Universe View Through Gravitational Waves

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Flow of the Talk

- Electromagnetic View: Composition, distances, isotropy and expansion;
- General Relativity: Dynamical space-time, compact objects, waves;
  - Compact sources $\rightarrow$ GW view ;
  - Non-Compact sources $\rightarrow$ GW background;
- FLRW Background: Epochs, anisotropies, scales;
- GW perturbations: Modes, evolution, nature of perturbations;
- Pulsar Timing Arrays: Nano Hertz frequencies.

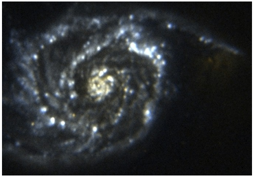
Electromagnetic View

- Look up in the sky and see planets, stars, the Milky Way ;
- Use telescopes and see star clusters, galaxies, galaxy clusters;
- Look at a galaxy through different filters. Here is the M51.

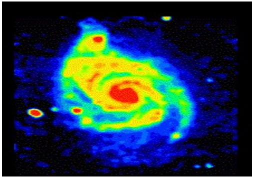
The Whirlpool Galaxy, M51

infrared

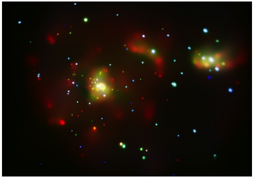
visible



ultraviolet



radio

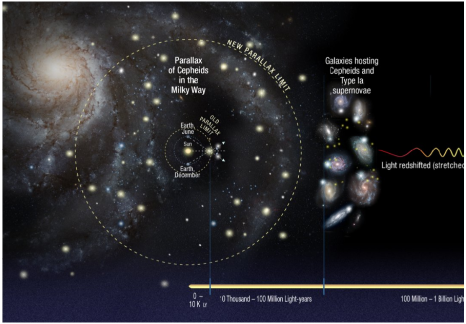


x-ray

GW?

http://www.physci.mc.maricopa.edu/Astronomy/astlabs/ast114/galaxy-lab/m51.htm
http://coolcosmos.ipac.caltech.edu/cosmic_classroom/multiwavelength_astronomy/multiwavelength_museum/m51.html

The Cosmological Distance Ladder



Nearby stars	Triangulation/parallax	~ 30 pc
Star clusters	main sequence photometry	$\sim 10^5$ pc
Galaxy	variable stars	$\sim 10^6$ pc
Inter-galaxy	Novae, brightest stars	$\sim 10^7$ pc
Galaxy clusters	supernovae, brightest galaxy	$\sim 10^{10}$ pc

Isotropy and Expanding Universe

If you look at only the objects beyond about 200 Mpc, then their sky distribution is isotropic. May be the same is true if viewed from different galaxies.

The Cosmological principle: *On the large scale, universe is (spatially) homogeneous and isotropic.*

Hubble Law: Hubble observed that the light from distant galaxies is red-shifted and *the red-shift increases linearly with the distance to the galaxies.*

Our universe is expanding (everywhere).

General Relativity

• GR views gravitational force as a manifestation of curved space-time geometry which is generically dynamical;

$\rightarrow$ Natural framework for an expanding universe.

• GR is an extension of special relativity to gravitation.

$E = mc^2 \Rightarrow$ all forms of energy can gravitate.

$\rightarrow$ Highly compact objects, WD, NS, BH can exist.

• GR predicts that gravitational effects can propagate at finite speed conveying energy, momentum and angular momentum.

$\rightarrow$ Compact binary systems can merge!

View through GWs

It is important that compact objects such as NS and BH exist. Only they can come close enough and merge quickly enough.

Black holes and their mergers are brand new processes which are not visible in an electromagnetic view.

When GW astronomy i.e. localization of mergers and distances to them, matures, we will have new features in our sky maps.

The blank box will display the GW view of M51.

GW Background

The mergers and other localized sources would have been producing GWs all through the history of the universe. These would contribute to a background ‘hum’ of gravitational waves. Primordial gravitational waves would also hum.

The famous CMBR is an electromagnetic hum left over from the photon decoupling era, about 3×10^5 years after the Big Bang. A similar hum is expected (not observed) from the era of neutrino decoupling, about 1 second after the Big Bang.

GWs interact so weakly that they are essentially always decoupled. Their hum from early universe (if observed) will be from the Big Bang itself.

GWs: compact sources

Einstein's Quadrupole formula:

$g_{ij} \approx \eta_{ij} + h_{ij}$, $h_{ij}(r,r) = \frac{2G}{c^4}\frac{}{r}\ddot{Q}_{ij}(t - r/c)$, $\frac{dE}{dt} = \frac{G}{5c^5}(\dddot{Q}_{ij})^2$.

Merely mathematical solns or do convey energy/momentum?

Bondi-Pirani-Robinson (1957-60) and Bondi-Burg-Metzner (1962): Einstein eqns with localized sources, admit Radiative solutions carrying away energy with accompanying loss of the source mass.

Prediction and confirmation

Orbital decay (Nobel 1993)

Gravitational 'chirps' (Nobel 2017)

GWs: Non-compact sources

These are the sources producing the GW background. It has two component (i) the astrophysical sources which are distributed all over and (ii) the primordial fluctuations.

- We can only proceed through a linearization about some homogeneous background;
- Only statistical information may be inferred.

If $\delta\phi(t, \vec{k})$ denotes the Fourier coefficient of a generic linearized perturbation, then we presume,

$$\langle \delta\phi(t, \vec{k}) \rangle = 0 \ , \ \langle \delta\phi(t, \vec{k}) \delta\phi(t, \vec{l}) \rangle = \delta^3(\vec{k} - \vec{l}) \Delta^2_{\phi}(t, k)$$

FLRW background

$$\Delta s^2 = -\Delta t^2 + a^2(t)\Big(\sum_{i=1}^{3}(\Delta x)_i^2\Big) \quad , \quad T_{\mu\nu} = \rho(t)u_\mu u_\nu + P(t)(u_\mu u_\nu + g_{\mu\nu}) \ .$$

$$3\frac{\ddot{a}}{a} = -4\pi(\rho + 3P) \qquad \text{(Raychaudhuri equation);}$$

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi\rho \qquad \text{(Friedmann equation);}$$

$$\dot{\rho} = -3(\rho + P)\frac{\dot{a}}{a} \qquad \text{(Conservation equation).}$$

$$P = 0 \ , \ \rho a^3 = \text{constant} \ , \ a(t) = C_M t^{2/3} \qquad \text{(Matter dominated);}$$

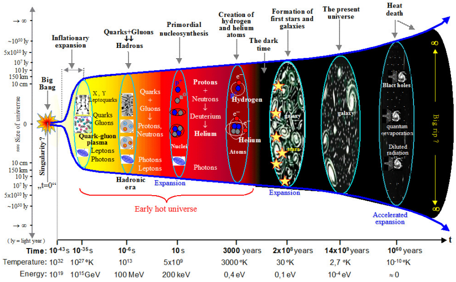
$$P = \frac{1}{3}\rho \ , \ \rho a^4 = \text{constant} \ , \ a(t) = C_R\sqrt{t} \qquad \text{(Radiation dominated);}$$

$$P = -\rho = -\frac{\Lambda}{8\pi} \ , \ a(t) = C_{dS}e^{\sqrt{\Lambda/3}t} \qquad \text{(de Sitter).}$$

conformal time: $\eta := \int^t a^{-1}(t')dt'$.

η is negative for acceleration and positive for deceleration, always increases with t.

Main epochs



The universe expands at a certain rate and thus sets a length scale. It is called the Hubble Radius $:= H(t)^{-1} = \frac{a}{\dot{a}}$. The physical length scales or sizes however increase as $a(t)$.

Their ratio controls much of the structure formation.

Cosmological Anisotropies

The CMB anisotropies confirm the existence of perturbations. Their noteworthy properties are:

(a) $\frac{\Delta T}{T} \sim 10^{-5}$. As per COBE normalization, the power spectrum, $P_S(k)(k/k_*)^{n_S-1}, A_S \sim 10^{-9}, A_T \sim 0.036 A_S$;

(b) The angular distribution resolution: ~ 10 arc-min. At LSS,

$$10' \leftrightarrow 30 \text{ Mpc},\ 180^\circ \sim 36 Gpc,\ H_0^{-1} \sim 4 \text{ Gpc}.$$

Experiment	Sensitivity window (f)	wavelength range
Ground based interferometer	$1 - 10^3 Hz$	$300 - 3 \times 10^5\ km$
Space based interferometers	$10^{-5} - 0.1 Hz$	$10^6 - 10^{10}\ km$
Pulsar Timing Arrays	$3 \times 10^{-9} - 10^{-6} Hz$	$10^{-1} - 30\ lyr$
CMB	$3.4 \times 10^{-19} - 7 \times 10^{-18} Hz$	$4 - 10^2\ Gpc$

Gravitational Perturbations

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \epsilon h_{\mu\nu} \ , \ T_{\mu\nu} = \bar{T}_{\mu\nu} + \epsilon T^{(1)}_{\mu\nu}$$

$$h_{\mu\nu} = (\bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu) \ , \ T^{(1)}_{\mu\nu} = \mathcal{L}_\xi \bar{T}_{\mu\nu} \quad \text{(gauge transformations)}$$

$$\bar{\Box} h_{ij} = 0 \ , \ \bar{\nabla}_j h^j{}_i = 0 = \bar{g}^{ij} h_{ij} \quad \bar{g}_{ij} = a^2(t)\delta_{ij} \quad \text{(gauge fixed);}$$

$$h_{ij}(t,\vec{x}) = \sum_{\sigma=+,\times} \int \frac{d^3k}{(2\pi)^{3/2}} e^{-i\vec{k}\cdot\vec{x}} \left[h_\sigma(\vec{k},t) \varepsilon^\sigma_{ij}(\vec{k}) \right] \ , \ k^j e^\sigma_{ji} = 0 = \delta^{ij} e^\sigma_{ij}$$

$$H_{ij}(\vec{k},\eta) := a(\eta) h_{ij}(\vec{k},\eta) \leftrightarrow H_\sigma(\vec{k},\eta) := a(\eta) h_\sigma(\vec{k},\eta) \Rightarrow H''_\sigma(\vec{k},\eta) + \left(k^2 - \frac{a''}{a}\right) H_\sigma(\vec{k},\eta) = 0$$

$$a(\eta) = \eta^\alpha \Rightarrow \frac{a''}{a} = \frac{\alpha - 1}{\alpha}(aH)^2 \geq 0 \ , \ (aH) \sim \eta^{-1} \ \forall \ \alpha = 2, 1, -1.$$

$$\frac{k^2}{a^2} \ll H^2 \ \leftrightarrow \ (k\eta)^2 \ll 1 \ \leftrightarrow \ \lambda_{phy}(t) \gg d_H(t) \quad \text{(super-Hubble)}$$

$$\frac{k^2}{a^2} \gg H^2 \ \leftrightarrow \ (k\eta)^2 \gg 1 \ \leftrightarrow \ \lambda_{phy}(t) \ll d_H(t) \quad \text{(sub-Hubble)}$$

Evolution and Power Spectrum

Super-Hubble modes remain constant while sub-Hubble decay as $\sim a^{-1}(\eta)$.

During acceleration $|\eta|$ decreases $\Rightarrow$ super-Hubble remains so.

During deceleration η increases $\Rightarrow$ sub-Hubble remains so.

de Sitter epoch Illustration

$$h_\sigma(\eta, \vec{k}) = z_\sigma(\vec{k})\ e_k(\eta) + z^*_\sigma(-\vec{k}) e^*_k(\eta)\ ,\ e_k(\eta) = \frac{\alpha_k}{a(\eta)}\Big(1 - \frac{i}{k\eta}\Big) e^{-ik\eta}$$

$$\langle z_{\vec{k}} \rangle = 0 = \langle z^*_{\vec{k}} \rangle\ , \langle z_{\vec{k}} z^*_{-l} \rangle := \mathcal{A}_h(\vec{k}) \delta^3(\vec{k} - \vec{l})\ \Rightarrow$$

$$\langle h_\sigma(\eta, \vec{x}) h_\sigma(\eta, \vec{x}) \rangle = \int_0^\infty \frac{dk}{k}(2)\left[\frac{k^3}{2\pi^2}\mathcal{A}_h(k)|e_k(\eta)|^2\right]$$

$$\text{with } |e_k(\eta)|^2 = \frac{|\alpha_{\vec{k}}|^2}{a^2(\eta)}\Big(1 + \frac{1}{k^2\eta^2}\Big) \text{ and}$$

$$\Delta^2_h(\eta, k) = 2\mathcal{A}(k)|e_k(t)|^2\ ,\ P_h(\eta) := \frac{k^3}{2\pi^2}\mathcal{A}_h(k) \text{ Power spectrum.}$$

Nature of Perturbations

The power spectrum has free functions of k: α_k and $\mathcal{A}_h(k)$.

$h_\sigma(\eta, \vec{x})$	stochastic function	$\alpha, \mathcal{A}$ are free;
$h_\sigma(\eta, \vec{x})$	classical field	$\lvert\alpha_k\rvert = \frac{1}{\sqrt{2k}}, \mathcal{A}$ free;
$h_\sigma(\eta, \vec{x})$	quantum field	$\mathcal{A}$ determined by quantum state.

For Bunch-Davies vacuum,

$$\begin{aligned} P_h(k,\eta) &= \frac{k^3}{2\pi^2}|e_k(\eta)|^2 = \frac{H^2}{4\pi^2}(1+k^2\eta^2) \\ &\to \frac{H^2}{4\pi^2} \text{ for super-Hubble } k \end{aligned}$$

Scale invariant spectrum for super-Hubble modes.

AGWB: Pulsar Timing Arrays

Pulsars are rotating neutron stars whose magnetic field beams in the radio frequencies. The Millisecond Pulsars are extremely stable with fractional change $\delta\nu/\nu \sim 10^{-15}$. The radio waves suffer various time delays as they reach earth. The Time-of-Arrival of the pulses is recorded. Modeling the known sources of time delays and comparing with the delays inferred from TOA, gives the residual time delay. This potentially is due to the AGWB. How is this identified?

PTA Observable

A pulsar at sky location $\hat{p}$ with a passing GW $h_{ij}(t, \vec{x})$.

$$T_{elapsed}(t) = \int_0^L d\alpha (1 + \tfrac{1}{2} h_{ij}(t, \vec{x}(\alpha\vec{p}))\hat{p}^i\hat{p}^j).$$

Fractional change in the pulse periods is:

$$z(t) := \frac{\Delta T}{T}(t) \approx \frac{\hat{p}^i\hat{p}^j\varepsilon^{\sigma}_{ij}(\hat{k})}{2(1+\hat{k}\cdot\hat{p})} h_\sigma(t)$$

t TOA

$\hat{k}$ direction of GW $\Rightarrow$

σ polarization

$$\langle z_a z_b \rangle = \int \frac{dk}{k}\left[\frac{k^3}{2\pi^2}\mathcal{A}\right]\left(4\pi\mu_{Hellings-Downs}(\gamma_{ab})\right), \; cos\gamma_{ab} := \hat{p}_a \cdot \hat{p}_b$$

$$\mu_{HD}(\gamma) = \frac{1}{4} + \frac{1}{12}cos(\gamma) + \frac{1}{2}\left\{(1-cos\gamma)ln\left(\frac{1-cos\gamma}{2}\right)\right\}$$

Summary and references

Waves are messengers from processes taking place far away. Gravitational waves fill-in where EM waves step aside. They do so in the form of chirps, as a background hum and also leave imprints on the polarization anisotropies of CMB.

For further reading

- Valerie Domcke, TASI lectures, arxiv:2409.08956 ;
- Azadeh Maleknejad, MPA Lectures on GWs in Cosmology;
- Caprini and Figueroa, CGWB, arxiv:1801.04268;

Thank You