How far are stars? How do we know?
(The Cosmological Distance Ladder)

Ghanashyam Date
The Institute of Mathematical Sciences, Chennai
http://www.imsc.res.in
shyam@imsc.res.in

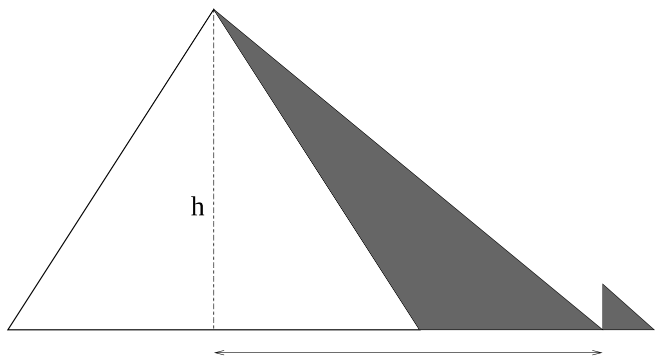
Introduction:

In this age of information, we are told a lot of 'facts': that our universe has trillions of galaxies each with trillions of stars; that there are objects called *white dwarfs, neutron stars, black holes*; other exotic stuff called *dark matter, dark energy*; that the universe is about 13.7 billion years old and so on. This is rather remarkable especially when contrasted with other set of facts: we did *not* know that galaxies existed till about a hundred years ago; that the planet Jupiter has moons till about 400 years ago; that the earth is round till about 2500 years ago! We cannot even leave our solar system. How then did we (do we) gather knowledge about our universe?

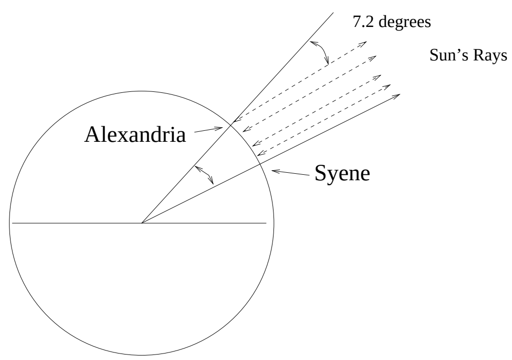
We can of course *watch, think and theorise.* A crucial ingredient in the process of building up the knowledge of our universe is the estimation of distances to the heavenly bodies. We are going to see some of the milestones in the estimation of the distances.

Two historical examples:

The simplest way to measure a distance between two points is to lay meter sticks between them (or tie a rope between them if the points are not on the land) and count them. Now imagine that you are asked to find the height of the historical Great Pyramid of Giza, Egypt. Obviously you cannot drop a plumb line and measure its length. But you notice that every object casts a shadow and know all about *similar triangles.* The pyramid casts a shadow and so does a pole. The lengths of the shadows cast on the ground is proportional to their heights. Hence the ratios of the heights is the same as the ratios of the lengths of the shadows. The shadow lengths are known and so is the hight of the pole. From this the height of the pyramid is computed. The Schematic illustration indicates the logic. Around 600 BC, Thales determined the height to be about 150 meters.



The second example is more challenging. The ancient Greeks were aware that the earth is actually round like a sphere which must be very large. An obvious question is: what is its radius? Again we cannot use any 'direct method'. It was known that as you go in the northern direction, the Sun never comes exactly overhead i.e. the shadow of a pole will never have zero length. But on a given day it has the shortest shadow. It was also known that the day in which the shadow is shortest in Alexandria (northern Egypt) is the same as the day in which the shadow is of zero length in Syene (modern day Aswan, southern Egypt). On this special day, the angle made by Sun's rays with the pole was measured. The distance between the two cities was known. If the angle is θ radians and the distance is d kilometers, then Earth's radius is d/θ kilometers. The figure illustrates the logic of the computation.



This is how Eratosthenes determined Earth's radius to be about 6400 kms which is surprisingly close to the present value of about 6,378.1370 km (equatorial radius)[1].

Now we jump to the times closer to the modern era. The next natural distances to be determined are: distance to the Moon, to the Sun (*the astronomical distance*) and distances to the other planets. These methods make use of observations of *eclipses* (solar and lunar)

[1] A story is that Poseidenius determined a smaller value of about 4480 kms and the great Ptolemy favoured the smaller value which was accepted as the correct value for over 1000 years. The smaller values encouraged the shipping expeditions to be sponsored!

and *transits* of planets. These are little lengthier to explain in a short space and I refer you to the reference at the end.

We will assume that these distances have been determined, in particular the Astronomical Unit (Earth-Sun mean distance of about 150 million km) and go to the next challenge: distances to the stars.

Nearby Stars: The Parallax Method

Straighten your arm and hold up the index finger. Look at the index finger once from the left eye and once from the right one. The finger seem to move *relative* to the scene ahead of you. The amount of shift depends on the distance between your eyes, the distance between your eyes and the finger and the distance between your finger and the distant scenery. The *angular measure* of this apparent shift of foreground object (your finger) relative to the background objects (the scenery) is called *parallax* of the foreground object. knowing the distance between the eyes and measuring the angular shift, gives a determination of the distance to your figure.

Now scale-up everything. Replace the finger by a particular 'nearby' star of interest, replace the two eyes by observing the star from two diametrically opposite points on the earth's orbit around the Sun and the scenery by the 'distant stars' (these do not appear to move). Measuring the two angular positions gives the angular shift while 2 times the Astronomical Unit provides the length scale. Here is the sketch explaining the geometry.

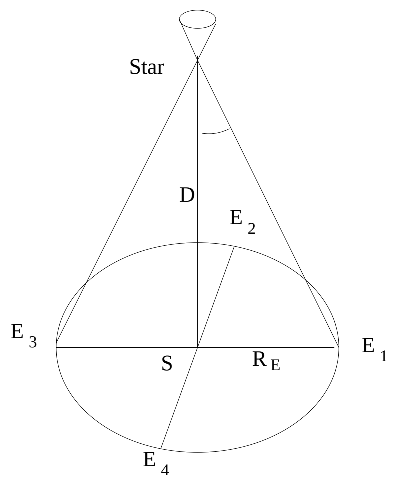


FIG. 1: (Annual) parallax := $\pi_* = R_E/D$.

Search for parallax:

The determination of stellar parallax took over 150 years! Here are some of the milestones.

The Copernican model of the solar system is already in place i.e. planets go around the Sun. A coordinate system to mark the positions of stars is also in place namely, there is the *Celestial Sphere* which is an imaginary sphere of indefinite radius, concentric with the centre of earth and with a longitude (*Right Ascension*) and latitude (*Declination*) grid chosen such that the two equators match. Stars are thought to be 'stuck' to the inner side of the celestial sphere or distributed within such that positions are fixed. The model implied that there should be apparent motion of 'nearby' stars mirroring the motion of the earth around the Sun. However in the 15th century, no one had seen *any* apparent motion - either the model is wrong or the stars must be very far. Galileo searched for parallax but found none.

Stars must be very far:

The first modern attempt was made by Robert Hooke (1669). Assuming that the bright star γ Draconis to be nearer and hence with a larger parallax, he built a specialised telescope to observe it. However, his observations were unreliable.

Flamsteed (1712) proceeded to make a precise determination of positions of some 3000 stars within 10 arc-seconds (1 arc-second is 1/3600th of a degree) precision and observed Polaris ('Dhruva') for several years. Nonetheless, these efforts did not lead to discovering parallax. Stars must be really far away.

Comparing brightness of the brightest star Sirius ('Vyadha') with that of Sun and assuming both to be equally bright intrinsically, Huygens estimated Sirius to be about 27,664 AU away. This is qualitative comparison and very reliable. Newton compared Saturn and Sirius and making assumptions about how much light of the Sun is reflected by Saturn, estimated Sirius to be about 800,000 AU. This indicates a parallax of about 0.26 arc-seconds, well below the capability of the instruments available then. It was indeed a great challenge.

But astronomers continued in their efforts, and in the process discovered something else equally important in the subsequent development.

Discovery of Proper motion, Aberration of light and Binary systems:

Halley realised that the star catalogues must be more precise. He compared the ancient catalogues prepared by the Greeks and the one prepared by Flamsteed and found that while most positions were unchanged, Sirius, Arcturus ('Swati'), Betelgeuse and Aldebaran ('Rohini') had moved. He had discovered *proper motion* of stars ($\sim$ 1720). Proper motion refers to shift in the *angular position* of stars. Stars were not stuck to the celestial sphere after all. There is no celestial sphere with stars attached to its inside!

James Bradley observed γ Draconis (a conveniently located bright star) and found that its declination kept changing every night reaching a maximum in September and its minimum in March. This looked like parallax except that the min/max should have been reached December and June respectively. Eventually he realised that this apparent shift is not due to parallax but due to *finite speed of light* (already argued by Romer) and earth's velocity in its orbit. He termed the effect as *Aberration of light* ($\sim$ 1730). What is this effect?

We are familiar with waking in the rain. As we walk faster, we tilt the umbrella forward to avoid getting wait. If we change our velocity (direction and/or magnitude), the tilt has to change. Compare the rain with light coming from a star, our walking with the motion of earth around the Sun and the umbrella as the telescope pointing towards the star. Just as we change the tilt of the umbrella when we change our motion, the earth based telescopes must change tilts as earth changes its motion in the orbit. The amount of tilt is determined by the speed of light and the speed of earth and has nothing to do with either the proper motion or the parallax. Due to the Aberration of light[2], the star catalogues must be revised. Bradley's observations established that earth does go around the Sun, speed of light is finite and determinable from earth's orbital speed and the fact that parallax was not discovered meant that the γ Draconis was farther than 200,000 AU.

Herschel began hunting for parallax using the *differential parallax method* (relative parallax of two objects) and identified many stars which looked close to each other (within 2 arc-second separation). Bur are these pairs of stars bound together (in which case they are close to each other and differential parallax method is of no use) or were these just line-of-sight coincidences? By measuring the positions of a binary system *Castor*, he actually concluded that the stars go around each other with a period of several hundred years. Binary stars were discovered. They showed that Newton's law of gravitation was applicable to stars as well!

This discovery of binary stars lead Struve to set criteria for selection of candidates for parallax determination - they should be *bright with a large proper motion* or should be *wide binaries.*

By about 1830, instruments improved, catalogues were better and definite criteria for search were developed. This culminated in Bessel discovering the annual parallax of 61 Cygni (1838) and within a few weeks Struve discovered parallax of α *Centauri.* The distances were: α Centauri 278,000 AU, 61 Cygni 719,000 AU and Vega 1,650,000 AU.

[2] This has nothing to do with aberrations in optical instruments such as lenses which are due to imperfect shapes, impurities in the media etc.

A new unit of measure is called for - *parsec-* the distance at which the annual parallax would be 1 arc-second. This is about 3.26 light years. The three nearest stars are at 1.35, 3.46 and 8 pc away. The parallax method is reliable in estimating distances up to about 90 parsecs.

We have exhausted the relatively direct method based on geometry to estimate distances and *many more stars are still left!*

Stellar Spectra:

We can do more than just note the angular positions of the dots in the sky, we can pass the star light through a prism and take its spectrum. What information does it reveal? *Firstly*, the general tinge of the light is correlated with the *surface temperature* of the star (recall red-hot coal, blue flame of a burner, ...). Astronomers use this to introduce a spectral classification of stars: stars are grouped in to classes labelled $O, B, AFGKM(RNS)$ in decreasing order of temperature from 30,000 to 3,000 ^{0}K. This plays a role in the so-called *Hertzsprung-Russell Diagram* (more on this below). *Secondly,* the the spectral lines can be compared with the atomic spectra taken in the laboratories to deduce the atomic elements present near the surface of the star. This plays a role in developing models of *stellar evolution* and arrive at an estimate of the *age* of the star. *Thirdly,* and more directly important in distance estimation, is that the *Doppler shift* in the atomic spectral lines relative to those on earth, gives an estimate of the *radial velocity* (along the line of sight) of the star relative to us. Proper motion measurements on the other hand give *transverse, angular velocities.* If we can somehow infer the *transverse, linear velocity* of a star, then we can infer its distance from us (recall transverse linear velocity = radial distance $\times$ transverse angular velocity). This is achieved by the *Moving Cluster Method* which forms a crucial step in estimating even larger distances.

Moving Cluster Method:

If one considers the process of star formation from a dust cloud, it stands to reason that stars originating in the same cloud will have approximately the same age and similar chemical content. One can identify such clusters of stars from their age estimate and chemical composition (inferred from the spectral analysis). These come in two varieties: *open clusters* (young and bright stars) and *globular clusters* (old and dimer stars). Globular clusters typically contain over 100,000 stars while open clusters contain a few hundred to a thousand stars. The best example of open cluster is *Pleiades* ('Krittika'). The one which is studied most is the *Hyades Open Cluster* consisting of about a 300 - 400 stars with similar age and chemical content. It happens to be the nearest open cluster and is identifiable by its bright stars forming a ‘V’, in the Taurus constellation. These cluster stars appear to be moving together as exhibited by their proper motion (angular shifts).

Suppose now that the cluster stars are all moving roughly in the same direction like a flock of birds. Due to perspective effect, the parallel motion appears to converge (or diverge) to some point. For a wider cluster, this point can be determined quite accurately. If θ denotes the angle between the inferred direction and the line of sight, then the common transverse velocity would be $\tan(\theta) \times$ the radial velocity. The radial velocity is inferred from the Doppler shift. The transverse velocity then determines the distance from the proper motion. In practice, the identification of moving cluster stars as well as the inference of the common direction involves a statistical analysis of the proper motion data.

This method gives the distance to the Hyades cluster to be about 46 pc.

This cluster has stars of many different spectral types and is useful for the next method.

Main Sequence Fitting Method:

Now we begin to make use of the inverse square fall-off of intensity. If a source of light has an *absolute luminosity*, I_0 (energy output per second) and we receive the light at a distance R from the source, then the *flux*, $I(R)$ (energy received per second per unit area) is given by $I_(R) = I_0/(4\pi R^2)$. Astronomers use *magnitudes* to refer to the intrinsic and apparent brightness whose exact relation to luminosity and flux are not important for us. The flux is what is measured by our instruments directly. *If* we could know the absolute luminosity of a star, we can immediately infer its distance. But this is also an unknown. Once again we have to construct subtler methods.

Let us appeal to models of evolution of stars which begins its life when nuclear fusion is ignited which produces enough heat and pressure to halt the gravitational collapse and makes the star shine. At some stage (after billions of years for a star like our Sun), the hydrogen gets essentially converted to Helium and next level of nuclear fusion reactions start and so on till all the nuclear fuel is burnt up. The core then collapses to form a *white dwarf or a neutron star or a black hole* depending upon the mass of the collapsing core. At this stage the star has died. Evidently, the energy out put as well as the surface temperature are correlated with age of the star and we could expect to see some regularity. This is uncovered by making a plot luminosity vs temperature. This is the *Hertzsprung-Russell Diagram* displayed below. Notice that different types of stars, populate different regions of the plot.

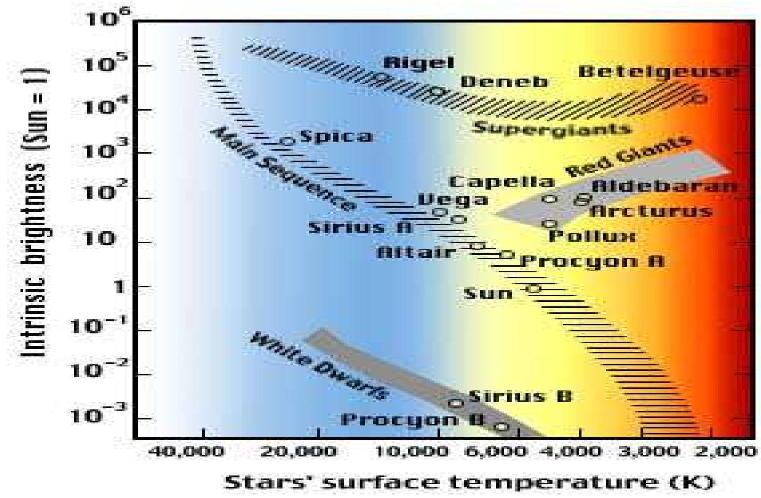


FIG. 2: *Hertzsprung-Russell Diagram:* Plot of absolute luminosity versus spectral type. observe.arc.nasa.gov/nasa/space/stellardeath/stellardeath_1ai.html

How is this plot constructed? Thanks to the distance estimates done by the geometric and the moving cluster methods, we have the distances to a large sample of stars whose fluxes are also known. Hence we know their absolute luminosities. Surface temperature is also known by direct observation, thus the H-R diagram is constructed.

Now construct a similar diagram for another cluster, but now in terms of flux vs surface temperature because we don't know their absolute luminosities. We can identify the main sequence stars of this cluster. They must be similar to the main sequence stars of the know cluster such as the Hyades and hence must have have similar absolute luminosities. We already know their fluxes and hence we know the distance to that cluster! We have appealed to the universality of laws of physics governing stellar evolution and the inverse square fall-off law.

This method can be used to estimate distance up to about 100,000 pc and still more objects are left.

Variable Stars:

During the nuclear fuel burning stage in a star's life, some of the stars undergo a periodic variation of their energy out-put. Their brightness varies periodically. These are called *variable stars*. Depending upon their periods, these are grouped in to three classes: *Mira Variables* with periods in months, *Cepheid variables* with periods in days and the *RR-Lyrae* with periods in hours. The names refer to the constellations where such variable stars were discovered.

In 1907, Henrietta Leavitt discovered a *brightness-period power law relation* for the Cepheid variables in the *small Magellanic Cloud* - our sister galaxy. By 1913, Hertzsprung calibrated the relation. Now measuring the period of a Cepheid class variable star, immediately gives the absolute luminosity and hence the distance estimate.

In 1923, Hubble observed Cepheid variables in the *Andromeda Nebula* and estimated its distance to be 275,000 pc. This is larger than the size of our galaxy. The Andromeda nebula is extra-galactic. Soon it was recognised as a galaxy like our Milky Way and thus we became aware of existence of other galaxies.

By now the strategy should be clear. We look for some regularity in a population of stars (or clusters or galaxies), to ascertain its absolute luminosity and thereby infer the distance. Such features are called *Standard Candles.* The main sequence stars, Cepheid variables are examples of these. These suffice to give distance to about a million parsecs. Beyond this, these candles are too faint to be of use. *Novae, brightest stars of a galaxy, brightest galaxies of a cluster of galaxies, Super-Novae* are the brighter candles which build up the distances to about 10^{10} pc.

Here is a summary table of the distance ladder.

Rung	Distances up to	Method
I	< 30 parsecs	Kinematic (Parallax) Methods
II	$< 10^5$ parsecs	Main Sequence Photometry
III	$< 10^6$ parsecs	Variable Stars
IV	$< 10^7$ parsecs	Novae, Brightest Stars etc
V	$\sim 10^{10}$ parsecs	Brightest Galaxies, Super-Novae etc

Summary:

Let us recapitulate.

- The distances to stars and beyond (and hence the size of our universe) is estimated as a series of steps of a ladder. The lower rungs estimates are crucial for the estimates at the next rung. The lowest rung uses relatively direct, geometric methods.

- Observations by relatively small telescopes are still useful as they contribute information at lower lower rungs and to statistical methods. Many of the early discoveries were made with small telescopes.

- The assumption of universality of laws of physics as well as technological advances improving observational precision are central to building up the knowledge of our universe.

References:

1. Most of the material contained in the article is beautifully discussed in the book: *Measuring the Universe: The Cosmological Distance Ladder* by Stephen Webb, Springer-Praxis series, 1999.

2. For those who wish to play with viewing the sky virtually, there is a nice software called *Stellarium* which is available free, goggle will give you the exact link.