range of values, we have a *Reissner-Nordstrom Black Hole*. For $M^2 = Q^2$, it is known as an *extremal* black hole while for $M^2 < Q^2$ ($r_\pm$ is complex), one has what is known as a *naked singularity*. As before, the Riemann curvature components blow up *only* as $r \to 0$ and since there is no one way surface cutting it off from the region of large r, it is called a naked singularity. We will concentrate on the black hole case.

A kruskal like extension is carried out in a similar manner. The tortoise coordinate r_* is now given by,

$$r_*(r) \;=\; r + \frac{r_+^2}{r_+ - r_-}\ell n\left|\frac{r - r_+}{r_+}\right| - \frac{r_-^2}{r_+ - r_-}\ell n\left|\frac{r - r_-}{r_-}\right| \; . \tag{5.109}$$

There are now *three* regions to be considered:

$$\begin{array}{lllll} A & : & 0 < r < r_- & \leftrightarrow & 0 < r_* < \infty \quad \text{(Stationary)} \\ B & : & r_- < r < r_+ & \leftrightarrow & -\infty < r_* < \infty \quad \text{(Homogeneous)} \\ C & : & r_+ < r < \infty & \leftrightarrow & -\infty < r_* < \infty \quad \text{(Stationary)} \end{array}$$

The Kruskal-like coordinates, U, V are to be defined in each of these regions such that the metric has the same form and then “join” them at the chart boundaries $r_\pm$. The corresponding *Penrose Diagram*, can be obtained by specializing the figure 8.1 and is discussed in section (8.1).

Not for Circulation

5.4.2 The Stationary (non-static) Black Holes

The Kerr-Newman Black Holes

It turns out that for the Einstein-Maxwell system, the most general stationary black hole solution – the Kerr-Newman family – is characterized by just *three* parameters: mass, M, angular momentum, J and charge, Q. For $J = 0$ one has spherically symmetric (static) two parameter family of solutions known as the *Reissner-Nordstrom* solution. The $J \neq 0$ solution is axisymmetric and non-static. This result goes under the title of ‘uniqueness theorems’ and is also referred to as *black holes have no hair*. The significance of this result is that even if a black hole is produced by any complicated, non- symmetric collapse it settles to one of these solutions. All memory of the collapse is radiated away. This happens *only* for black holes!

The black hole Kerr-Newman space-time can be expressed by the following line element [17, 29]:

$$ds^2 \;=\; -\frac{\eta^2\Delta}{\Sigma^2}dt^2 + \frac{\Sigma^2 sin^2\theta}{\eta^2}(d\phi - \omega dt)^2 + \frac{\eta^2}{\Delta}dr^2 + \eta^2 d\theta^2 \qquad \text{where,} \tag{5.110}$$

$$
\begin{array}{lllllll}
\Delta & := & r^2 + a^2 - 2Mr + Q^2 & ; & \Sigma^2 & := & (r^2 + a^2)^2 - a^2 sin^2\theta\Delta \\
\omega & := & \frac{a(2Mr - Q^2)}{\Sigma^2} & ; & \eta^2 & := & r^2 + a^2 cos^2\theta
\end{array}
$$

$$
\begin{array}{lllll}
a = 0 & , & Q = 0 & : & \text{Schwarzschild solution} \\
a = 0 & , & Q \neq 0 & : & \text{Reissner-Nordstrom solution} \\
a \neq 0 & , & Q = 0 & : & \text{Kerr solution}
\end{array}
$$

These solutions have a true curvature singularity when $\eta^2 = 0$ while the coordinate singularities occur when $\Delta = 0$. This has in general two real roots, $r_\pm = M \pm \sqrt{M^2 - a^2 - Q^2}$, provided $M^2 - a^2 - Q^2 \geq 0$. The outer root, r_+ locates the event horizon while the inner root, r_- locates what is called the *Cauchy horizon*. When these two roots coincide, the solution is called an *extremal* black hole.

When $\Delta = 0$ has *no real root*, one has a *naked singularity* instead of a black hole. A simple example would be negative mass Schwarzschild solution. The name *naked* signifies that the true curvature singularity at $\eta^2 = 0$ can be seen from far away. While mathematically such solutions exist, it is generally believed, but not conclusively proved, that in any realistic collapse a physical singularity will always be covered by a horizon. This belief is formulated as the "cosmic censorship conjecture". There are examples of collapse models with both the possibilities. The more interesting and explored possibility is the black hole possibility that we continue to explore.

Not for Circulation

We can compute some quantities associated with an event horizon. For instance, its area is obtained as:

$$
A_{r_+} := \int_{r_+} \sqrt{det(g_{ind})} d\theta d\phi \; = \; \sqrt{\Sigma^2} \int sin\theta d\theta d\phi \; = \; 4\pi(r_+^2 + a^2) \tag{5.111}
$$

For Schwarzschild or Reissner-Nordstrom static space-time we can identify $(-g_{tt} - 1)/2$ with the Newtonian gravitational potential and compute the 'acceleration due to gravity' at the horizon by taking its radial gradient. Thus, *for* $a = 0$,

$$
\text{Surface Gravity, } \kappa \; := \; -\frac{1}{2}\frac{dg_{tt}}{dr}|_{r=r_+} \; = \; \frac{r_+ - M}{r_+^2} \; = \; \frac{r_+ - M}{2Mr_+ - Q^2} \tag{5.112}
$$

Although for rotating black holes 'surface gravity' can not be defined so simply, it turns out that when appropriately defined (see equation (8.8) it is still given by the last equality in the above expression.

There is one more quantity associated with the event horizon of a rotating black hole – the angular velocity of the horizon, Ω. For the rotating black holes we have two Killing vectors: $\xi := \partial_t$ (the Killing vector of stationarity) and $\psi := \partial_\phi$ (the Killing vector of axisymmetry). Their $(norms)^2$'s are given by $g_{tt}, g_{\phi\phi}$ respectively. Both are *space-like*