Title: Hydrogen's challenges to Quantum Theory.

Author: N.D. Hari Dass

Affiliation: TIFR-TCIS, Hyderabad.

Abstract: Of the three routes to quantum theory i.e i. black body radiation, ii. specific heats, and, iii. atomic spectra, it was the last that led to the full development of QM as we understand it today. The spectrum of Hydrogen, the simplest of the elements, played a critical role. Starting with Bohr's model for circular orbits it progressed through the Bohr-Sommerfeld model for generic orbits. Both these gave identical results a striking feature of which was the *accidental degeneracy.* Roots of this degeneracy already existed in the classical Keplerian orbits themselves.

The real breakthroughs came with Heisenberg's Matrix Mechanics and Schrödingers Wave Mechanics. It was straightforward to treat the Hydrogen atom in Schrödinger's approach getting the same result as earlier. But Heisenberg was unable to do this in Matrix Mechanics. It was Pauli who solved this in a most elegant manner exploiting the properties of Runge-Lenz vectors. Shortly later, Dirac treated the problem, also in Matrix Mechanics wherein he developed the action-angle methods for quantum theories. Though he did not complete the problem, he gave methods with which arbitrary central potentials could be handled.

Nearly 25 years after the original breakthroughs, Feynman developed his radically new approach of *Path Integrals.* The impact of this line of thinking was immediate and widespread. However, Feynman could not solve the Hydrogen atom problem this way. This was deeply frustrating to him and he was highly embarassed by this failure. He posed this as a problem to Hagen Kleinert and the two made some attempts which went nowhere. In 1979, 31 years after Feynman's discovery of the path integral method, Duru and Kleinert found a way out. Through a combination of *local rescalings* of time as well as a novel mapping(which had been around for sometime) of 3-d Coulomb problem to a 4-d oscillator problem, called the *Koustanheimo-Stieffel map*, they managed to give a path-integral representation that was doable.

But they did not actually evaluate it. Also, it was in the so called *Phase space representation*, and suffered from a number of flaws. Finally, in 1982, Ho and Inomata succeeded in solving the problem exactly in the *Configuration space representation.* The Hydrogen problem had also been reduced to that of four-dimensional harmonic oscillators albeit with constraints.