GRAVITY

(From Newton to Einstein to . . .)

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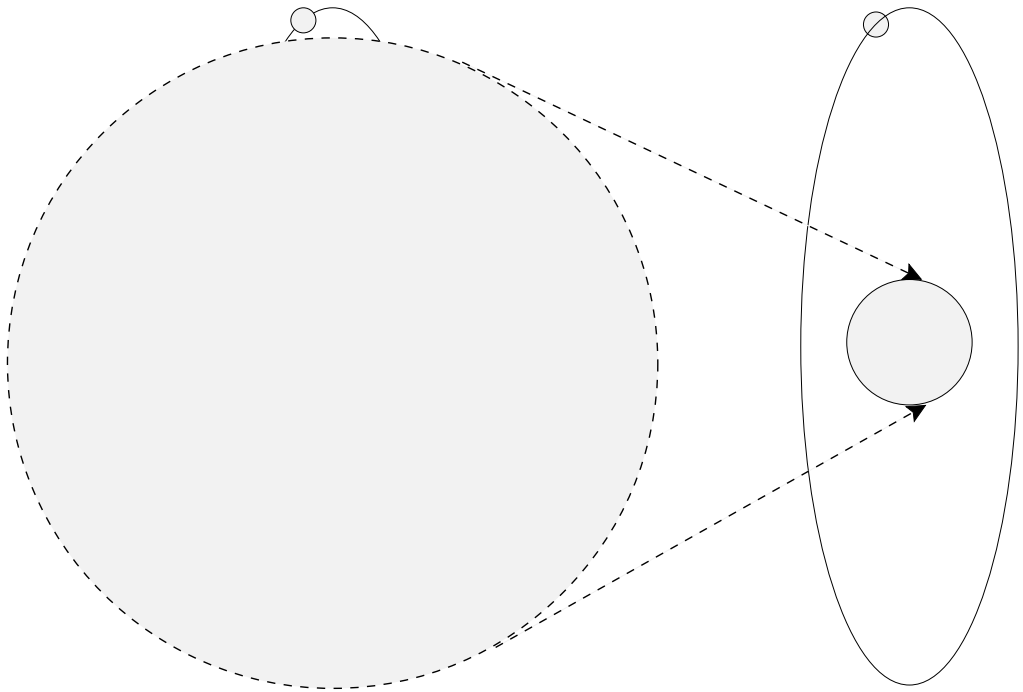

Figure: Projectile and Moon

Explains planetary motions, terrestrial tides, star formations

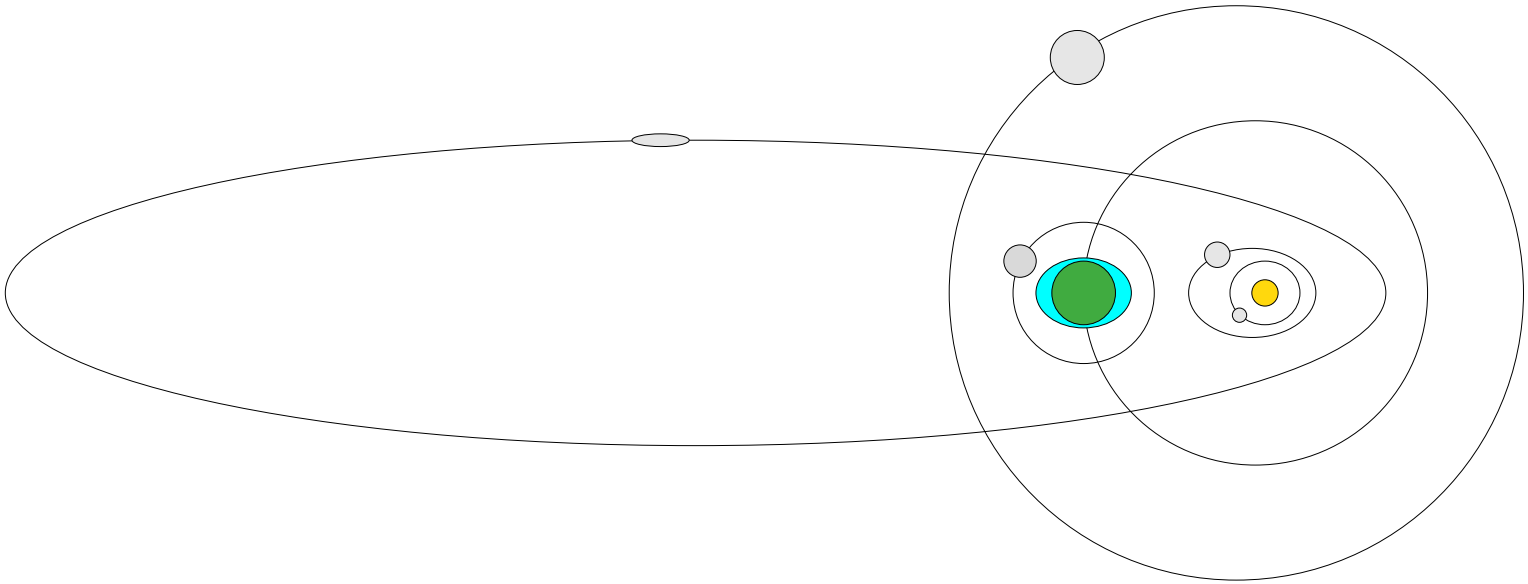

Figure: Planetary orbits and Tides on Earth

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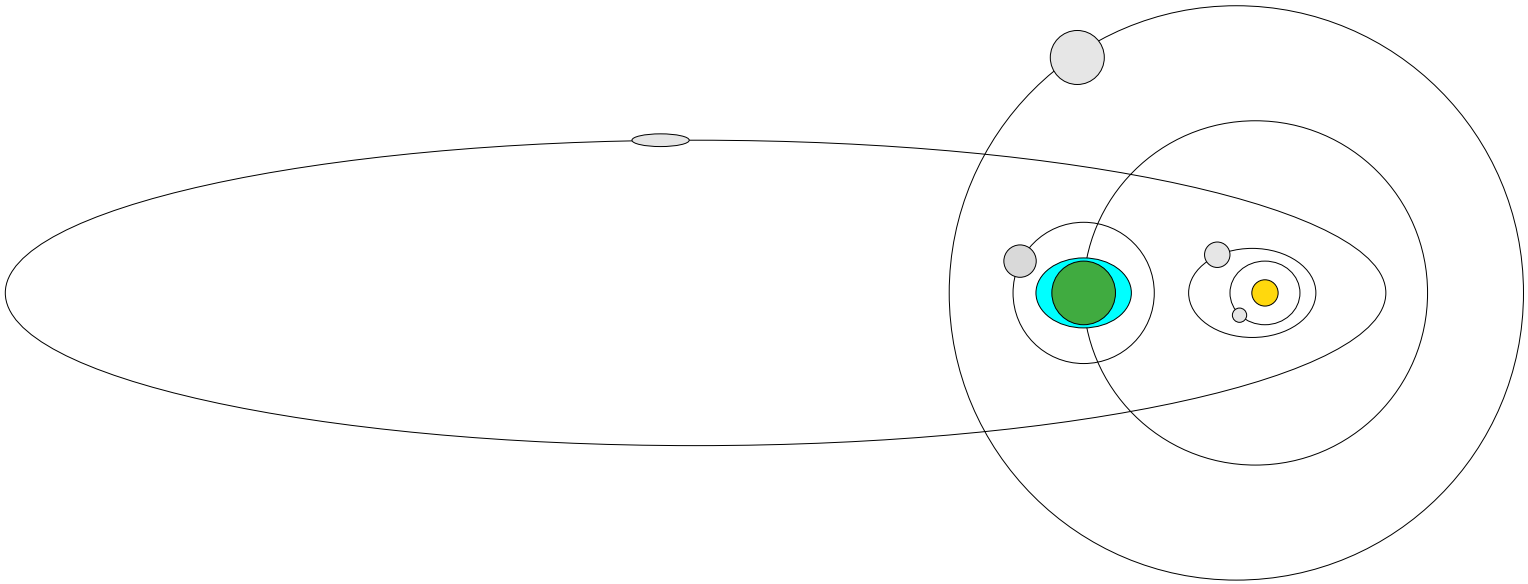

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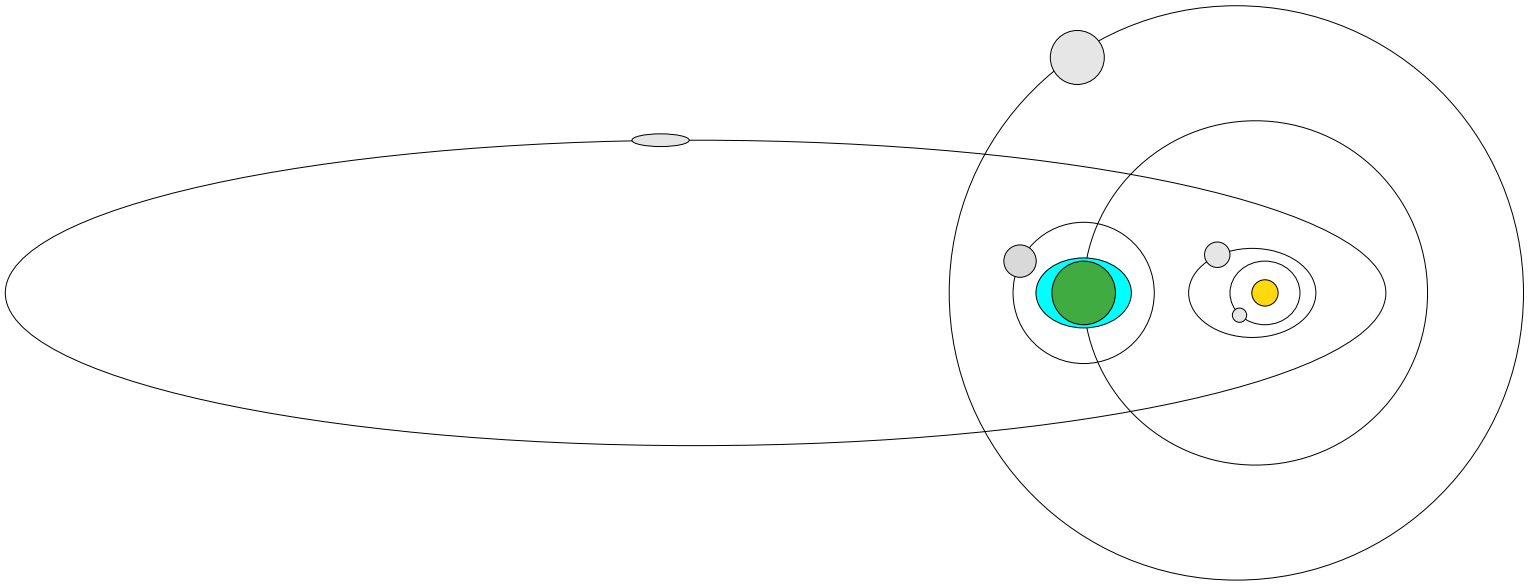

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The Force acts universally and instantaneously.

A curious coincidence - $m_{\text{inertial}} = m_{\text{gravitational}}$.

Challenge from Special Relativity

Speed of light in vacuum is independent of the state of uniform motion of the source or the detector. Kinematics must be adapted to reflect this property.

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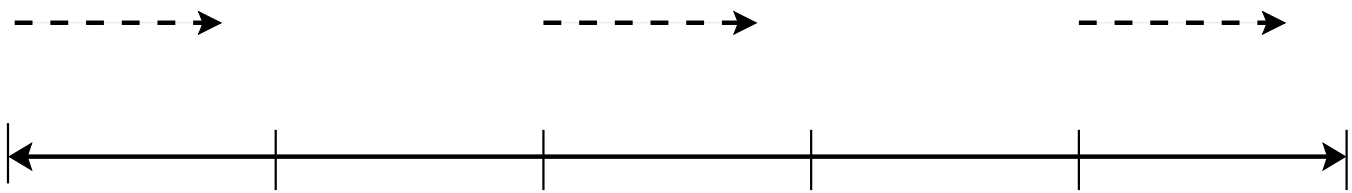
$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - \Delta t^2$$

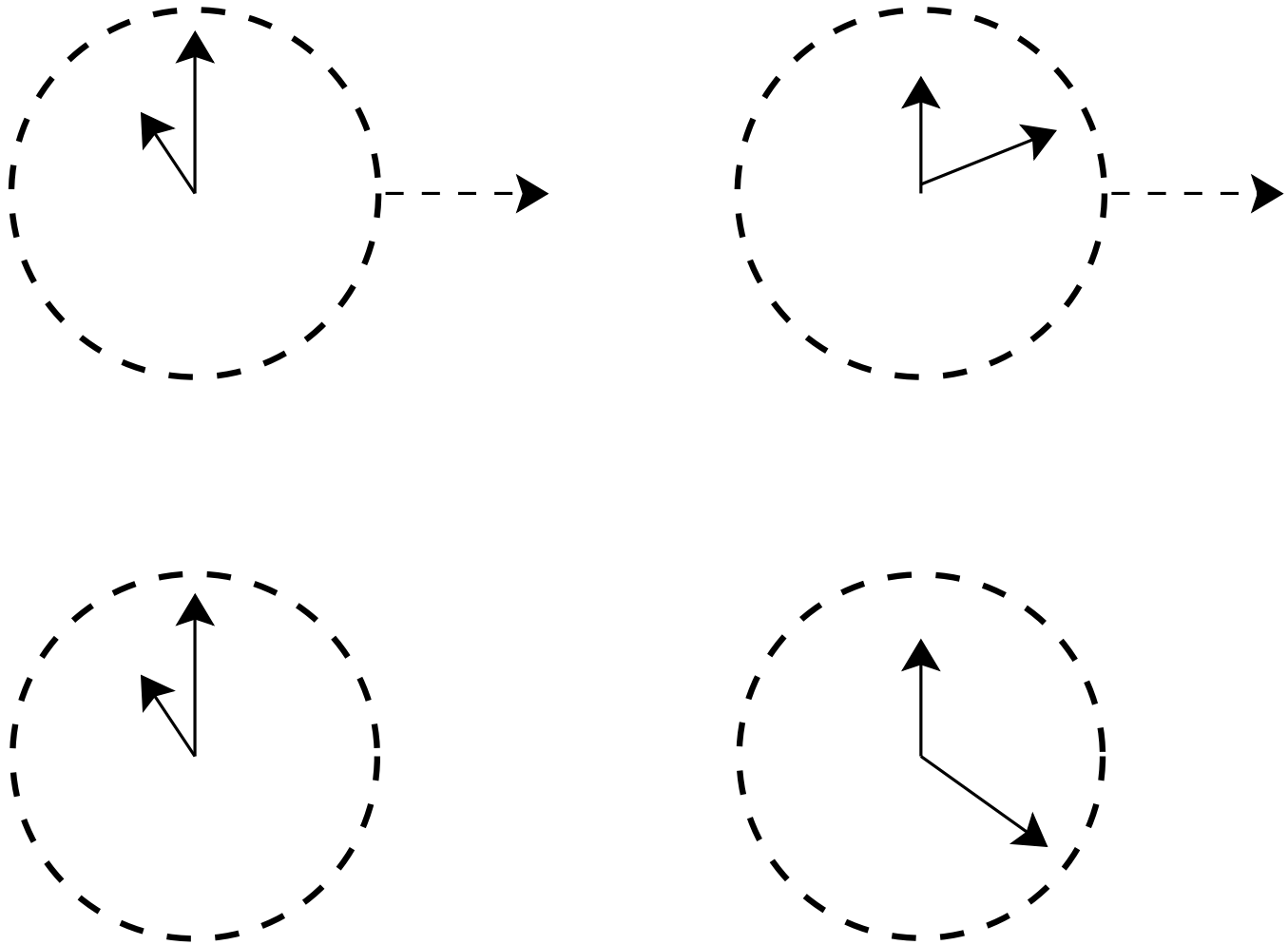
Length determination, time stamping are observer dependent.

Challenge from Special Relativity . . .

Moving objects contract along the direction of motion



Moving clocks run slower



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The rotating disk simulates a non-uniform gravitational field. And also has a non-Euclidean geometry.

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$$\Delta s^2 = \sum_{i,j=0}^{3} g_{ij} \, \Delta x^i \, \Delta x^j$$

(Pseudo -) Riemannian Geometry

Since accelerated observers are plenty and diverse, so must be non-Euclidean geometries. What decides the geometry in a given physical situation? Distribution of mass-energy and material stresses.

Quantitatively determined by the Einstein Equation!

$$\nabla^2 \Phi_N(x) = -4\pi\rho \quad \rightarrow \quad G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\frac{d^2 x^i}{dt^2} = -\delta^{ij} \frac{\partial \Phi_N}{\partial x^j} \quad \rightarrow \quad \frac{dx^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

Much richer theory

(Pseudo-) Riemannian Geometry

Clocks run slower in stronger gravitational field
(Tested everyday in GPS devices)!

Straight lines are *geodesics* $\leftrightarrow$ free motion is along a geodesic
$\Rightarrow$ planetary orbits precess;

Paths of light are 'curved' too! $\Rightarrow$ Light 'bends' near
gravitating bodies (Gravitational lensing);

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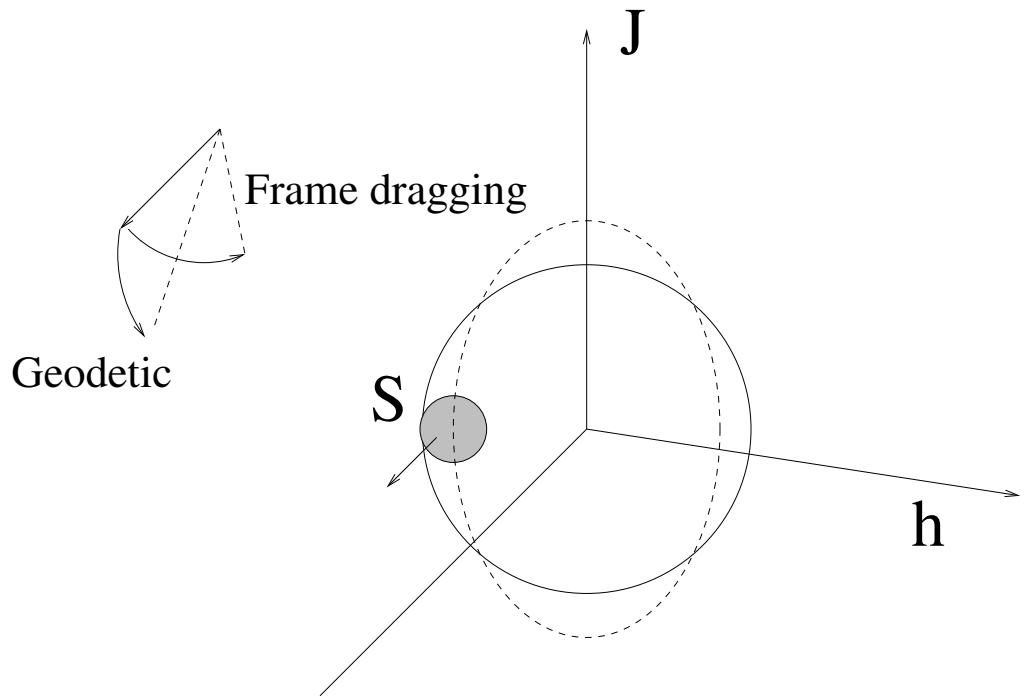


Figure: Spinning gyroscope in free fall near rotating earth

Effects on Stars

Every form of energy gravitates, not just the mass. $\therefore$
Einsteinian gravity is 'stronger' than Newtonian one.

In a star, this affects the *stability* of the hydrodynamic equilibrium in stars and leads to

Black Holes!

Dynamic Cosmology

The possibility of a dynamical space-time accommodates the Hubble law leading to the Expanding Universe.

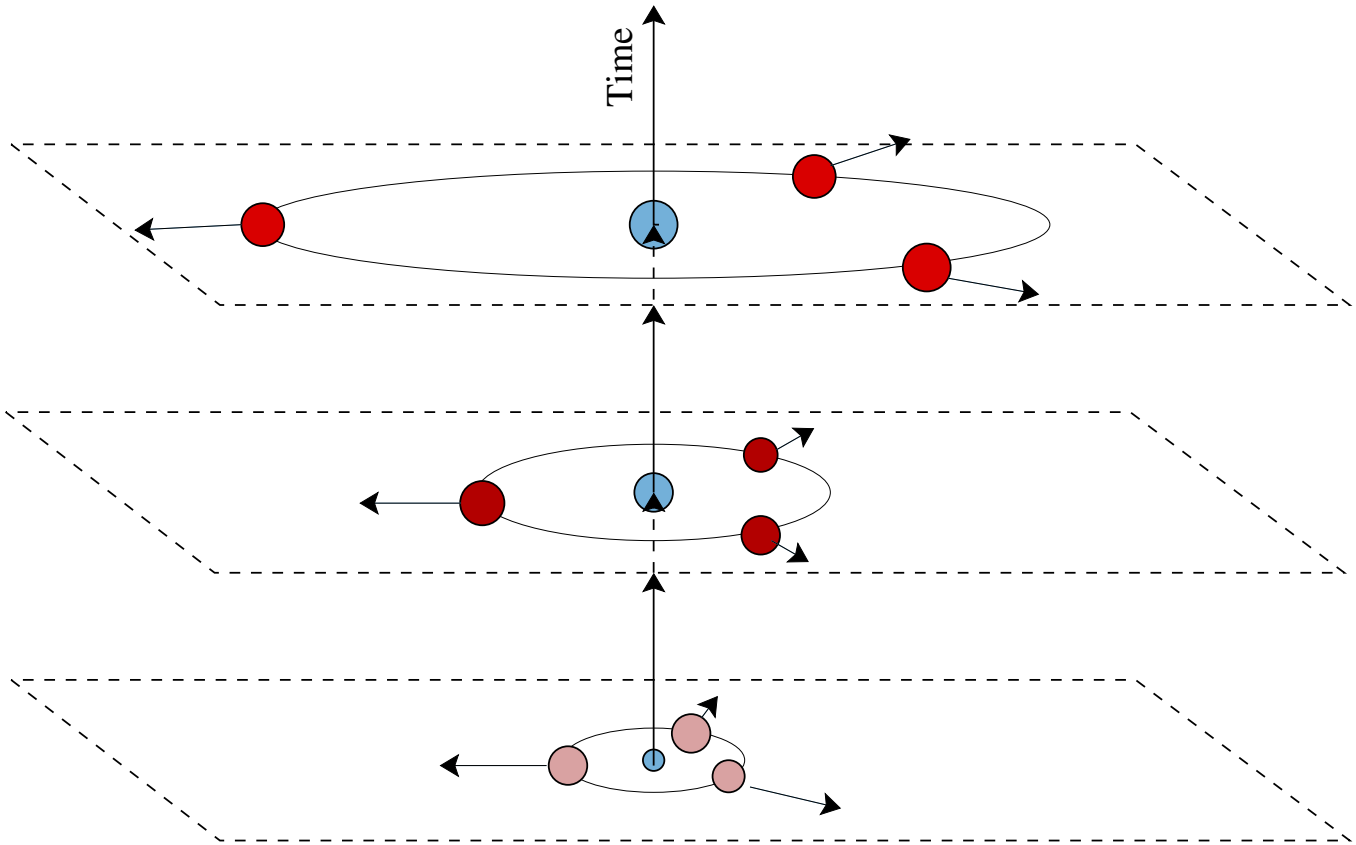


Figure: Red Shifts and Stretching of Space

In conjunction with other forces of nature - electromagnetic, strong and weak - leads to a detailed picture of Hot Big Bang in which structures of various types and scales can form.

Here are some pictures of our universe according to our current 'concordance Model' of Cold Dark Matter with Cosmological Constant.

Figure: http://spacecoalition.com/wp-content/uploads/2013/03/planck4.jpg

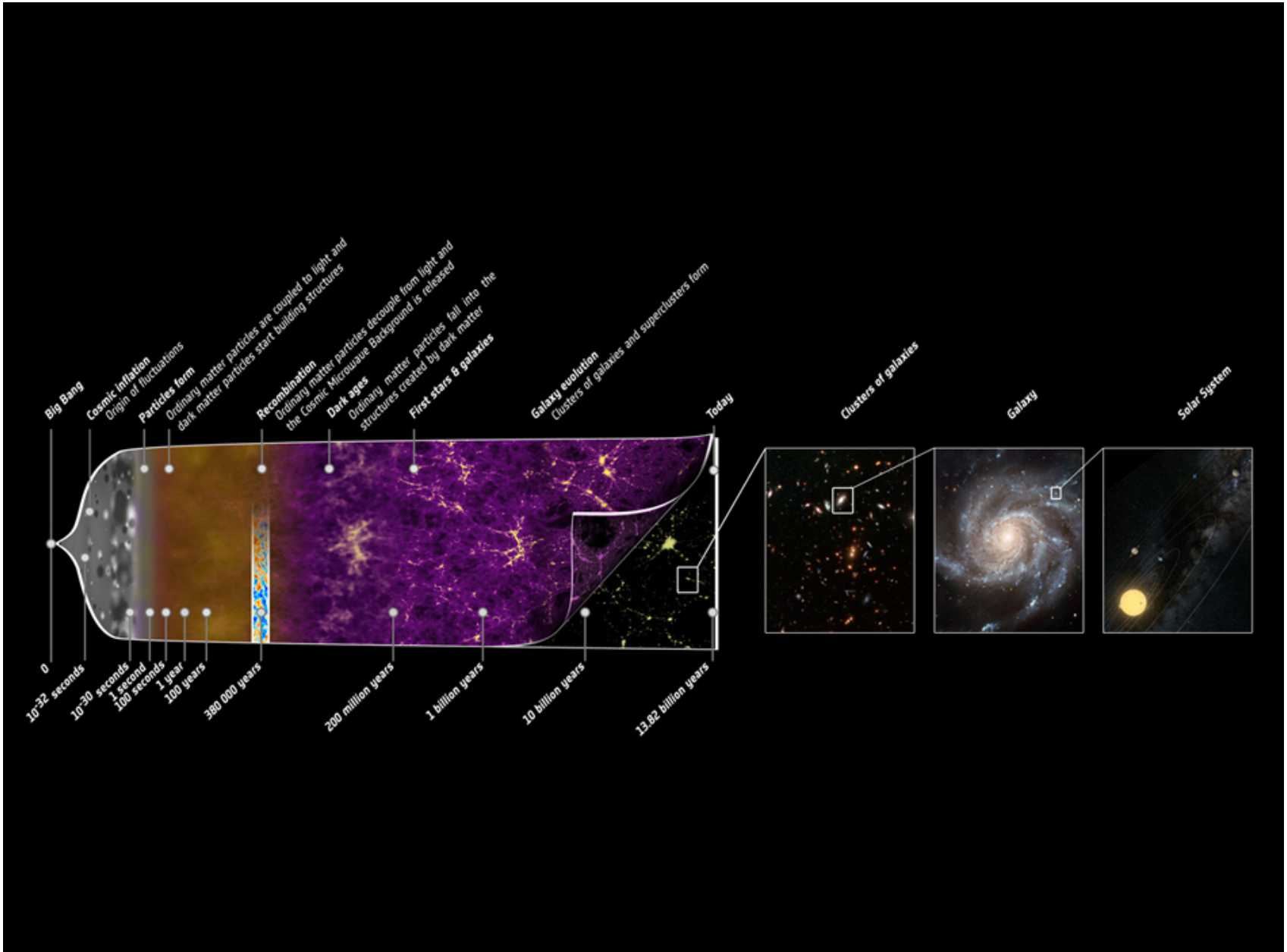
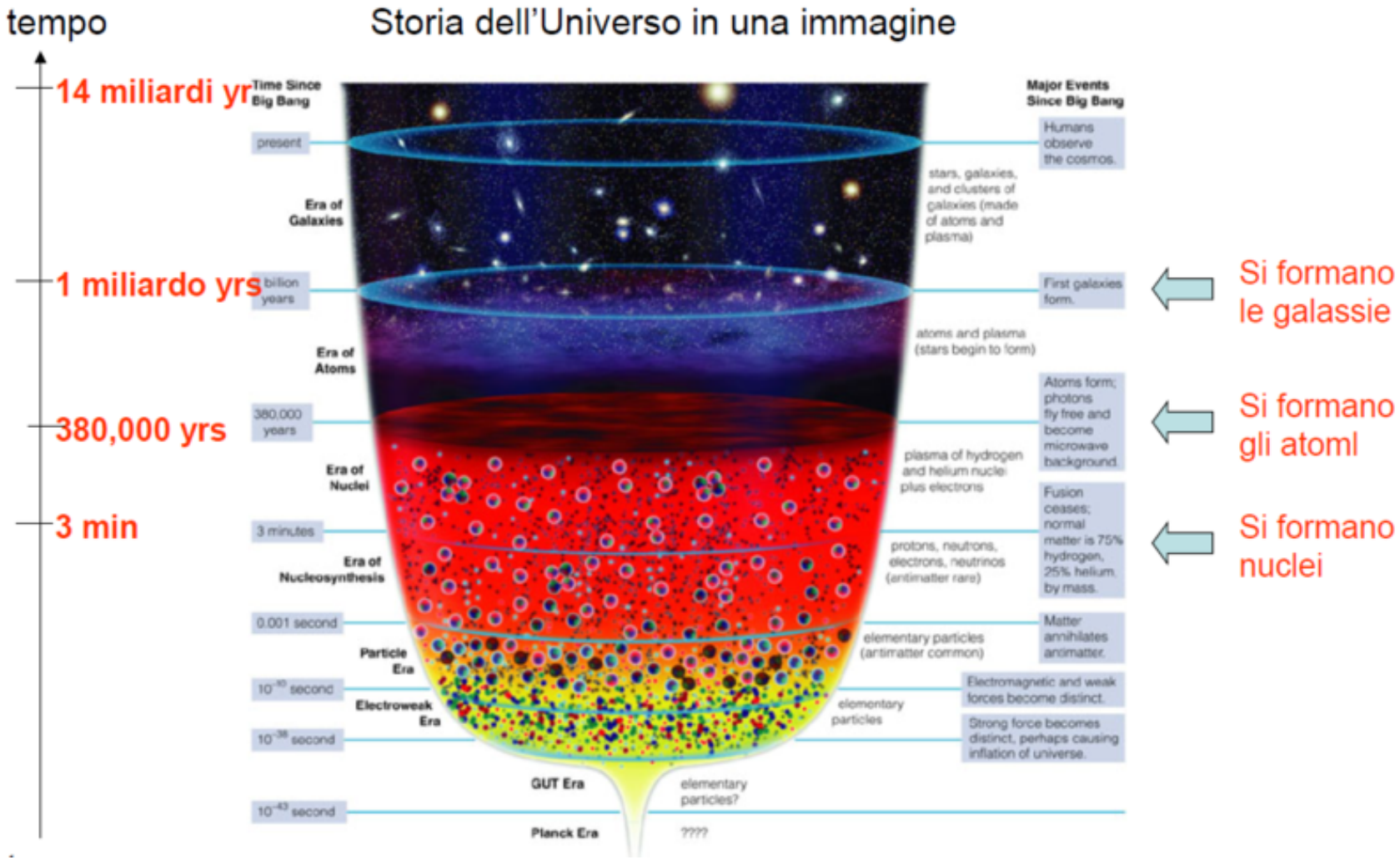


Figure: http://img854.imageshack.us/img854/4650/univ.png



Gravitational Waves

Brand New feature of gravitation having its own, independent degrees of freedom - Gravitational field can exist and evolve independent of material mass-energy.

Just as accelerated charges radiate away energy in the form of electromagnetic waves, so do accelerated masses $\Rightarrow$

gravitationally bound bodies can suffer orbital decay (Hulse-Taylor binary Pulsar).

'Ringing down' black holes emit unique waveforms. If detected, will unambiguously confirm existence of gravitational waves.

Gravitational Waves

If gravitational astronomy can be achieved, we will see completely different 'sky' - gravitationally 'bright' sources will be completely different from the electro-magnetically bright (optical, radio, x-ray,...) sources.

World-wide effort for direct detection - and Indian Initiative in Gravitational-Wave Observations - collaboration in India.

http://www.gw-indigo.org

Beyond Einstein?

Widely believed that General Relativity is inadequate to handle some of its own predictions! (Singularity Theorems);

Black Hole Thermodynamics suggests microscopic degrees of freedom;

Quantum framework suggests that classical horizons may change drastically (Hawking effect - Information loss puzzle).

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Challenging Frontier!

Gravity Probe B: Spin Precession Estimates

Polar circular orbit with $r \sim 400$ miles;

Duration of experiment $\sim 12 - 14$ months;

Estimated Geodetic precession rate ~ 6.6 arcsec/yr;

Estimated Frame-dragging precession rate ~ 40.9 milliarcsec/yr;

GW Power Estimates

For a linear mass distribution of length scale L, mass M and angular speed Ω, the quadrupole radiated power:

$$P \sim 10^{-40} M^2 L^4 \Omega^6 \text{ Watts } \Rightarrow$$

For binary star: $L \sim 10^9, M \sim 10^{30}, \Omega \sim 10^{-4}$ $P \sim 10^{36}$;

For a Pulsar: $L \sim 10^4, M \sim 10^{30}, \Omega \sim 10^{+4}$ $P \sim 10^{53}$;

For a charge q, in a circular orbit of radius r and rotational frequency ω, $P \sim q^2 r^2 \omega^4$;

Atom collapse durations are 10^{-13} sec. and 10^{36} sec. for electromagnetic and gravitational instability.

Hulse-Taylor Pulsar Speed-up Graph

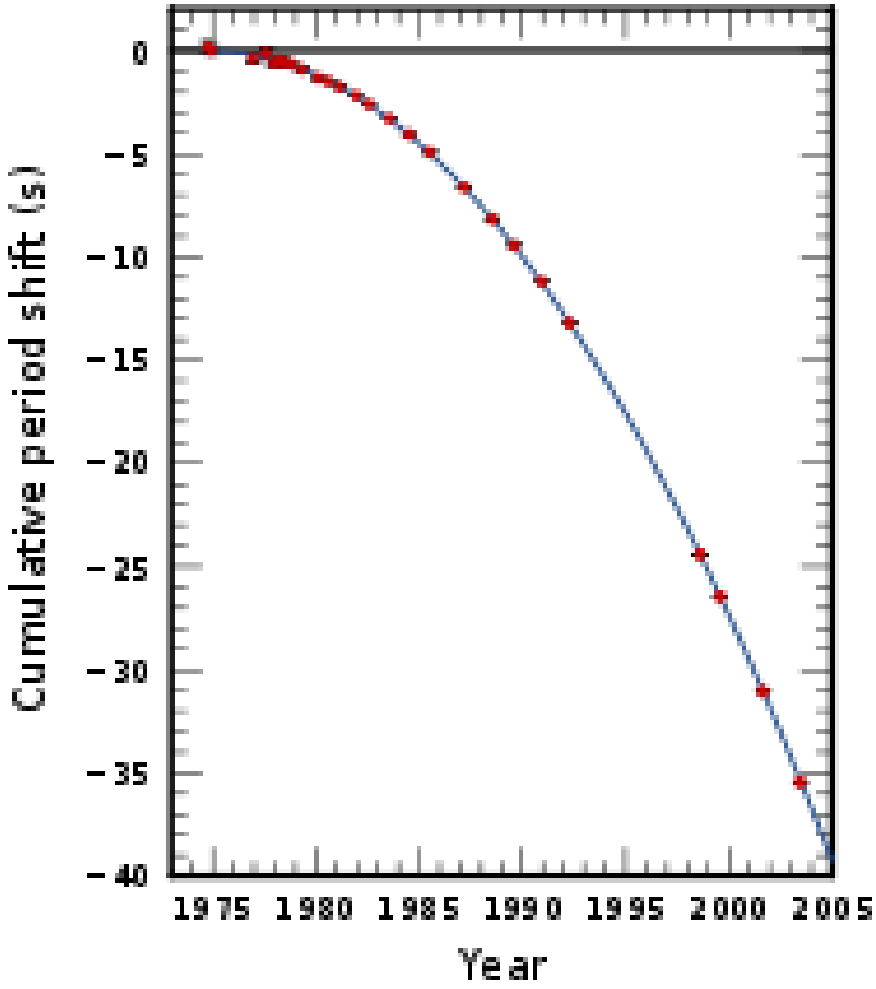

Figure: http://en.wikipedia.org/wiki/PSR_B1913%2B16