Black Holes and General Relativity
(Physics Nobel Prize 2020)

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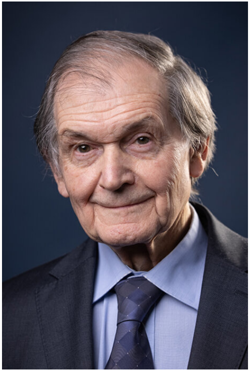
March 25, 2022 Online Colloquium Series "Nobel Talk" Physics Dept, BITS-Pilani

The Laureates and the Citations



Roger Penrose

Reinhard Genzel

Andrea Ghez

Roger Penrose "for the discovery that black hole formation is a robust prediction of the general theory of relativity"

Reinhard Genzel and Andrea Ghez "for the discovery of a supermassive compact object at the centre of our galaxy."

Credits: https://www.nobelprize.org/prizes/physics/2020/summary/

Strands of the Story . . .

General Relativity	Astrophysics	Astronomy
Schwarzschild Singularity (1916)	Chandrasekhar Limit (1930-31)	Birth of Radio Astronomy (1933-39)
Cosmological Singularities (1922-1955) Raychaudhuri Equation (1955)	Tolman-Oppenheimer-Volkoff Limit, Datt-Oppenheimer-Snyder (1939)	⋮
Kruskal Extension (1960)	⋮	Discovery of C-273 Quasar (1959)
The Kerr Solution (1963)	. . .	Energy output and size of Quasar
Trapped Surfaces (1965)	. . .	Identification of Sagittarius A^* (1974)
Singularity Theorems (1965-1975-. . .)	. . .	Supermassive Compact Object(1980-2020)

Continuation of the Story

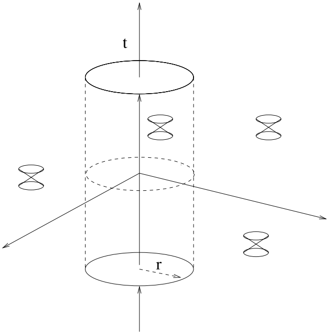
The singularity theory involves Global Structure of the Space-Time, Causality conditions, the Raychaudhuri equation, Energy Conditions and a definition of a "Singularity".

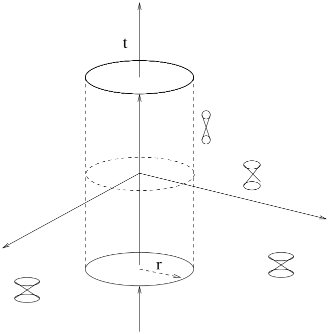
Establishing a Single, supermassive, compact Object involved Imaging the neighbourhood of Sgr A^* with greater and greater resolution, and tracking the orbiting stars over years of observations.

For this observational odyssey, the best source is the Nobel lectures by the laureates themselves at `https://www.youtube.com/watch?v=DWF1uNb9Q1Q`

I will confine to the theoretical part of the story.

Space-time Diagrams





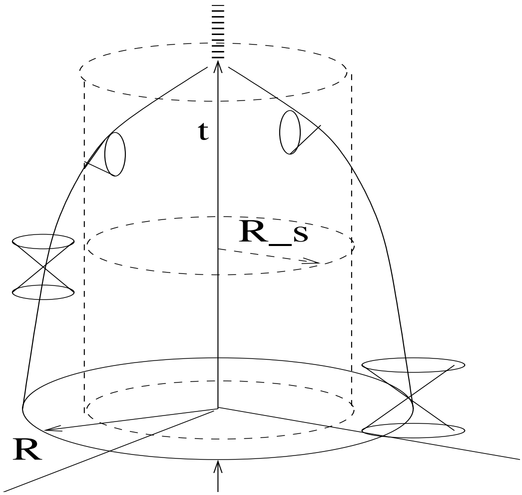
Minkowski Spacetime

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$

Schwarzschild Spacetime

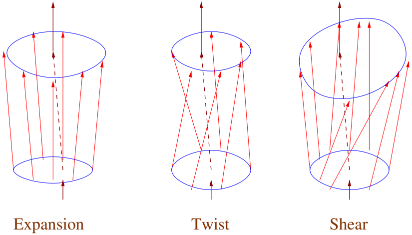
$$ds^2 = -\left(1 - \frac{R_s}{r}\right) dt^2 + \frac{1}{(1 - \frac{R_s}{r})} dr^2 + r^2 d\Omega^2$$

Unstoppable Gravitational Collapse



Oppenheimer-Snyder Collapse Spacetime

Raychaudhuri Equation and Conjugate Points



The Expansion θ, in particular varies according to,

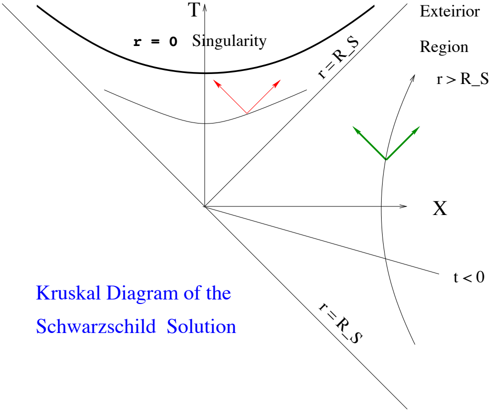
$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 + \mathrm{twist}^2 - \mathrm{shear}^2 - \boxed{R_{\mu\nu} v^\mu v^\nu}$$

It follows that if $\theta = \theta_0 < 0$ for some τ_0 and weak-energy condition is satisfied, then $\theta \to -\infty$ in a $\tau < \tau_0 + 3/|\theta_0|$.

The bundle of geodesics is said to focus and the geodesic is said to have a $\boxed{\text{conjugate point}}$.

Trapped Surfaces

Consider a sphere and flashes of light going outward and inward. These two bundles of null geodesics have their expansions $\theta_{\pm}$. A closed surface for which both the expansions are negative is a trapped surface. Here is an example.



Kruskal Diagram of the Schwarzschild Solution

Global Structure of Space-time

Einstein equations, $R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$, are *partial differential equations* and the different local solutions are to be patched up to form a global solution.

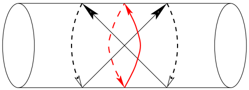
The global solutions may have Causal Pathologies and may lack of determinism. As an arena for doing physics, the space-times must be free of these pathologies.

But Einstein equations also depend on the matter stress tensor. This should also satisfy certain energy conditions e.g. that the matter energy density be positive and matter components obey the local speed limit of special relativity.

Only such pathology free solutions of the Einstein equation with physically reasonable stress tensor, are physically admissible.

Causal Pathologies

Causality



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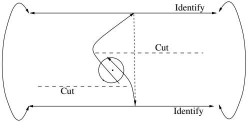
Strong Causality



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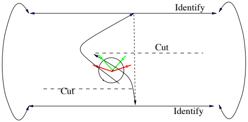
Stable Causality



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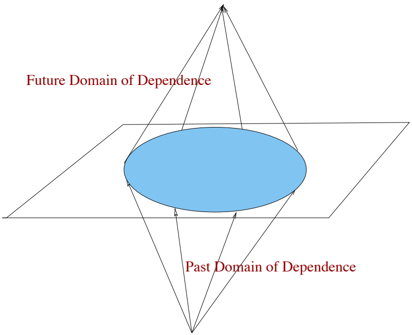
Determinism and Global Hyperbolicity



Globally Hyperbolic Space-times are those that can be represented as a Cauchy surface together with its past and future domains of dependence.

These are free of all causal pathologies.

Globally hyperbolic solutions of the Einstein Equations with stress tensor satisfying strong energy condition, constitute the set of our physically acceptable space-times.

Alas... there are still some hazards!

In globally hyperbolic space-times, between any two causally connected points, there exists a curve with maximum proper time. This curve is a geodesic with no conjugate points on it.

The contradiction between this result and implication of the Raychaudhuri equation, requires that a geodesic with a potential focusing point must be *incomplete*. When does such a contradiction arise?

If any solution among the physically admissible class has a trapped surface, then it has at least one incomplete, future directed time-like geodesic.

That is, if a gravitational collapse has proceeded to the extent of formation of a trapped surface, then it will lead to a singularity in finite time.

There are no special conditions on the initial data and hence the conclusion that formation of trapped surface signals occurrence of future singularity is robust.

Back to Sgr A^*

Having deduced the existence of a supermassive compact object Sgr A^*, the effort was to determine its mass and its size precisely. The initial estimates of mass $\sim 10^6 M_\odot$ and size $\sim 10^8$ km were arrived at by the spectral characteristics of the radio emission from its vicinity. It was also observed to have x-ray emission. These are due to accretion processes.

With the Very Long Baseline Interferometry VLBI radio observations, the position and proper motion of the central object could be tracked accurately while near infrared studies tracked individual stars orbiting the central object.

However, the central object's size was still about a couple of thousand Schwarzschild radii and as such could be made up of a cluster of some low mass stars, neutron stars, stellar mass black holes. Stability of such compact clusters against collapse or dispersion however indicated a time scale of about 10^5 years, ruling out such a possibility. The possibility of a ball of heavy fermions such as sterile neutrinos, gravitinos held together by degeneracy pressure too was ruled out by 2002. Thus,

Sgr A^* is a Black Hole of Mass $\sim 4.154 \times 10^6\ M_\odot$.

Other Black Holes

Apart from the supermassive black holes very likely lurking in the galaxy centres, there have been lighter candidate black holes. Typically these are suspected when a source is optically dim but x-ray bright. This is caused by accretion disc about the suspected black hole. The strategy again is to try to estimate the mass and the size of the suspected object.

Yet another method is based on the phenomenon of emission of gravitational waves. General relativity predicts that if the quadrupole moment (and higher) has a non-vanishing second time derivative, then such a mass distribution will loose energy by gravitational radiation. Binary systems thus loose energy and spiral towards each other and merge. The emitted wave form has a characteristic variation of amplitude with frequency - a chirp waveform.

This contains the information about the masses and their sizes. Using this the Gravitational Wave Observatories have detected over 80 binary black hole mergers with masses ranging from about 5 to 150 solar masses - a completely different mass range.

In Summary ...

Observationally, an object is a "black hole" if:

(a) the object is compact i.e. its radius is comparable to its Schwarzschild radius obtained from the estimate of its mass: $R_S := 2GM/c^2$;

(b) if the mass is larger than the maximum mass of the known stable, compact objects such as Neutron stars i.e. $\gtrsim 2 - 3M_\odot$.

In general relativity, a "black hole" is identified as a spatially compact region containing a trapped surface. A precise definition of its size requires a suitable definition of a "horizon".

These two criteria are linked together because formation of a trapped surface triggers a run away instability violating any upper mass limit. And we get,

Black Holes *and* General Relativity $\rightarrow$ Black Holes *in* General Relativity

$\boxed{\text{Thank You.}}$

For Further Reading

The book by Kip Thorn, *Black Holes and Time Warps*, W W Norton & Company, 1994, is a very readable and informative.

For the more technical minded, the article by José M. M. Senovilla, David Garfinkle, *The 1965 Penrose singularity theorem* is available on the arxiv server at https://arxiv.org/pdf/1410.5226