General Relativity: A Hundred Years Later

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Time-line of Development

1907 The Search begins
Equivalence Principle, Gravitational Red-shift, light bending;

1911 SR and Gravity both need to be replaced, EP holds locally;

1912 Time is warped but space is flat, gravity can gravitate too, Study of static field;

1912 Analogue of Electromagnetic induction in GR;

1913 Introduction of the metric tensor;

1914 Rotation and Gravitational field; Covariance properties;

1915 Mercury precession, post-Newtonian corrections, final form of the field equation GR is born.

Plan

- Space-Time Geometry, Gravity, Einstein Eqns;
- Implications of Geometry, an application and a tool;
- Stars, instability and Collapse;
- An Evolving Geometry for the Universe;
- Waves of Geometry;
- A breakdown? A clash with the quantum?
- A new extension . . .

Space, Geometry and Time

Space is a collection of locations, labelled by three numbers in an arbitrary, but one-to-one manner. These are local coordinates;

Geometry is a rule that translates coordinate differences into results of length measurements, say by a measuring rod. Eg.

$$\Delta\ell^2 \;=\; (\Delta r)^2 + r^2(\Delta\theta)^2 = (\Delta x)^2 + (\Delta y)^2$$

Time is a sequence of events, stamped by a clock.

Special Relativity: Space + Time → Space-time

Observation: Speed of light is constant, independent of motion of source and/or detector;

Location labels and time stamps of two observers in uniform relative motion are related by Lorentz Transformations.

⇒ Special Theory of Relativity: Space-time geometry is born.

$$(\Delta s)^2 = -c^2(\Delta t)^2 + \sum_{i=1}^{3}(\Delta x^i)^2$$

This is Minkowski Space-time. In this space-time,

Moving objects contract in the direction of motion.

Moving clocks tick at a slower rate.

Einstein's Happiest thought

Observation: The observed equality of inertial and gravitational mass implies that a freely falling observer cannot detect a uniform gravitational field.

This is Principle of Equivalence, later realized to be valid locally.

Equivalence of nearby observers with arbitrary relative motion provided gravitational field is also included.

Relatively rotating observers infer non-Euclidean geometry.

Space-time has a vastly more general Riemannian geometry!

Where is Gravity?

A general Riemannian space-time is curved, like a balloon. Due to the curvature, small bodies move along geodesics.

Nearby geodesics tend to converge towards each other as though being pulled by a gravitational force between them.

While the Newtonian picture of a gravitational force is an excellent approximation, more accurate picture is the geometrical one.

But what determines the geometry?

In the Newtonian view, gravitational field in a region is determined by the distribution of mass - via the Poisson Equation for the gravitational potential. Something similar must happen in the geometrical view.

Indeed it does! The space-time metric is determined by the Einstein equation:

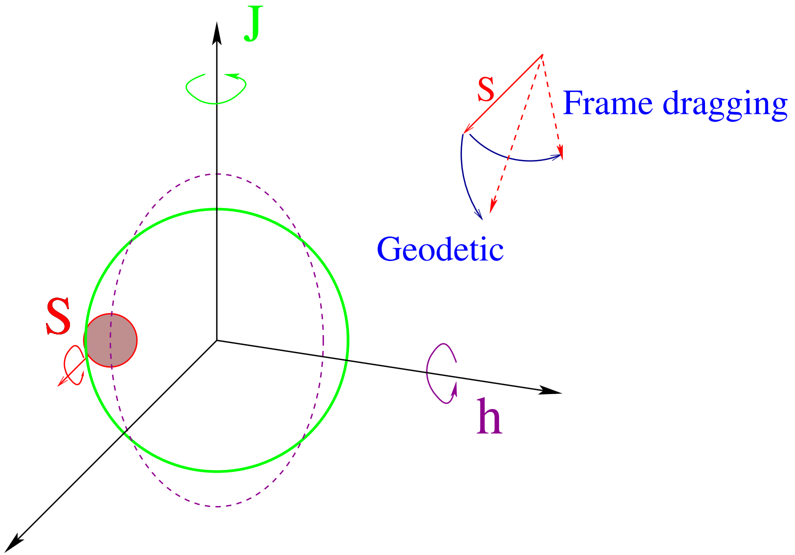
$$R_{\mu\nu}(g_{\alpha\beta}) - \frac{1}{2}R(g)\; g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Space-time is a Dynamical Entity.

Some Implications of the geometry:

- Clocks slow down near massive bodies;
- Light "bends" near massive bodies;
- Planetary orbits "precess" in general;
- Spin axis of a small gyroscope "precesses" near a massive body and with an additional precession if the body itself is spinning (frame dragging);

Spin Precessions



Implications of Clock Rates: GPS

Problem: Locate yourself on a map.

Solution: Determine the distances from four known beacons on the map.

Beacons = Satellites, Distance = time of travel,

clock synchronization is crucial!

Time dilations due to motion and due to geometry, both must be taken into account.

Light Bending $\rightarrow$ Gravitational Lensing

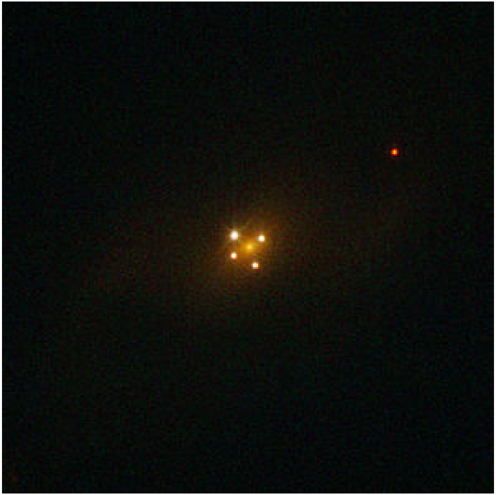
In our solar system, the light deflection angle is $\sim 1.75'' R_{\odot}/r_0$. Generally, matter distribution behaves as a Lens and produces distorted and/or multiple images of a light source.

The light bending is independent of the wavelength and is maximum closest to the scattering matter. It can produce multiple images (Einstein Cross), strong distortions (Einstein Ring), weak shape distortions and micro-lensing (transient brightening).

The theory of lensing has provided a new tool to estimate distribution of galaxies, clusters of galaxies, including invisible matter between a far away source and earth.

The weak lensing surveys (eg Dark Energy Survey) provides consistency checks on cosmological models.

Einstein Cross

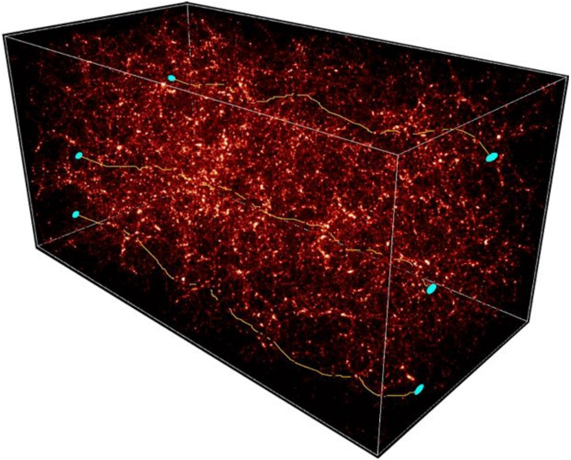


http://www.spacetelescope.org/images/potw1204a/

Einstein Rings

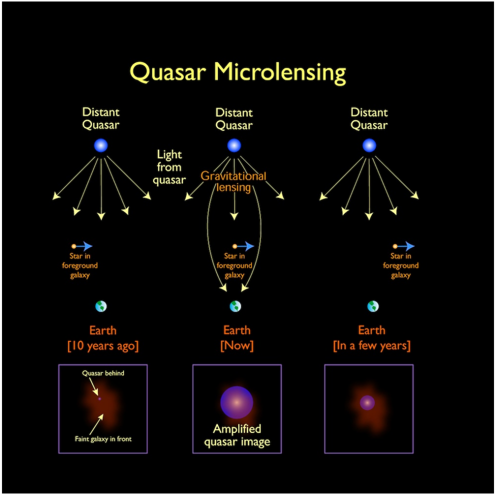
Loff, Sarah; Dunbar, Brian, "Hubble Sees A Smiling Lens". NASA.

Weak Lensing



http://gravitationallensing.pbworks.com/w/page/15553259/Weak%20Lensing

Microlensing



Jason Cowan, Astronomy Technology Centre/NASA

Confrontation with Matter

Imagine a spherical distribution of matter which distorts the geometry ("gravitational attraction") but is also supported by some thermodynamics pressure. The relevant equations are:

$$\Delta s^2 = -f(r)\Delta t^2 + g(r)\Delta r^2 + r^2(\sin^2\theta\Delta\theta^2 + \Delta\phi^2)$$

$$m(r) := 4\pi\int_0^r \rho(r')r'^2 dr' \, , \; g(r)^{-1} := \left(1 - \frac{2m(r)}{r}\right)$$

$$\frac{dP}{dr} = -\left[\frac{m(r)\rho}{r^2}\right]\left(1 + \frac{P(\rho)}{\rho}\right)\left[\frac{1 + \frac{4\pi r^3}{m(r)}P(\rho)}{1 - \frac{2m(r)}{r}}\right]$$

For relativistically moving matter, $P(\rho)$ is non-negligible (SR effect) while if the density and hence $m(r)$ gets larger, the $g(r)$ begins to get larger (GR effect).

New End State(s)

The GR effect may be interpreted as due to the feature that all forms of energy gravitate. To halt a collapse, higher pressure is needed which adds to the gravitational pull requiring still higher pressure. A runaway process is then possible.

Indeed, it is possible to establish that there can be no stable body with a mass M confined to a size $R < \frac{9}{8}\left[\frac{2GM}{c^2}\right] =: R_{min}$.

There is no such lower limit on size in Newtonian gravity.

Once a body contracts to a size smaller than R_{min}, the collapse is non-stoppable and the resultant state is expected to be a

Black Hole! (or a naked singularity?)

Our Visible Universe $1 \text{ ly} \sim 10^{12}$ km

Earth-Moon	$\sim$	10^{-7} ly
Earth-Sun	$\sim$	10^{-5} ly
Solar system size	$\sim$	10^{-1} ly
Nearest star	$\sim$	10^{0} ly
Open Clusters	$\sim$	10^{2} ly
Size of Milky Way	$\sim$	10^{4} ly
Distance to Andromeda Galaxy	$\sim$	10^{6} ly
Clusters of galaxies	$\sim$	10^{8} ly
Cosmological Scale	$\sim$	10^{9} ly

Isotropy and Cosmological Principle

Select objects which are about 200 MegaParsecs away. Their distribution on the Celestial Sphere is essentially uniform i.e.

On Cosmological Scales, visible universe is Isotropic.

Assume: This would be so if seen from any other object on the cosmological scale. This is the Cosmological Principle.

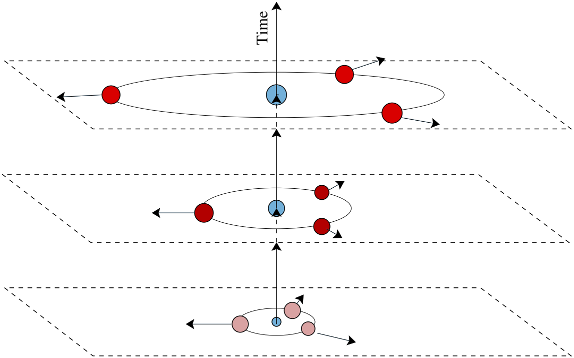
Appropriate space-time geometry is homogeneous and isotropic.

Naturally accommodates the

Hubble Law: Farther Galaxies have larger red-shifts.

Robertson-Walker Geometry

$$\Delta s^2 = -\Delta t^2 + a^2(t)\left[\Delta s_3^2\right]$$



Red Shifts and Expanding Universe

Evolution of Our Universe

Einstein Equation together with matter equation of state, imply that the scale factor was zero about 13.8 Gyrs ago!

It was a very hot soup of some elementary objects in mutual thermodynamic equilibrium. The expanding geometry forced a cooling which switched off various interactions at different epochs giving rise to the more complex structures we see today.

One such epoch with hydrogen atom + photon $\leftrightarrow$ proton + electron as dominant process, gave rise to the Cosmic Microwave Background Radiation.

Exquisite satellite based study of CMBR has given us very precise picture of the various epochs in the evolution.

The Dynamic Geometry plays a crucial role in shaping the universe and has revealed the mysterious Dark Energy.

Waves of Geometry?

Evidently geometry shows spatio-temporal scales of variations. Can localised features of geometry propagate? Do they "transport energy"? How may one check this?

Linearization of source free Einstein equation about a flat background, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, shows plane wave solutions which propagate at the speed of light and have two 'transverse polarizations' called $+$ and $\times$.

A circular arrangement of test masses are tidally distorted in a 'plus' and 'cross' form by a passing wave.

Einstein himself gave the Quadrupole Formula for the power radiated by a time varying Quadrupole Moment of a mass distribution.

$$P \sim \frac{G}{c^5} M^2 L^4 \Omega^6 \sim 10^{-54} M^2 L^4 \Omega^6 \quad \text{Watts}$$

New Mechanism for energy loss

Stars of a binary system constitute a time varying quadrupole moment and due to gravitational radiation come closer and rotate a bit faster. For a pulsar binary system, the change in period can be tracked. The Hulse-Taylor system, PSR B1913+16, was indeed tracked and the change in the period was found to be consistent with the quadrupole radiation within 1/2 a percent.

Hulse and Taylor received the Nobel prize in 1993.

Direct Detection of Gravitational Waves

Direct detection effort uses laser interferometry to detect tiny distortions in the arms of a Michaelson-Morley type arrangement, caused by a passing gravitational wave.

The Advanced-LIGO is expected to be operational later this year and may be we will have a direct detection of one of the crucial predictions of GR in the centenary year.

The effort is now to aim for a localization of sources to enter the era of Gravitational Wave Astronomy. IndIGO (http://www.gw-indigo.org), is a partner in this effort.

But(!), This is all Classical

GR, within a classical framework, establishes the autonomous existence of geometry as a physical entity on par with matter.

Matter reveals that a quantum framework is more accurate.

Might not the same be true with geometry?

Classical GR identifies regimes where its equations break down.

Given that (a) the 'always attractive' nature of gravity is retained and (b) only robust causal and predictive background space-times are acceptable, the Singularity theorems of GR establish that (i) towards the end phase of a complete gravitational collapse and (ii) at the beginning of an everywhere expanding universe,

GR breaks down.

Furthermore, the predicted black holes hint at being hot bodies embodying some entropy.

A Quantum Extension?

After Einstein replaced the ‘force view’ of gravitational phenomena by a manifestation of a dynamical, space-time geometry, many new phenomena were predicted and confirmed.

But the theory also made untenable predictions and revealed contradictory behaviour of black holes.

Search for its quantum extension began almost immediately and different aspects are taken as cornerstones to be preserved.

Should only the dynamical equations be quantum corrected or even the kinematical structure itself may change qualitatively?

The search is on . . .