# A Short Primer on Gravitational Waves

**February 11, 2016**

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### Gravity, Space-time and General Relativity

Conception of Gravitational force stems from the Newton's law of gravity which governs the fall of the proverbial apple (and the moon) towards the earth as well as the fall of planets towards the sun. The law also implies that if one of the bodies moves, its gravitational effect on the other body is felt *instantaneously* no matter how far the two bodies are. This works beautifully for planetary motion, predictions of lunar and solar eclipses, discovery of new planets etc.

**Einstein 1 (Problem):** The instantaneous action of gravitational interaction is incompatible with special relativity.

**Einstein 2 (Principle of Equivalence):** Uniform gravitational field is 'fictitious' i.e. can be traded for an accelerated observer. But the ocean tides are very 'real'.

**Einstein 3 (General Relativity):** Gravitation can be 'relativised' provided we recognize:

(a) the space-time has a geometry (that is, a rule which specifies how to compute the distance between two locations and the time interval between two events ) which is changeable,

(b) gravitation is a manifestation of effects of the *curvature* of the space-time geometry and

(c) the interaction of space-time and matter is specified by the Einstein field equations.

**Wheeler: Space-time tells matter how to move and matter tells space-time how to curve.**

The possible space-times are vastly richer. Not only does the new view give the planetary orbits more accurately, it indicates very strongly curved and exotic regions such as black holes and also accommodates an ever changing, evolving universe.

### What are Gravitational Waves?

There are geometries which very well approximate the gravity-free space-time, namely, the Minkowski space-time (ignoring a cosmological constant). These are given as solutions of the *linearized Einstein equations*. Among these are the space-time metrics which satisfy a *wave equation* with a propagation speed equal to the speed of light. These waves are *transverse* and have *two* independent *polarizations*. This means that if a wave is propagating the z-direction say, then it will move small test bodies in the x-y plane only and will do so in two independent patterns called the *plus* '+' polarization and the *cross* '$\times$' polarization.

Regardless of how the waves may be produced, they propagate with these characteristic properties: propagate at the speed of light and with two specific types of transverse polarization ("helicity 2"). They can also carry energy, momentum and angular momentum.

*Note: These are very specific properties and distinguish time dependent, even oscillatory, geometries from gravitational waves. For instance, the ocean tides produced by the moon and occurring twice a day, do not represent tides due to gravitational waves.*

**How are the produced?**

The solutions of linearized Einstein equation with *localized* matter, show that they are produced whenever the matter distribution is at least *quadrupolar* and the quadrupole moment (or any higher moment) has an *accelerated* time dependence.

In particular, spherical matter distribution (even if time dependent eg a breathing ball or a spherical, accelerated collapse) will *not* produce gravitational waves.

**Gravitational waves may also be produced by quantum fluctuations in the very early universe.**

To get an idea of orders of magnitude for waves produced by localized sources, we note:

$$(Amplitude) := h \sim \frac{G}{c^4}\frac{ML^2\nu^2}{D} \sim 10^{-44}\frac{M\ L^2\nu^2}{D} \qquad \text{(MKS units)}$$
$$(Power) := P \sim D^2(h\nu)^2 \sim 10^{-54}M^2L^4\nu^6 \qquad \text{(MKS units)}$$

Here, $M$ represents the source's *mass* (effective), $L$ represents source's *linear dimension* (effective), $\nu$ represents *frequency of time variation* (effective) and $D$ denotes the *distance* to the source. To be in the *wave zone*, the distance $D$ must be at least as large as the *wave length* given by the speed of light divided by the frequency. The quadrupole is represented as $ML^2$ while $\nu$ represents its time derivative. The power is proportional to the square of the time derivative of the amplitude. The amplitude is *dimensionless* while power is given in *watts*.

To get a sense of orders of magnitude:

• A 10 kg rod of length 10 meters, rotating about an axis perpendicular to its length, with 10 revolutions per second (10 Hz) will produce a wave of an amplitude of about $10^{-44}$ at a distance of 1000 km. This is also the fractional change in lengths it will induce in transverse direction. It will emit power at about $10^{-40}$ watts.

• A spinning neutron star with a bump on it surface, within our galaxy, will have $M \sim 10^{25}$kg, $L \sim 10^4$ meters, $\nu \sim 10^3$ Hz and $D \sim 10^{19}$ meters ($\sim$ 1 kilo-parsec), will give an amplitude of about $10^{-24}$ and will radiate at $10^{30}$ watts. The Sun radiates at about $10^{26}$ watts.

More promising sources for detection are binary systems of compact objects such as two neutron stars, two black holes or a neutron star and a black hole. They are typically within our galaxy (BH-BH from other galaxies could be detected as well).

• Binaries have *three* distinct phases: (1) in-spiral, (2) merger and (3) ring down. The last two last for short durations and have characteristic waveforms. During in-spiral, the amplitude is smaller at about $10^{-23}$ and the emitted power (for binaries within our galaxy), is about $10^{18}$. By contrast, for coalescing binaries the amplitude increases to about $10^{-19}$ and the power to about $10^{38}$. For galactic binaries of stellar scale black holes, these numbers rise to $10^{-16}$ and $10^{48}$ respectively. For binaries of super-massive black holes (extra-galactic), the amplitude gets better. Using the knowledge of astrophysics and various surveys, the estimate of detectable event rates range between a few per year to a few hundred per year.

**How are they detected?**

**The indirect method** relies on the loss of energy, momentum and angular momentum of a gravitationally bound system through gravitational radiation which in turn affects its motion. This can be tracked by astronomical observations.

The best studied example of this is the 1913+16 Hulse-Taylor binary pulsar. Over a thirty year tracking of its shrinking orbit, the match with the prediction based on energy loss from gravitational radiation is about 1 part in $10^3$, (**1993 Nobel Prize**).

At least qualitatively, the slow rotational speed of *young neutron stars* ($\sim$ 100 Hz) compared to the older *milli-second* pulsars, can be understood as due to *loss of angular momentum* through gravitational radiation.

**The direct detection** methods rely on the tidal distortions they produce in a detector eg. *resonant bars* (essentially single frequency) and *laser interferometry* which can detect a range of frequencies. The basic configuration of an interferometer has two arms at right angle along which split laser beams go back and forth and are subsequently recombined. If a gravitational wave passes through such an arrangement, there are delays introduced in the arrival times of the laser photons and the photon count on recombination carries a record of the passage of the wave.

The currently operational interferometers are: **LIGO** at Hanford, Washington state and Livingston, Louisiana, 4 km arm length; **VIRGO** near Pisa, Italy, 3 km arm length; and **GEO** near Hannover, Germany 600 m arm length.

Hopefully **INDIGO** at ?, India, 4 km arm length will materialize in the next few years.

These detectors aim for a strain *sensitivity* of $10^{-21}$ and a frequency range of about 10 Hz - $10^4$ Hz (wavelength of about $10 - 10^4$ km). For lower frequency waves, space based interferometers are needed.

This is also the 'audible' frequency range (20 - 20,000 Hz). Hence these sources of gravitational waves are sometimes called *'sirens'* and the merger phase waveform is called a *'chirp'*.

**What will the GWs show?**

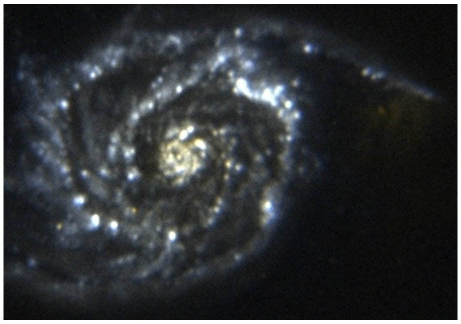
Their direct detection will spur development of **gravitational wave astronomy.** Because they interact very weakly (they can get lensed, like light), they carry essentially faithful information about their source. They cannot be masked, so nothing will remain 'opaque' ('singularities' in the centers of black holes are excluded).

As they are produced by bulk properties - as distinct from atomic level properties which cause the electromagnetic waves - they carry qualitatively different information. The sources which strongly radiate gravitational waves are generally weak radiators of electromagnetic waves. Consequently, the cosmos of GW sources would be very different, and *we have never seen it!*

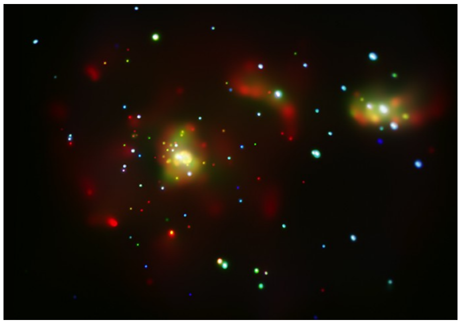
Here is an example of different views of the 'whirlpool galaxy' (M51), seen with different electromagnetic 'eyes'. *The last box is empty.*

infrared

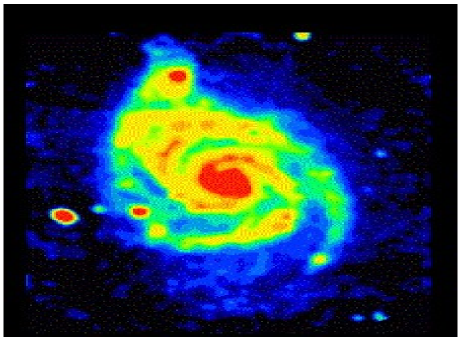
visible



ultraviolet



x-ray



Radio

GWs ?

*Credits:*

**http://www.physci.mc.maricopa.edu/Astronomy/astlabs/ast114/galaxy-lab/m51.htm**
The radio image is from:
**http://coolcosmos.ipac.caltech.edu/cosmic_classroom/multiwavelength_astronomy/ multiwavelength_museum/m51.html**

**To read more,** a good beginning source is the encyclopedia article by Bernard Schutz, available at http://arxiv.org/pdf/gr-qc/0003069.