vacuum (and usually disappear again) can get separated by the horizon and thus cannot recombine. The left over particle can be thought of as constituting black hole radiation. He in fact demonstrated that a black hole indeed radiates with the radiation having a black body distribution at a temperature given by $k_B T = \frac{\hbar\kappa}{2\pi}$! This provides the proportionality factor between surface gravity and temperature. Consequently, the entropy is identified as $S = \frac{k_B}{\hbar}\frac{A}{4}$. How much is this temperature? Restoring all dimensional constants the expression is [17]:

$$
\begin{aligned}
T &= \frac{\hbar c^3}{8\pi G k_B M_\odot}\left(\frac{M_\odot}{M}\right) \quad {}^0K \\
&= 6 \times 10^{-8}\left(\frac{M_\odot}{M}\right)
\end{aligned}
\tag{8.9}
$$

Notice that heavier black hole is cooler, so as it radiates it gets hotter and radiates stronger in a run-away process. A rough estimate of total evaporation time is about $10^{71}(M/M_\odot)^3$. The end point of evaporation is however controversial because the semi-classical method used in computations cannot be trusted in that regime. This is also the cause of the tension between general relativity which allows for black hole horizons and quantum theory which suggests evaporation thereby raising the possibility of pure quantum state evolving into a thermal density matrix - the *information loss problem*.

If the thermodynamic analogy is true, the statistical mechanics cannot be far behind and one way to ascribe micro-states to black hole horizons is to look for a quantum theory of gravity. A simple way to see that entropy *can be* proportional to the area is to use the Wheeler's 'it from bit' picture. Divide up the area in small area elements of size about the Planck area ($\ell_p^2 \sim 10^{-66} cm^2$). The number of such cells is $n \sim A/(\ell_p^2)$. Assume there is spin-like variable in each cell that can exist in two states. The total number of possible such states on the horizon is then 2^n. So its logarithm, which is just the entropy, is clearly proportional to the area. Of course same calculation can be done for volume as well to get entropy proportional to volume. What the picture shows is that the entropy being proportional to the area is suggestive of associating *finitely many* states to an *elementary area* of a black hole.

There are very many ways in which one obtains the Bekenstein-Hawking entropy formula. Needless to say, it requires making theories about quantum states of a black hole (horizon). Consequently everybody attempting any theory of quantum gravity wants to verify the formula. Indeed in the non-perturbative quantum geometry approach the Bekenstein-Hawking formula has been derived using the 'isolated horizon' framework (modulo the value of the 'Barbero-Immirzi' parameter being chosen for one black hole), for the so-called non-rotating horizons. String theorists too have reproduced the formula although only for black holes near extremality.

Recall that extremal solutions are those which have $r_+ = r_-$ which implies that the surface gravity vanishes. For more general black holes this is taken to be the definition of extremality. For un-charged, rotating extremal black holes $M = |a|$ while for charged, non-rotating ones $M = |Q|$. Since vanishing surface gravity corresponds to vanishing temperature one looks for the third law analogy. It has been shown that the version of third law, which asserts that it is impossible to reach zero temperature in finitely many steps, is verified for the black holes -it is impossible to push a black hole to extremality (say by throwing suitably charged particles) in finitely many steps. There is however another version of the third law that asserts that the entropy vanishes as temperature vanishes. This version is *not* valid for black holes since extremal black holes have zero temperature but finite area.

Black holes which began as peculiar solutions of Einstein equations have revealed an arena where general relativity, statistical mechanics and quantum theory are all called in for an understanding.

8.4 Quasi-local definitions of horizons

The various results on black hole mechanics/thermodynamics used event horizon as the *definition* of the black hole. This is unsatisfactory for two reasons. The event horizon definition refers to infinity *and* also needs the entire space-time to be known to identify it. At any spatial slice, an observer would not know if he/she is being engulfed by a surface which will be part of the event horizon! It is much more desirable, both from a conceptual and a practical angle eg in a numerical evolution, to characterise a black hole in a more local manner. Indeed such a characterization of black holes is available. For the stationary black holes it is captured by the notion of an *isolated horizon* [66, 67] while for a evolving black hole, it is captured by the notion of a *dynamical horizon* [68, 69]. Both these notions arose from the notion of *trapping horizons* [70] which are generalizations of the *apparent horizon* [18, 17]. Let us get a glimpse of these and note important results.

We have already defined trapped surfaces (6.5.1) as two dimensional, space-like sub-manifolds such that the expansions of both the orthogonally in-going and out-going null geodesics is negative. These played a role in establishing singularity theorems (6.5.5, 6.5.6). These surfaces are also related to the event horizon in a *strongly asymptotically predictable* space-times with $R_{\mu\nu}k^\mu k^\nu \geq 0 \; \forall \; k \cdot k = 0$, namely, any marginally trapped surface is contained in the black hole region B. This property also extends to certain three dimensional space-like submanifolds [17].

Let Σ be a any asymptotically flat Cauchy surface for $\tilde{V}$ - the region of the unphysical space-time $(\tilde{M}, \tilde{g})$ which is globally hyperbolic - containing the spatial infinity and being space-like there. Let C be a closed, three dimensional submanifold of $\Sigma \cap M$, with its two dimensional boundary $\dot{C}$. If the out-going null geodesics orthogonal to $\dot{C}$ have their expansion non-negative, then $\dot{C}$ is called *outer marginally trapped surface* $(\theta \leq 0)$