metric compatible derivative on Δ *induced* from ∇ on the space-time. Thus, only *some* components of the induced connection are required to be time independent. It follows that $\mathcal{L}_\ell \omega_a = 0$. Thus follows the *zeroth law of mechanics* for all WIH.

A WIH with vanishing surface gravity is said to be *extremal*.

- *Isolated Horizon,* $(\Delta, [\ell], \mathcal{D})$ (IH): requires further that *all* components of the induced connection be time independent i.e. in the definition of WIH, replace ℓ^b by an arbitrary V^b tangential to Δ. Every WIH is not necessarily an IH and generically, if a WIH admits IH structure, it is unique.

  Symmetries of a IH are determined by isometries of the induced metric on cross-sections of Δ. If it has rotational isometry i.e. there exist a Killing vector ϕ^a on Δ, the *angular momentum* of a WIH is defined to be $J_\Delta(\phi) := -\frac{1}{8\pi}\int \phi^a \omega_a \, \epsilon^{(2)}$. If there are matter gauge fields, there are further contribution to the horizon angular momentum. The above expression may thus be termed *purely geometrical* (or 'bare') angular momentum of Δ.

  Consider now a definition for mass of an IH. Every constant linear combination of ℓ and ϕ is an isometry of Δ and we may associate a horizon energy with such a Killing vector, $t^a(B, \Omega) := B\ell^a - \ \Omega\phi^a$, where B and Ω are constants. This is explored conveniently in the *covariant phase space formalism* i.e. employing the (pre-)symplectic structure on the space of solutions of the field equations. Every vector field on the space-time manifold, induces a vector field on the covariant phase space and this is required to be a Hamiltonian vector field in order to be able to define a *function* on the phase space. Not every vector field $t^a(B, \Omega)$ induces a Hamiltonian vector field on the phase space. Explicit computation shows that the surface gravity, $\kappa(B, \ell) := B\ell^a\omega_a$, and the angular velocity parameter Ω, must be functions of the two quantities defined on Δ, namely, the horizon area a_Δ and the horizon angular momentum J_Δ, satisfying,

$$\frac{\partial \kappa(B, \ell)}{\partial J_\Delta} \ = \ 8\pi G \frac{\partial \Omega}{\partial a_\Delta}$$

  This then implies that $\frac{\kappa(B,\ell)}{8\pi G}\delta a_\Delta + \Omega \delta J_\Delta = \delta E_\Delta(B, \Omega, \ell)$ which is the statement of the *first law of mechanics of (weakly) Isolated Horizon*.

  As in the case of the dynamical horizons, there are infinitely many first laws. Thanks to the uniqueness theorems for (electro-)vacuum black holes, we have a unique dependence of the surface gravity on area and angular momentum, namely that obtained in the Kerr-Newman solution. Choosing $\kappa(a_\Delta, J_\Delta)$ to be this function, the above integrability conditions can be solved to give the angular velocity and the E_Δ for the isolated horizons.

Finally, imagine that a dynamical horizon 'relaxes' to an isolated horizon so that H and Δ are 'joined' at some boundary. It turns out that the two notions of energy and angular momentum defined on H and Δ agree at the boundary.

This completes our basic summary of the quasi-local generalizations of black holes. There are many other interesting aspects of these and several delicate points which should be seen in the references [70, 68, 69, 66, 67].