

Contents

I The Basics	1
1 From Newton to Einstein: Synthesis of General Relativity	3
1.1 Space, Time, Observers	3
1.2 General Relativity and Space-Time Arenas	6
2 Examples of Space-times	11
2.1 No Gravity (Minkowski Space-Time)	18
2.2 Uniform Gravity (Rindler Space-Time)	20
2.3 Centrifugal Gravity (Uniformly Rotating Platform)	22
2.4 Spherical Gravity (The Schwarzschild Space-Time)	24
2.5 Cosmological Gravity (Robertson-Walker Space-times)	26
2.6 Undulating Gravity (Gravitational waves)	28
3 Dynamics <i>in</i> Space-time	31
3.1 Particle motion including spin	31
3.2 Wave motion	33
4 Dynamics <i>of</i> Space-time	39
4.1 Einstein Equation	39
4.2 Elementary properties and peculiarities	42
4.3 The Stress Tensor and Fluids	45

4.4	Operational determination of the metric	48
5	Elementary Phenomenology	51
5.1	Geodesics and the classic tests	51
5.1.1	Geodesics	54
5.1.2	Deflection of light	55
5.1.3	Precession of perihelia	57
5.2	Relativistic Cosmology	58
5.2.1	Friedmann-Lamaitre-Robertson-Walker Cosmologies	60
5.2.2	Digression on Big-Bang Cosmology: Thermal History	65
5.2.3	The Cosmic Microwave Background Radiation	70
5.3	Gravitational Waves	74
5.3.1	Plane Waves	76
5.3.2	Gravitational Radiation	77
5.3.3	Radiated energy and the quadrupole formula	81
5.4	Black Holes - Elementary Aspects	86
5.4.1	The Static Black Holes	86
5.4.2	The Stationary (non-static) Black Holes	91
5.4.3	Observational Status	93
5.5	Stars in GR	94
II	The Beyond	99
6	The Space-Time Arena	101
6.1	Preliminary notions and results	102
6.2	Causality	107
6.3	Determinism and Global Hyperbolicity	109
6.4	Geodesics and congruences	113

6.5	Singularity Theorems	122
7	Asymptotic Structure	127
7.1	Vicinity of the null infinity	132
7.2	Vicinity of the spatial infinity	140
8	Black Holes	143
8.1	Examples of extended Black Hole solutions	144
8.2	General Black Holes and uniqueness theorems	148
8.3	Black Hole Thermodynamics	151
8.4	Quasi-local definitions of horizons	154
9	Cosmological Space-Times	163
10	Gravitational Waves	171
10.1	Conceptual Issues	171
10.2	Observational issues	173
11	Field Equation: evolutionary interpretation	177
11.1	The $3 + 1$ decomposition	177
11.2	Initial value formulation	181
11.3	Hamiltonian Formulation (ADM)	183
12	Numerical Relativity	191
13	Into the Quantum Realm	197
13.1	Gravity is ‘Emergent’	198
13.2	The quantum gravity paradigm	200
13.2.1	String Theory: The Unification Paradigm	201
13.2.2	Loop Quantum Gravity: The background independence demand	202

14 Epilogue	205
15 Mathematical Background	207
15.1 Basic Differential Geometry	207
15.2 Sets, Metric Spaces and Topological Spaces	207
15.3 Manifolds and Tensors	210
15.4 Affine Connection and Curvature	215
15.5 Metric tensor and Pseudo-Riemannian geometry	219
15.6 Summary of Differential Geometry	221
15.7 Theorems on Initial Value Problem	239
15.8 Petrov Classification	242
Index	261