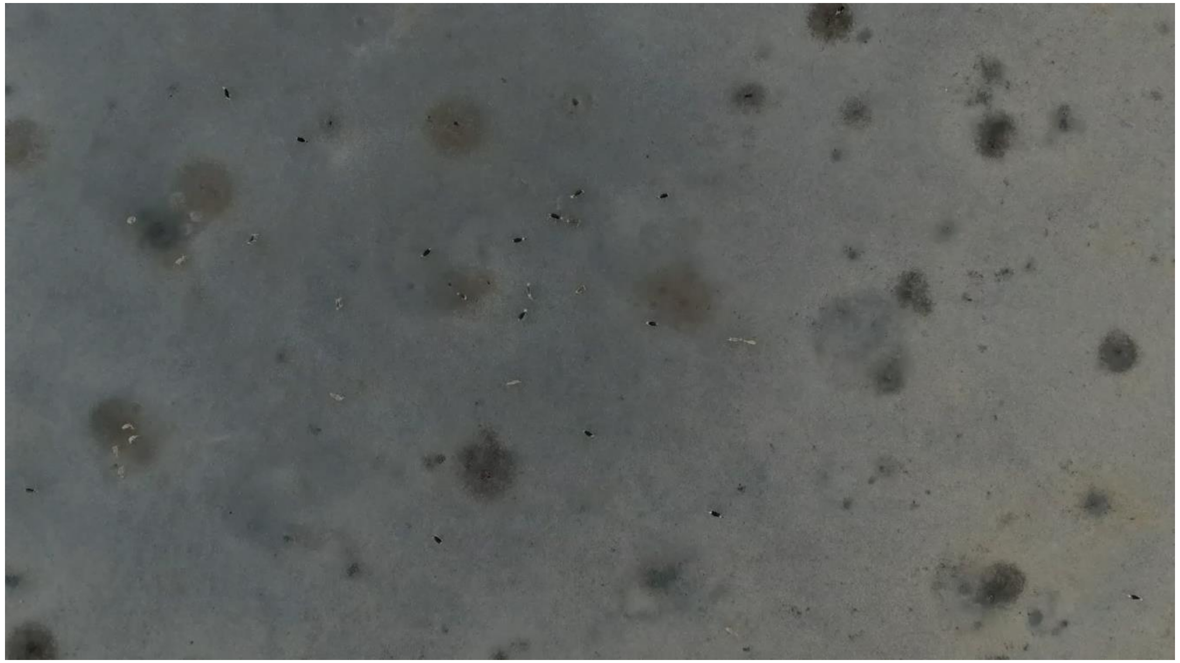
Summary so far: Noise and synchrony

- Broad classes of dynamics of synchrony:
  - Noise-induced Order
  - Bistable Deterministic Order
  - Unistable Deterministic Order
- Typically: The disordered state exhibits more noise, and pushes system away from the disorder.
- Methodological novelty: Data-driven stochastic differential equations.

Collective Escape Dynamics: Jadhav et al 2024

Collective Escape Dynamics: Jadhav et al - Ongoing

Rathore et al - Ongoing



Lekking - a social mating system
Rathore et al, 2023, Phil. Tran. Roy. Soc B

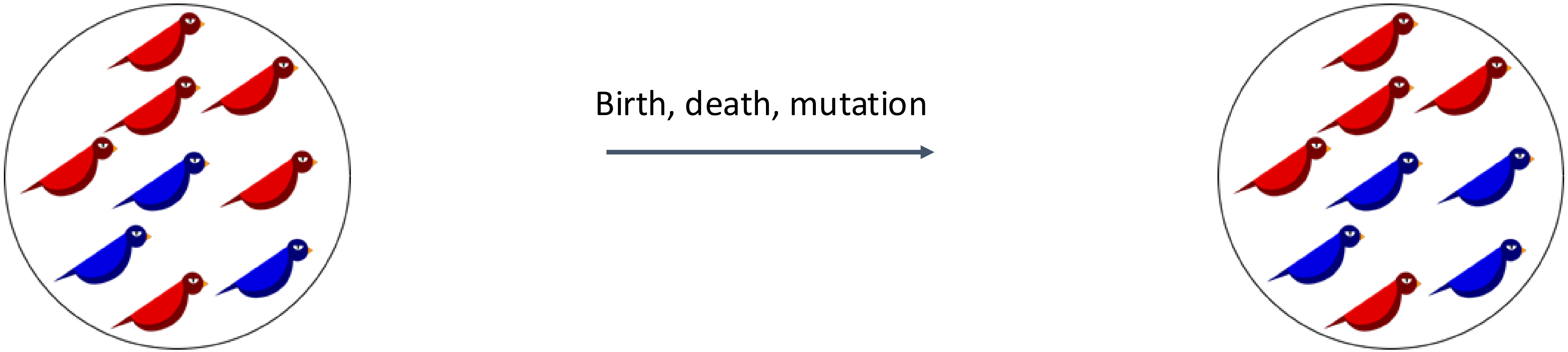
Outline for today's talk: Noise to Order!

- Theme 1: Deciphering dynamics of fish schools
  - Noise helps characterize the nature of synchrony.
  - Experiments of fish schools

- A novel method: Data-driven equation discovery

- Theme 2: Evolution and Noise

Simplest case of evolutionary dynamics: two types



p = proportion of type *1*

Changes over time, following the replicator equation!

$$\frac{dp}{dt} = p(1-p)(w_1 - w_2)$$

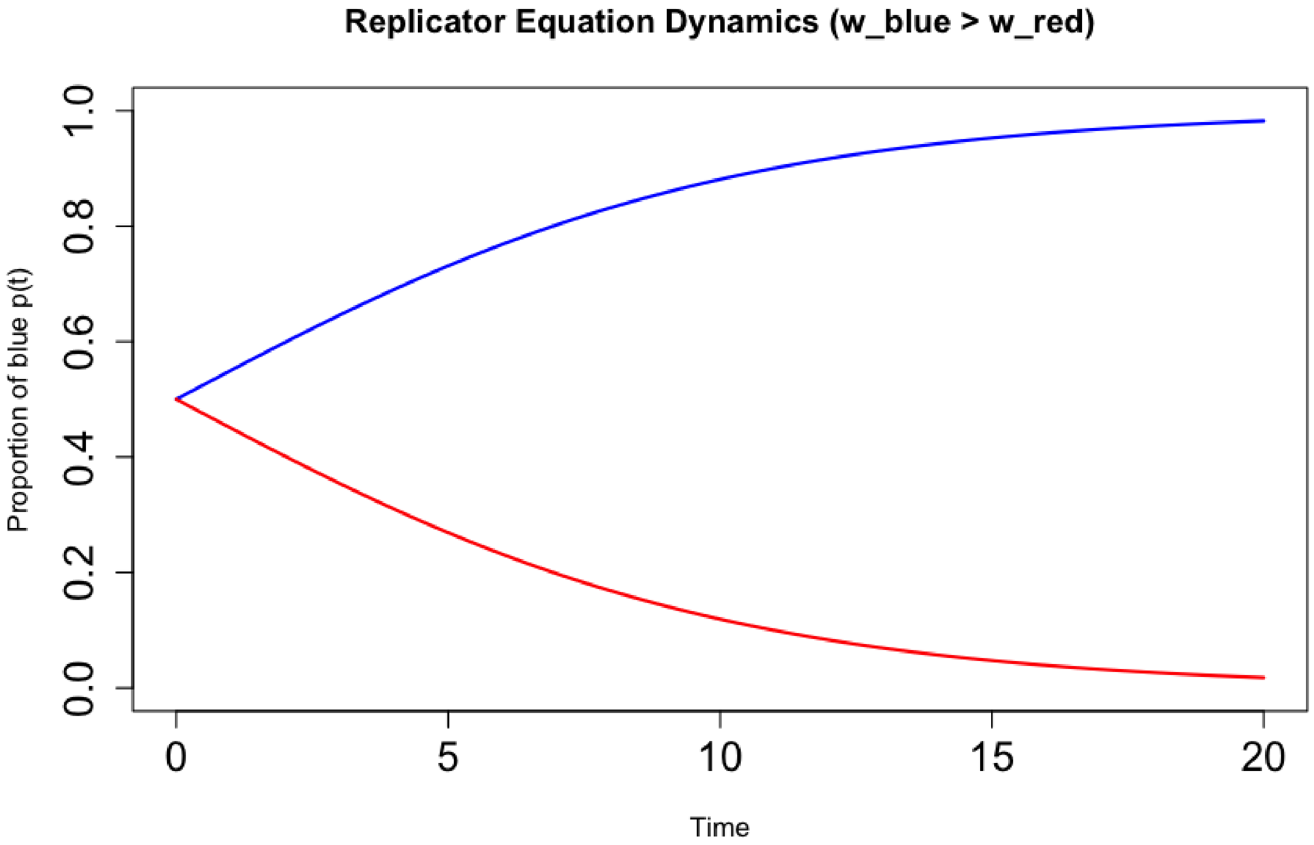
w_i : fitness of type *i*

$$w_i = b_i - d_i$$

Natural selection

- ‘Survival of the fittest’:
  - If $w_{blue} > w_{red},\ p \to 1$ over time.

Replicator equation

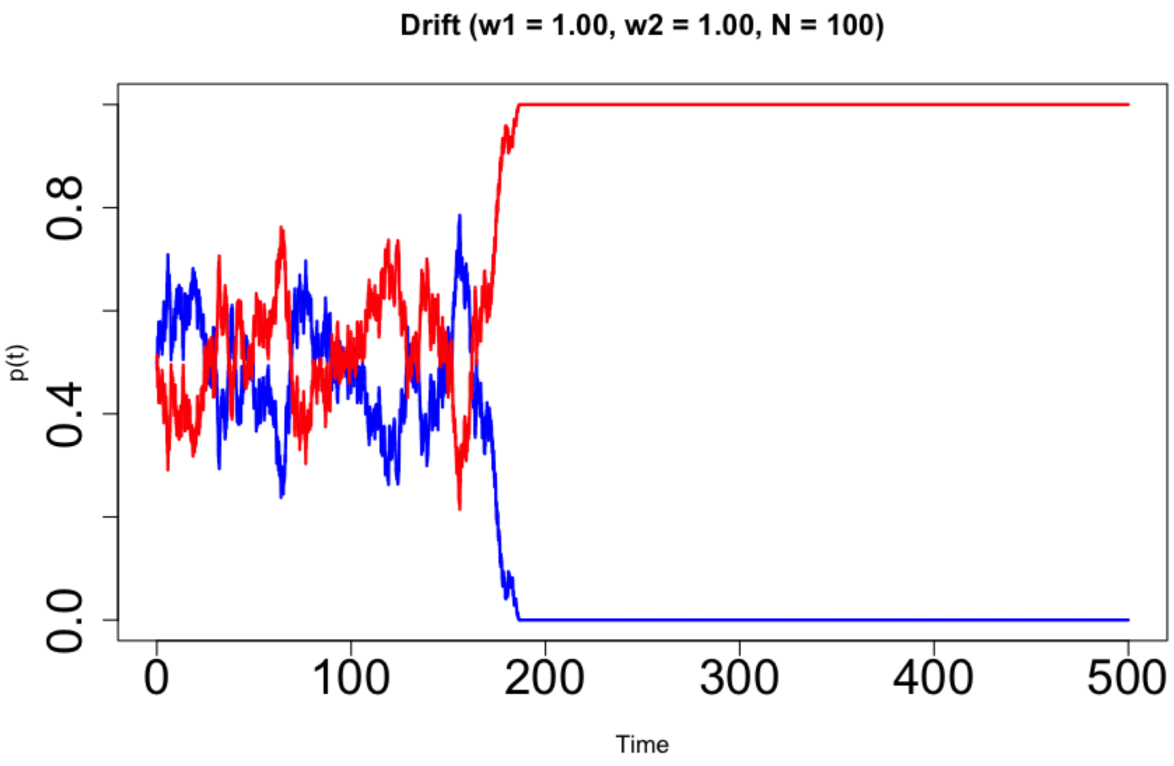


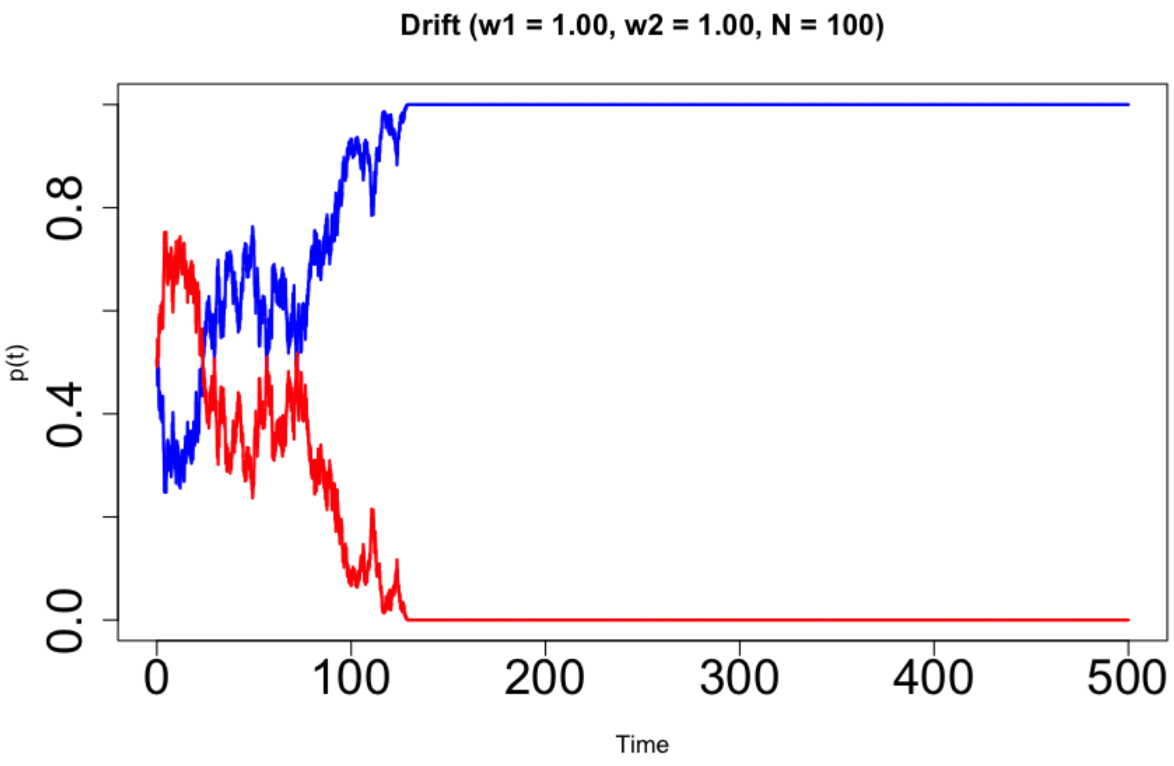
$$\frac{dp}{dt} = p(1-p)(\,w_1\ - w_2)$$

Implicit assumption: Non-structured and Infinitely large population, thus stochasticity can be ignored.

Random genetic drift: finite populations

- Birth and death are stochastic.
  - We can expect stochastic fluctuations even if two traits/alleles are of equal fitness $w_{blue} = w_{red}$
  - **Prob of fixation of red (or blue) = Initial frequency of red (or blue)**

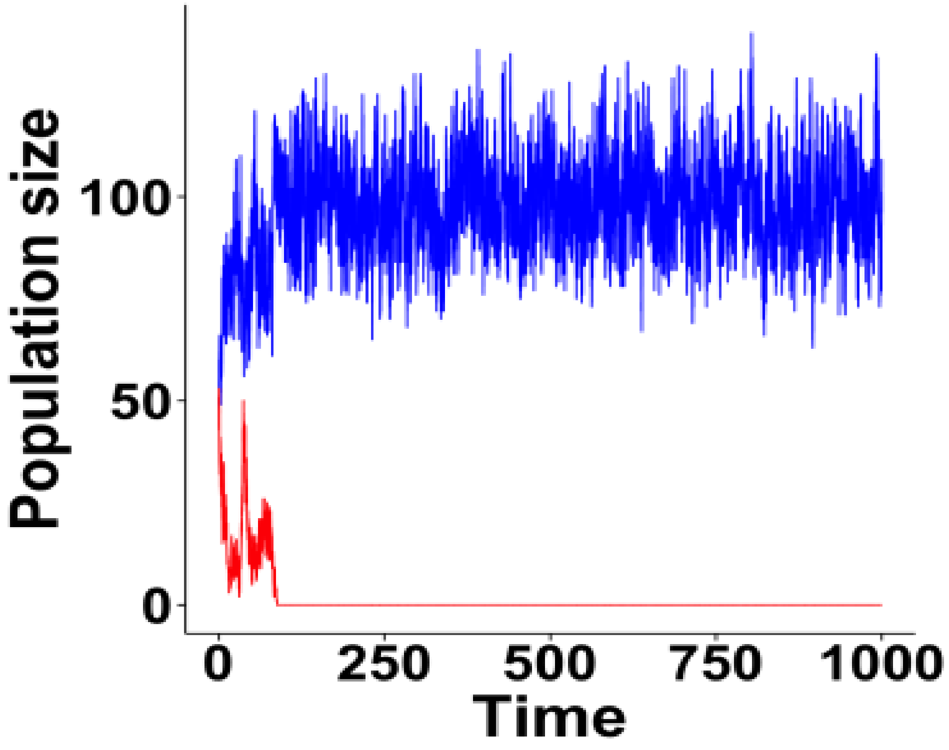




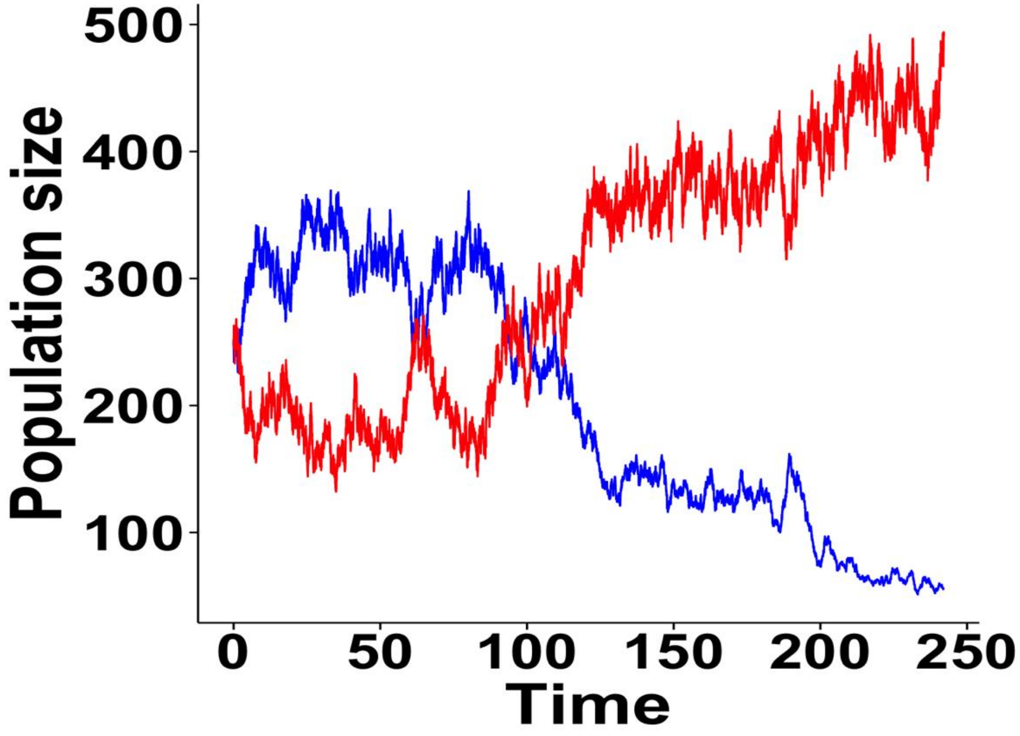
However:

- Much of finite population theory assumes: Population size is fixed.
- Due to stochastic birth and death:
  - p (frequency) fluctuations over time
  - N (population size) also fluctuates over time
- Most classic models ignore that N (population size) also fluctuates over time.

If we allow *N* to fluctuate: Eco-evolutionary dynamics



K = 100



K = 1000

Small populations: Blue fixation
(Reversal of selection; not merely drift)

Selection: Large populations: Red fixation

An illustration of how noise and ecology may affect evolution

- Assume that Fitness (A) > Fitness (B)
  - Selection prefers A.

- Caveat: This is guaranteed only in infinite populations!

- In finite fluctuating populations:
  - B may be preferred over A: IF trait B has slower turnover rate (i.e. slow birth and slow death rates).
  - *Noise-induced reversal of selection*

Bhat and Guttal, *2025*

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SYNTHESIS

Eco-Evolutionary Dynamics for Finite Populations and the Noise-Induced Reversal of Selection

Ananda Shikhara Bhat[1,2,*] and Vishwesha Guttal[2]

Assumption: We can define per-capita birth and death rates

Population density of type i individuals

Per-capita birth rate of type i individuals

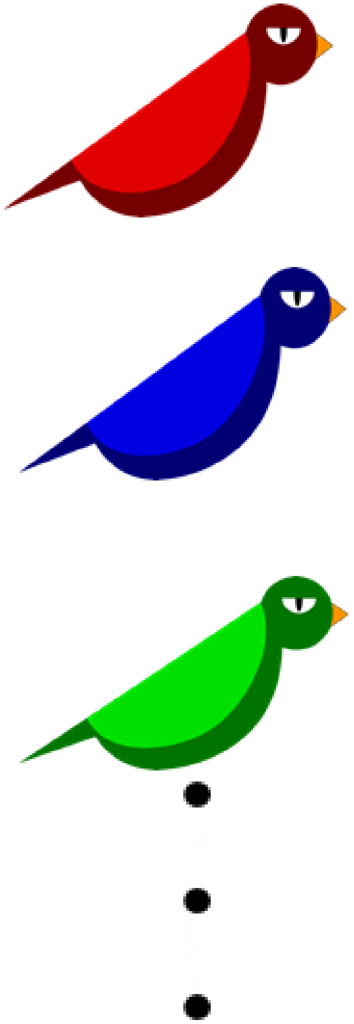
$$b_i(\mathbf{x}) = x_i \beta_i(\mathbf{x})$$
$$d_i(\mathbf{x}) = x_i \delta_i(\mathbf{x})$$

Per-capita death rate of type i individuals

Fitness of the i^{th} type $w_i(\mathbf{x}) = \beta_i(\mathbf{x}) - \delta_i(\mathbf{x})$

Turnover rate of the i^{th} type $\tau_i(\mathbf{x}) = \beta_i(\mathbf{x}) + \delta_i(\mathbf{x})$

An equation for population density dynamics



Fitness controls 'deterministic part'

Turnover controls 'stochastic part'

$$dx_i = x_i w_i(\mathbf{x})dt + \frac{1}{\sqrt{K}}\sqrt{x_i \tau_i(\mathbf{x})}dW_t^{(i)}$$

$\approx N(0, dt)$ random variable

$$\mathbb{E}[dx_i] = w_i x_i$$

$$\mathrm{Var}[dx_i] = \tau_i x_i / K$$

Example: If there are two alleles (red, blue)

- Assume finite *N* and no selection $(w_{blue} = w_{red})$
- **Classic expectation:** $dp = \text{noise}$
  - Random genetic drift: red and blue equally likely to fix (if p(t=0)=0.5).
  - TRUE only if N = fixed.
- Our formalism predicts, if N is also fluctuating:

$$dp = -(\tau_{blue} - \tau_{red})/\mathrm{N}\,\mathrm{dt} + \text{noise}$$

- **The allele with a lower turnover rate has an advantange!**

Recall: Turnover rate of the i^{th} type $\tau_i(\mathbf{x}) = \beta_i(\mathbf{x}) + \delta_i(\mathbf{x})$

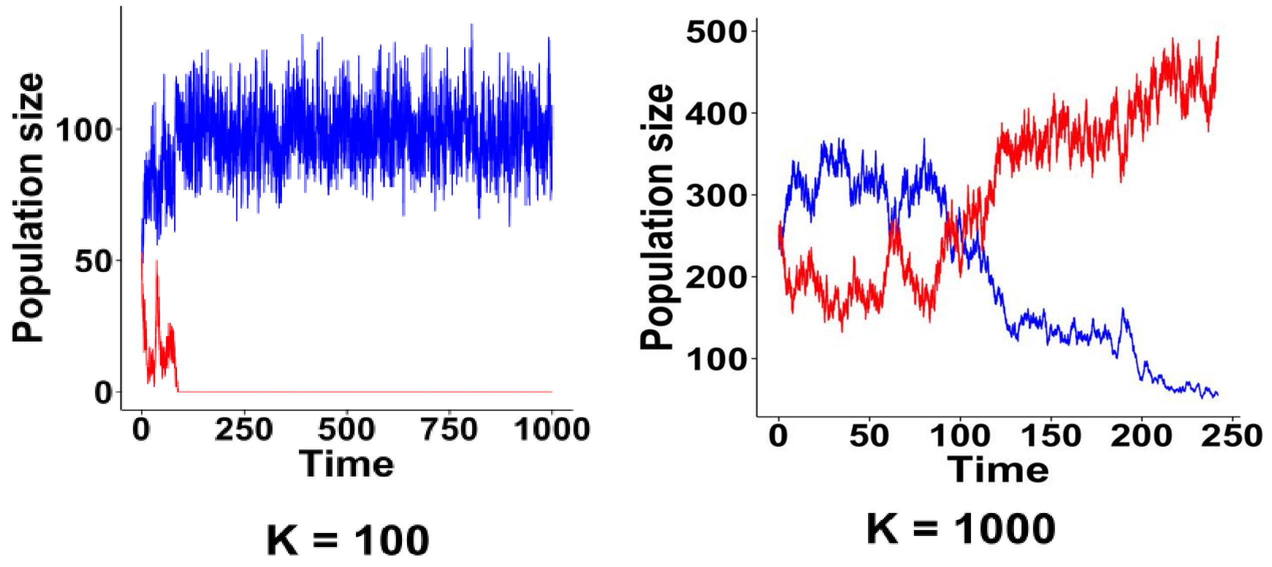
- Turnover measures `pace of life'

- Key predictions are:

  - Outcomes of larger and smaller populations need not be same, if turnover rates are not same.

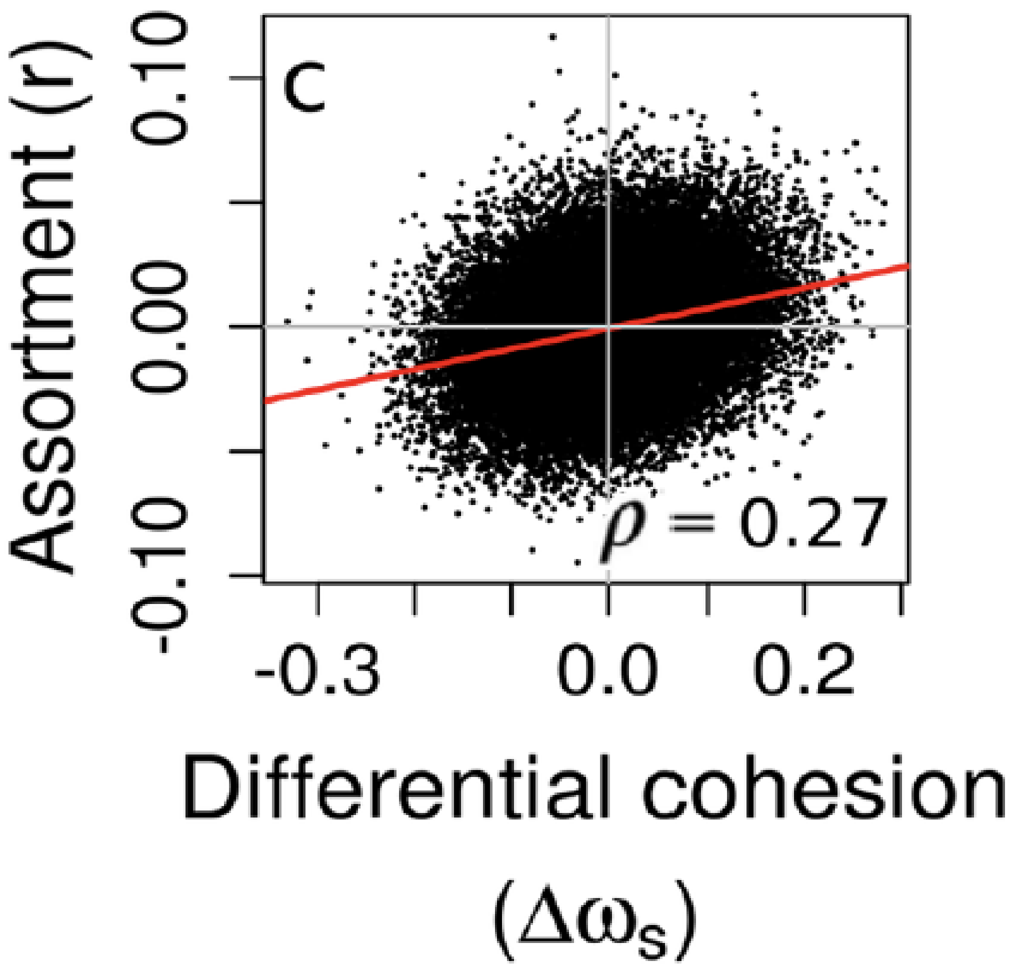
  - In smaller populations, slower paced type (those with lower value of τ) has an advantage, even if fitness are equal!

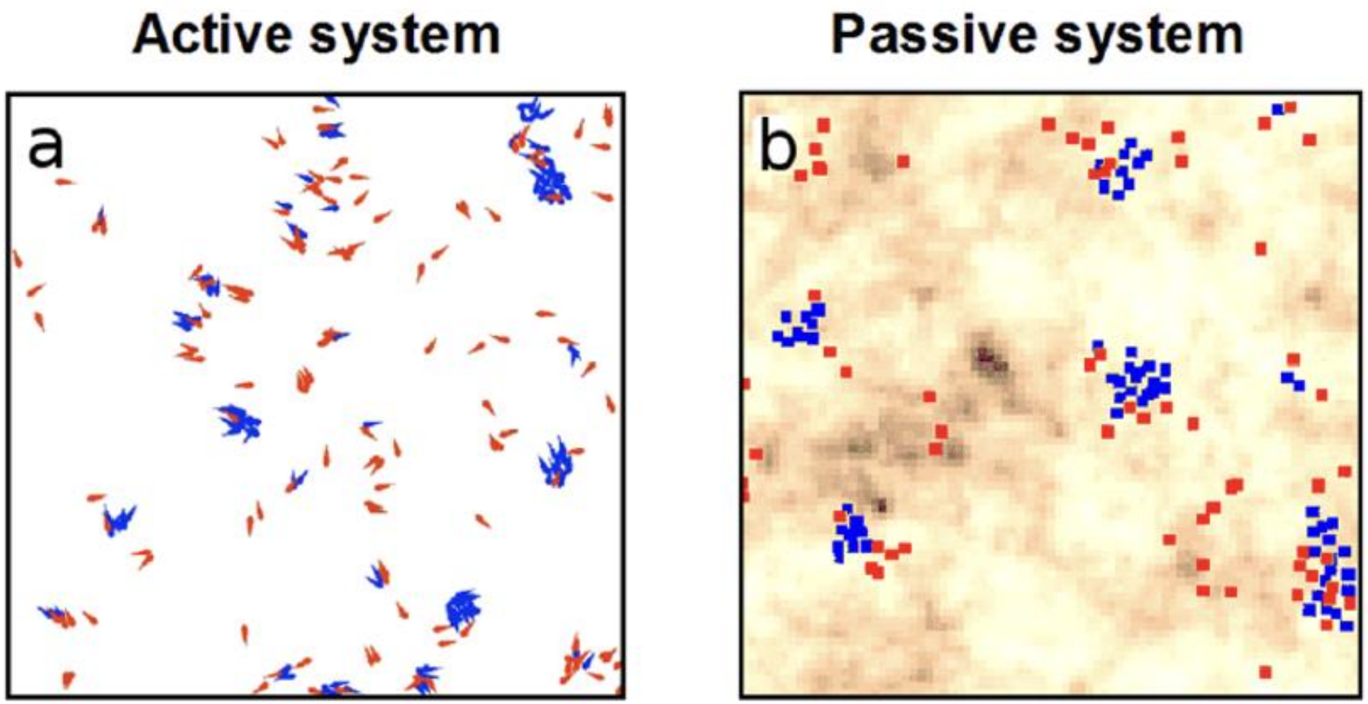
  - A neutral evolution requires traits to equal turnover rates, not just equal fitness.



Bhat and Guttal, 2025, Am Nat

Some older work: Demographic Noise and Evolved differential cohesion leads to assortment of cooperators





Joshi, Couzin, Levin, Guttal 2017 PloS Comp Biology

Joshi and Guttal, 2018, Evolution

Synthesis: Noise – of finite systems -- is intriguing!

- Finite systems => Intrinsic noise.
  - Strength of noise depends on system size and the state of the system!
- Flocking Systems: Three broad classes of collective motion in finite flocks
  - Noise-induced order
  - Bistable disorder and order
  - Deterministic order
  - Typically: The disordered state exhibits more noise, and pushes system away from the disorder.
- Finite-size noise can reverse the direction of selection
  - Can facilitate the evolution of cooperation and flocking; Multi-level selection
  - Turnover rate (birth + death rates) becomes important.

Thank you all – for the invitation and for attending the talk