away via gravitational waves.

This established that gravitational radiation does exist with all the physically expected attributes.

How does this help in the *detection* of gravitational waves? The above analysis establishes that general relativity *does* make a physically sustainable prediction of gravitational waves and puts the quadrupole formula, which is used for estimation of expected amplitudes of gravitational waves, on a conceptually reliable footing. The analysis also provides analytical checks on numerical simulations involving strong field regimes (eg merger and ring down phases), for instance, by computing the news function for correlating energy loss with back reaction on source motions.

Finally, the observation of orbital decay of the Hulse-Taylor binary pulsar has established that the decay is fully consistent with the energy radiated as per the quadrupole formula. This constitutes an evidence for gravitational radiation *and* also empirically shows that binary systems, even though only under gravitational forces, *can* be a source of gravitational radiation.

10.2 Observational issues

Unlike the indirect conformation of gravitational waves, which is based on the loss of energy due to gravitational radiation, the approach for direct detection of gravitational waves is based on the *time varying tidal effects* caused by a passing gravitational wave. Consequently, the primary issues are: (i) the types of sources together with the characteristic amplitude and time dependence of their signal and (ii) the choice of test body/detector system and their *sensitivity parameter*. The secondary issues involve the identification of a signal and an estimate of expected detection rate. We will briefly describe these aspects. For a recent review and an excellent textbook please see [28, 27].

Sources: As seen in the section (5.3), any matter distribution which is at least quadrupolar and has an accelerated motion is a potential source of generation of gravitational waves. Cataclysmic short duration events such as a supernova and other gravitational collapse produce a *burst* signal while long duration binary systems of compact objects produce *periodic* signals during their in-spiral phase. There are many individually sub-detection level sources which could produce a stochastic background. A cosmic gravitational background is also expected from the very early universe.

Amplitude and frequency estimates: These are based on the quadrupole formula, $h^{TT}_{ij}(t) = \frac{G}{c^6}\frac{2}{r}\frac{d^2}{dt^2}\int_{source}\rho x_i x_j$ which gives the amplitude *at the detector* when the source is at a distance r. Here $\rho = T_{00}$ is the energy density and we have restored the factors of G, c. The amplitude h_{ij} is *dimensionless*. There are three parameters associated with a localised (as distinct from a stochastic background) source - *mass* M of the source, a *length scale,* L associated with the quadrupole and a *time scale,* T characteristic of the time variation. Hence on dimensional grounds, we can write for a typical component of the amplitude,

$$h \sim \frac{G}{c^4}\frac{ML^2T^{-2}}{r} \sim 10^{-44}\frac{ML^2T^{-2}}{r}$$

Here r is the distance to the source and we have used the mks units. The M, L, T are not always the total mass or the 'radius' or a period but are some fractions of these. Such geometry dependent numerical factors are absorbed in the M, L, T parameters. For short duration sources such as gravitational collapse, it is easier to estimate the total energy released E, the frequency of the gravitational waves, f, the duration over which the source is observed, T. Then using $\dot{h} \sim hf$ and average power $\sim E/T$, leads to an estimate of the amplitude as, $h \sim \frac{1}{\pi r f}\sqrt{\frac{E}{T}}\sqrt{\frac{G}{c^3}} \sim \frac{10^{-18}}{\pi r f}\sqrt{\frac{E}{T}}$ in MKS units [28].

The supernovae sources are at distances in the Kpc $\sim 10^{19}m$ (in our galaxy) to Mpc (in other galaxies) range. Their estimated event rate is quite low - roughly once in fifty years or so for a Milky Way type galaxy. The properties of supernovae with regards to frequency of the gravitational waves and the energy carried by them, is estimated from simulation which indicate the typical numbers to be $\frac{E}{c^2} \sim 10^{-7}M_\odot \sim 10^{23}$ Kg, $f \sim$ kHz, $T \sim$ millisecond. This leads to an amplitude of about $h \sim 10^{-21}$.

For isolated pulsars as well as binaries of stars and stellar mass black holes, the distance is again in the 10 Kpc - Mpc range. For pulsars, the effective mass parameter would be about $10^{-3}M_\odot \sim 10^{27}kg$, $L \sim 10^4 m$ and $T \sim 10^{-3}s$ leading to $h \sim 10^{-23}$. For long duration, sources of periodic signal, the effective amplitude is actually larger thanks to matched filtering method of signal extraction. If n is the number of cycles of the signal contained in the observation period of T, then the effective amplitude is $h_{eff} \sim \sqrt{n}h$. For a signal of frequency f, observed for time T, the number of cycles is $n = fT$.

The most promising and studied candidates are binary systems. For a mass M spherical object, the radius of last stable circular orbit is about 3 times the Schwarzschild radius. For binaries made up of neutron stars or black holes, the binaries could be quite tight with L closer to the radius of the last stable circular orbit. These are called *coalescing binaries.* For binaries involving white dwarfs or normal stars, the L would be quite large and are called *in-spiraling binaries.* We can eliminate the binary radius by the angular frequency using Kepler's law to get $h \sim \frac{10^{-55}}{r}M^{5/3}\Omega^{2/3}$. For $M \sim M_\odot$ and $\Omega \sim 10^{-4}$, we get $h \sim 10^{-28}$. For the last stable orbit, the angular frequency for a solar mass object would be about $\Omega \sim 10^4$ leading to $h \sim 10^{-22}$. Although the amplitude for white dwarf binaries is quite small, they are nearer and amenable to enhancement through matched filtering. Coalescing binaries of super-massive black holes too are candidates at frequencies of the order of mHz. For coalescing binaries, there is also the possibility of merger into a black hole which then *rings down* to its stationary state. These ringing frequencies, called quasi-normal modes, are characteristic of the black hole parameters. The amplitudes during this phase turn out to be sizable and vary between about $10^{-21} - 10^{-17}$ even over several hundred mega-parsec distances.

Finally, there are the stochastic gravitational waves which are made up of incoherent superposition of a large number of sources as well an expected *isotropic* component as a relic from the very early universe. Here the study is not by measuring an amplitude, but rather by studying the frequency spectrum of the gravitational energy density or more precisely the quantity, $\Omega_{gw}(f) := \rho_c^{-1} \frac{d\rho_{gw}}{dlog f}$ where $\rho_c := \frac{3c^2 H_0^2}{8\pi G}$ is the cosmological critical energy density. The expected frequencies range over $10^{-18} - 10^{9}$ Hz [27].

The upshot is that the expected amplitude or effective amplitude from various sources is about $h \sim 10^{-21}$ or smaller while the frequencies vary between mHz to kHz for individual sources but over a vast range for stochastic background.

Detection methodology: Since the detection method is based on tidal distortions of bodies, the earliest method proposed by Weber, was to use a *Resonant bar*. The idea is that a strain produced in a system will make the system vibrate with its fundamental frequency. For an aluminium cylinder of length ~ 3 meters and mass of 1000 kg has its resonant frequency in the range of 500 - 1500 Hz. The amplitude of this vibration will be set by the gravitational wave to be $\sim 10^{-21} \times 10^{3}$ meters. This is very tiny and is smaller than or comparable to three main sources of triggers - thermal excitations, noise in the amplification process and the quantum uncertainty. Even at low temperatures of tenth of a Kelvin, the rms amplitude of thermal fluctuations is about 6×10^{-18} meters. With a very narrow bandwidth around the fundamental frequency (Q factor of $\sim 10^{6}$), it is possible to have the duration of the signal to be short (10^{-3} sec.) enough so that the noise amplitude reaches only about a thousandth of its rms value, thereby permitting a signal detection of $h \sim 10^{-20}$. The noise in the amplification process can also be managed for lower frequencies $\sim 10^{2}$ Hz. The quantum mechanical zero point fluctuation $\sim \sqrt{\hbar/(2M\omega)} \sim 10^{-21}$ meters. So as thermal noise is reduced, the quantum noise begins to challenge. Squeezing of uncertainty in a different observational procedure is a possible option. Apart from the standard bar configuration, spherical resonant bodies have also been used which can have more mass in a smaller volume and also have sensitivity in all directions.

Another type of detector uses light beams between a transmitter and a receiver at different locations and attempts to detect the slight fluctuations in the arrival rates due to the distortion in the *physical path length*[1]. A passing gravitational wave causes the proper length traversed by the light beam to change and hence its arrival time. The rate of light pulses received therefore changes from the rate of emitted pulses. Measuring this rate gives a detection of a gravitational wave [28, 88].

Clearly, this depends on availability of very precise time-stamping. The best clocks with stability of few parts in 10^{16} can detect an amplitude of about 10^{-15}. Pulsars

[1] The form of the gravitational wave is the simplest in the TT-gauge. This gauge corresponds to a freely falling coordinate frame with the coordinate time being the proper time of the freely falling observer. In this gauge, the spatial coordinates of a particles initially at rest, do not change as can be seen from the geodesic equation. The physical lengths however do change [27].

themselves are comparably stable and hence can be used for time-stamping for the emitted pulses. Simultaneous observations of several pulsars over long periods can detect very low frequency ($\sim$ nano Hertz) gravitational waves.

Essentially the same logic holds in interferometric detectors whose two arms change their physical lengths by different amounts producing interference fringes. Many collaborative efforts are built around Michelson-Morley type interferometer using lasers. Since the expected highest frequency is about kHz, the wavelengths are larger than 300 kms. It is impossible to build an interferometer with comparable arm length. The arms of the current earth based interferometers are in the range of 300 meters (TAMA) and 600 meters (GEO) meters to 4 kilometers (LIGO). It is possible to effectively increase the arm length a hundred fold by making the laser beam make a 100 traversals in a Fabry-Parot cavity before producing fringes. A longer arm length has the advantage that length determination needs to be within about 10^{-16} meters which is smaller than the size of a nucleus! No mirror can be ground to this degree of smoothness. Here the fact that the laser beam has a width means that individual rays reflect from different irregularities on the mirror surface. Averaging the lengths over the beam cross-section, measures the coherent movement and tiny changes in these averages averages can be determined. Being fixed to the earth, there are many sources of noise eg fluctuations in the gravitational field due to seismic shifts and other movements of mass. These are controlled with suspensions and filtered out selecting frequency windows. As mentioned above for the resonant bars, thermal noise is reduced by keeping the mirrors at cryogenic temperatures or by choosing material for suspension fibres. The quantum noise due to Poisson statistics obeyed by the laser photons, called the *photon shot noise* is a limiting noise which is sought to be minimised by using squeezing. For an extensive discussion of possible noise sources and their control or avoidance, please see [27].

Suffice it to say that extracting an unambiguous signal of gravitational waves from some astronomical source from a variety of noises larger than the signal, is a daunting task requiring sophisticated data analysis techniques as well as a 'bank of templets' of expected waveforms for use of matched filtering. The requirements are being met and there is talk of gravitational wave astronomy using data from multiple detectors.