CHAOTIC GROUP ACTIONS ON THE RATIONALS

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Dedicated to Professor Peter Zvengrowski on the occasion of his seventieth birthday

Abstract. We characterize groups which can act chaotically on $\mathbb{Q}$. We show that there are $2^{\aleph_0}$ many distinct conjugacy classes of chaotic actions on $\mathbb{Q}$ of certain families of groups.

1. Introduction

The notion of chaotic action of a group on a Hausdorff topological space X was introduced by Cairns et al. [2], generalizing Devaney's notion of chaotic maps in topological dynamics [5]. (See also [1]). It was shown in [2] that, given a group G, there exists a Hausdorff topological space on which G acts chaotically if and only if G is residually finite; in fact the space can be assumed to be compact. Interesting examples of chaotic group actions on manifolds have been constructed by Cairns et al. in [2], [3],[4]. See also [10]. In this paper, we study chaotic group actions on $\mathbb{Q}$, the space of rational numbers, with its usual topology.

Definition (Cf. [10])*Let $\mathcal{S}$ be a collection of subgroups of a group G. We say that $\mathcal{S}$ satisfies condition $\mathcal{C}$ if it is a countable collection of finite index subgroups of G such that $\bigcap_{H\in\mathcal{S}} H = \{1\}$. We say that G satisfies condition $\mathcal{C}$ if G has a collection of subgroups satisfying condition $\mathcal{C}$.*

The definition of chaotic group action will be recalled in §2. We now state the main results of this paper.

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Theorem 1. *Let G be an infinite group. Then G satisfies condition $\mathcal{C}$ if and only if there exists an effective action of G on $\mathbb{Q}$ which is chaotic.*

P. M. Neumann [11, §4] has shown that there are $2^{\aleph_0}$ distinct conjugacy classes of self-homeomorphisms of the rationals which permute the points in a single cycle. Using the construction in the proof of Theorem 1 we establish the following.

Theorem 2. *There are $2^{\aleph_0}$ distinct conjugacy classes of chaotic G actions on $\mathbb{Q}$ where G is one of the following groups: (i) any finitely generated infinite abelian group, (ii) direct sum and direct product of countably many copies of $\mathbb{Z}$, (iii) any finitely generated torsion-free nilpotent group, (iv) free groups of rank at most $2^{\aleph_0}$, and, (v) the groups $F_2 * H$, $F_n \times H, n \geq 2$, where F_n denotes the free group of rank n and H is any group that acts chaotically on $\mathbb{Q}$.*

The following remarkable theorem, whose proof can be found in [11], will be exploited in our proofs.

Sierpiński's Theorem: *If X is a countable metrizable topological space with no isolated points, then X is homeomorphic to $\mathbb{Q}$.*

Theorem 1 subsumes that part of Theorem 3.7 of [10] whose proof was omitted. Theorems 1 and 2 are proved in §2. We study in §3 some closure properties of the class of all groups satisfying condition $\mathcal{C}$.

2. Proof of main results

We begin by recalling the definition of chaotic group action.

Definition 3. [1]. *Let G be a group which acts on a Hausdorff topological space X. We say that the action of G is* chaotic *if the action is effective and following properties hold:*
(i) Topological transitivity: *Given any two non-empty sets $U, V \subset X$ there exists an element $g \in G$ such that $g(U) \bigcap V \neq \emptyset$.*
(ii) Density of finite orbits: *The set of all $x \in X$ whose G-orbit is finite is dense in X.*

For example, the usual action of $\mathrm{SL}(2,\mathbb{Z})$ on the torus $\mathbb{R}^2/\mathbb{Z}^2$ is chaotic.

We now prove Theorem 1.

Proof of Theorem 1: Suppose that G acts chaotically on $\mathbb{Q}$. Let $S \subset \mathbb{Q}$ be the set of all periodic points (i.e., points whose G-orbits are finite). For each $x \in S$, let $G_x \subset G$ denote the isotropy at x. Then $\mathcal{S} := \{G_x \mid x \in S\}$ satisfies condition $\mathcal{C}$.

Conversely, suppose that $\mathcal{S}$ is a countable collection of finite index subgroups of G satisfying condition $\mathcal{C}$. Since any finite index subgroup of G is contained in a finite index *normal* subgroup of G, we assume without loss of generality, that every member of $\mathcal{S}$ is normal in G. Furthermore, by expanding the collection $\mathcal{S}$ if necessary, the cardinality of the resulting collection remains countable and so we may (and do) assume that $\mathcal{S}$ is closed under finite intersections. Write $\mathcal{S} = \{N_i \mid i \in \mathbb{N}\}$.

We begin with the G-space $X := \{0,1\}^G$ considered in [2]. Cairns et al. showed that the usual G-action on X, defined as $(\gamma.f)(x) = f(\gamma^{-1}x), f \in X, \gamma, x \in G$, is chaotic.

Let Q be the space of all $f \in X$ such that f is constant on the N_i-cosets for some $i \geq 1$. Observe that Q is countable. Indeed, denoting by η_i the canonical quotient map $G \longrightarrow G/N_i$, one has $Q = \bigcup_{i\in\mathbb{N}} \eta_i^*(\{0,1\}^{G/N_i})$. Here $\eta_i^*(f) = f \circ \eta_i$ for $f\colon G/N_i \longrightarrow \{0,1\}$. The above expression for Q exhibits Q as a union of finite G-stable sets. Hence Q consists entirely of points whose G-orbits are finite.

Next we establish topological transitivity of the G-action on Q. As observed in [2] it is easily seen that the G-action on X is topologically transitive. So the assertion would follow if we show that Q is dense in X. This will also prove that the action of G on Q is effective. Let U be a basic open set which consists of $f \in X$ with prescribed values at finitely many distinct elements, say, $x_1, \ldots, x_n \in G$. Since $\bigcap_{i\in\mathbb{N}} N_i = \{1\}$, and since $\mathcal{S}$ is closed under finite intersections, there exists a natural number k such that $x_i^{-1}x_j \notin N_k$, for $i \neq j$. Thus $x_1, \ldots, x_n$ belong to distinct N_k-cosets of G. Let $h\colon G/N_k \longrightarrow \{0,1\}$ be any set-map such that $h(x_iN_k) = f(x_i), 1 \leq i \leq n$. Then $\eta_k^*(h) \in Q \cap U$. Thus G acts chaotically on Q.

It remains to show that Q is metrizable. (This is obvious if G is countable, because in this case X is a Cantor space.) Let $S_i \subset G$ be a complete set of pairwise distinct coset representatives for G/N_i. Thus S_i is finite for each i. Now let $S = \bigcup_{i\in\mathbb{N}} S_i$. Let $X_S = \{0,1\}^S$ and let $\pi : X \longrightarrow X_S$ denote the restriction map $f \mapsto f_{|S}$. Note that X_S is metrizable—indeed it is a Cantor space—and that π is an open map. *We claim that $\pi_{|Q}$ is an imbedding.* Metrizability of Q follows from the claim as X_S is metrizable. To establish the claim, first we show that π is one-to-one. Suppose that $\pi(f) = \pi(f'),\ f, f' \in Q$. Thus $f_{|S} = f'_{|S}$. We must show that $f = f'$. Let $f = h \circ \eta_i, f' = h' \circ \eta_j$ for some i, j. Since $\mathcal{S}$ is closed under finite intersections, $N_i \cap N_j = N_k$ for some k. Since S contains S_k and since $f_{|S} = f'_{|S}$, we see that $f_{|S_k} = f'_{|S_k}$. Since f and f' are constant on N_k-orbits, it follows that $f = f'$. Clearly $\pi_{|Q}$ is continuous and, since π is open, $\pi_{|Q}$ is also open. Therefore $\pi_{|Q}\colon Q \longrightarrow \pi(Q)$ is a homeomorphism, establishing the claim.

It is readily observed that there are no isolated points in Q. Thanks to Sierpiński's theorem, we have $Q \cong \mathbb{Q}$ and the proof is complete. □

Remark 4. Let $\mathcal{H} = \{H_j\}$ be a countable collection of subgroups of countably infinite index in G such that no two of them are conjugate in G. The above proof can be modified to allow for infinite G-orbits Gf_j in our model space for $\mathbb{Q}$, where the isotropy at f_j equals H_j. We shall only outline the changes needed to allow in our model space when $\mathcal{H}$ is a singleton $\{H\}$. Let T be a complete set of pairwise distinct left coset representatives for G/H. Replacing the set S in the above proof by $\widetilde{S} := S \cup T$, note that the resulting space $X_{\widetilde{S}}$ is again a Cantor space. Denote by $\chi_H\colon G \longrightarrow \{0,1\}$ the indicator function of $H \subset G$. Then the isotropy at $\chi_H \in X$ equals H. The same proof as above shows that G-action on the space $\widetilde{Q} := Q \cup G\chi_H \subset X$ is chaotic. Again, by Sierpiński's theorem, $\widetilde{Q}$ is homeomorphic to $\mathbb{Q}$. Observe that the G-space $\widetilde{Q}$ has exactly one infinite orbit.

We now turn to proof of Theorem 2.

Proof of Theorem 2. (i) Let P be any non-empty set of primes which contains the finite set F of primes which divide the orders of torsion elements of G. An integer n is called P-primary if all its prime divisors are in P. We keep the notations used in the proof of Theorem 1.

Let $\mathcal{S}_P$ denote the family of all subgroups of G having (finite) P-primary index. Note that $\mathcal{S}_P$ satisfies condition $\mathcal{C}$. Denote the chaotic G-action on $\mathbb{Q}$, obtained as in the proof of Theorem 1, corresponding to $\mathcal{S}_P$ by ϕ_P. It has the property that the cardinality of each orbit is P-primary. Conversely, if n is P-primary, then there exists a subgroup $H \subset G$ of index n. The orbit of the point in $\mathbb{Q} \cong Q \subset X$ corresponding to the indicator function χ_H has cardinality n. In particular, there exists an orbit of cardinality a prime p if and only if $p \in P$. Since the existence of an orbit of a given cardinality depends only on the conjugacy class of ϕ_P, we see that ϕ_P is not conjugate to $\phi_{P'}$ if $P' \neq P$ is another set of primes containing F, completing the proof in this case.

(ii) The proof is similar to (i) above and so we omit the details.

(iii) First let $G = N(n, \mathbb{Z})$ the group of unipotent upper triangular $n \times n$ matrices over $\mathbb{Z}$. For any non-empty set of primes P, the collection $\mathcal{S}_P := \{\Gamma_k \mid k \text{ is } P\text{-primary}\}$, where $\Gamma_k := \ker(G \longrightarrow N(n, \mathbb{Z}/k\mathbb{Z}))$, satisfies condition $\mathcal{C}$. Again, as in the proof of Theorem 1, we obtain a chaotic G-action ϕ_P on $\mathbb{Q}$ corresponding to the collection $\mathcal{S}_P$. As in the proof of (i) we see that, if $P' \neq P$ is another set of prime, then ϕ_P is not conjugate to $\phi_{P'}$.

If G is an arbitrary finitely generated torsion-free nilpotent group, by a theorem of P. Hall [9, Ch. 2 §4.2], it can be imbedded in $N(n, \mathbb{Z})$ for some n. Thus, we may regard G as a subgroup of $N(n, \mathbb{Z})$. Let P be any non-empty set of primes. Intersecting G with the subgroups $\Gamma_k \in \mathcal{S}_P$ of $N(n, \mathbb{Z})$, we get a collection $\mathcal{N}_P$ of subgroups of G which satisfies condition $\mathcal{C}$. The rest of the proof is similar to the case of $N(n, \mathbb{Z})$ and omitted.

(iv) Recall that there exist pairwise non-isomorphic two generator infinite groups $H_\alpha, \alpha \in \mathbb{R}$ (see [8, Ch. 4, §3]). Thus there exist normal subgroups $N_\alpha \subset G, \alpha \in \mathbb{R}$, such that $G/N_\alpha \cong H_\alpha$. Note that there is no automorphism of G which maps N_α onto N_β for $\alpha \neq \beta$.

Now let ϕ be a chaotic G action on $\mathbb{Q}$ consisting only of points having finite G-orbits. By Remark 4, for each $\alpha \in \mathbb{R}$, there exists a chaotic G-action ϕ_α on $\mathbb{Q}$ having exactly one infinite orbit, say Gx_α, with isotropy at $x_\alpha \in \mathbb{Q}$ being equal to N_α. Since $N_\alpha \neq \gamma(N_\beta)$ $\forall \gamma \in \text{Aut}(G)$ for $\alpha \neq \beta$, a straightforward argument shows that the images of the monomorphisms $\phi_\alpha, \phi_\beta : G \longrightarrow \text{Homeo}(\mathbb{Q})$ determine distinct conjugacy classes of subgroups of $\text{Homeo}(\mathbb{Q})$, completing the proof in this case.

(v) Observe that, with notation as in (iv) above, there exist surjections $\eta_\alpha \colon G \longrightarrow H_\alpha, \alpha \in \mathbb{R}$, where $G = F_2 \times H$ or $F_2 * H$. It can be seen that G satisfies property $\mathcal{C}$. We set $N_\alpha := \ker(\eta_\alpha)$ and proceed as in the proof of (iv) above to complete the proof. □

Remark 5. I do not know if, any infinite group G satisfying condition $\mathcal{C}$ admits continuously many chaotic actions on $\mathbb{Q}$ which belong to distinct conjugacy classes. In particular, I could not settle this question when G is the group of p-adic integers, or the group $(\mathbb{Z}/p\mathbb{Z})^\omega$, the direct product of $\aleph_0$ many copies of $\mathbb{Z}/p\mathbb{Z}$ where p is a prime. However, it is clear that arguments used in the proofs can be suitably modified and applied to other examples of groups for which the answer is in the affirmative.

3. Groups satisfying condition $\mathcal{C}$

Denote by $\mathcal{G}$ the class of all groups satisfying condition $\mathcal{C}$. In view of Theorem 1, infinite groups belonging to $\mathcal{G}$ act chaotically on $\mathbb{Q}$. In this section we establish certain closure properties of $\mathcal{G}$. Note that all finite groups are in $\mathcal{G}$ and, if $G \in \mathcal{G}$, so does any subgroup of G.

We begin by establishing the following lemma.

Lemma 6. (i) *A group G belongs to $\mathcal{G}$ if and only if G can be embedded in a direct product of countably many finite groups.*
(ii) *$\mathcal{G}$ contains all countable residually finite groups.*
(iii) *Let $G_n \in \mathcal{G}, n \in \mathbb{N}$. Then the direct product $\prod_{n \geq 1} G_n$ is in $\mathcal{G}$.*
(iv) *$\mathcal{G}$ contains a free group of rank $2^{\aleph_0}$.*

Proof. Statements (i) and (ii) are easy to prove.

(iii) The assertion follows immediately from (i) because if $H := \prod_{i\geq 1} H_i$ where each $H_i = \prod_{j\geq 1} H_{i,j}$ is a countable direct product of finite groups, then $H = \prod_{i,j\geq 1} H_{i,j}$ itself is a direct product of a countable family of finite groups.

(iv) Let F_∞ be a free group of rank $\aleph_0$. Then F_∞ is countable and is residually finite. By (ii) and (iii), it follows that the group F_∞^ω, the countable direct product of a countably infinitely many copies of F_∞, belongs to $\mathcal{G}$. It is known that F_∞^ω contains a free group F_c of rank $c := 2^{\aleph_0}$ (cf. [12]). For the sake of completeness we sketch a proof of this fact in the remark below. Hence F_c is in $\mathcal{G}$. □

Remark 7. (1) We now sketch the proof that F_c imbeds in F_∞^ω. This result seems to be folkloric and is certainly well-known to experts. The proof given here is, as far as I am aware, due to A. Blass. Let $\{x_j \mid j \in \mathbb{N}\}$ be a basis for F_∞. If $\mathbf{a} = (a_n)$ is a sequence of natural numbers, we set $x_\mathbf{a} := (x_{a_n}) \in F_\infty^\omega$. For each real number $\alpha > 1$ in $\mathbb{R}$ let $\mathbf{a}(\alpha)$ be the sequence $(\lfloor 10^n\alpha \rfloor)_{n\geq 1}$. Given any finite collection of real numbers, $\alpha_1, \ldots, \alpha_k$, the n-th terms of $\mathbf{a}(\alpha_1), \ldots, \mathbf{a}(\alpha_k)$ are all distinct for sufficiently large n. For such an integer n, the n-th coordinates of $x_{\mathbf{a}(\alpha_1)}, \ldots, x_{\mathbf{a}(\alpha_k)} \in F_\infty^\omega$ are *pairwise distinct* basis elements of F_∞. We claim that $S = \{x_{\mathbf{a}(\alpha)} \mid \alpha \in \mathbb{R}\} \subset F_\infty^\omega$ generates a subgroup F with basis S. Indeed, if w is any reduced (non-vacuous) word in the $x_{\mathbf{a}(\alpha)}$'s, then there exists an n large such that the composition $F \subset F_\infty^\omega \xrightarrow{\eta_n} F_\infty$, where η_n is the projection onto the n coordinate, sends w to the *same* word in certain *basis* elements x_j's of F_∞. Therefore $w \neq 1$ in F and hence F is free with basis S.
(2) It is clear that assertion (i) in the above proposition can be reformulated as:
(i)' *$G \in \mathcal{G}$ if and only if G is a subgroup of $\prod_{n\geq 2} S_n$ where S_n is the symmetric group on n letters.*
(3) Assertion (ii) yields plenty of examples of groups in $\mathcal{G}$. For example, any finitely generated subgroup of $\mathrm{SL}(n, \mathbb{F})$ for any field $\mathbb{F}$ is in $\mathcal{G}$. A theorem of Baumslag asserts that the automorphism group $\mathrm{Aut}(G)$ of a finitely generated residually finite group G is again residually finite. Since $\mathrm{Aut}(G)$ is also countable we see that $\mathrm{Aut}(G) \in \mathcal{G}$. See [8] for proofs of these statements. There are also interesting examples

arising in geometric topology. The mapping class group of any closed oriented surface [6] and fundamental groups of a large class of compact three dimensional manifolds which includes Haken manifolds [7] are all residually finite and countable hence belong to $\mathcal{G}$.

Another example of a group belonging to $\mathcal{G}$ is the absolute Galois group $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$. Indeed it is the inverse limit of the finite Galois groups $\mathrm{Gal}(K/\mathbb{Q})$ as K varies over (the countable family of) finite Galois extensions of $\mathbb{Q}$.

We shall now establish the following

Proposition 8. (i) *Let $G_n \in \mathcal{G}, n \in \mathbb{N}$. Then the free product $G := *_{n\geq 1} G_n$ is in $\mathcal{G}$.*
(ii) *If G is in $\mathcal{G}$ then so is the free product of $*_{\alpha\in\mathbb{R}} G_\alpha$ where each $G_\alpha \cong G$.*

Proof. (i) Denote by Γ_n the kernel of the canonical retraction $\eta_n \colon G \longrightarrow H_n := G_1 * \cdots * G_n$. Then $\bigcap_{n\in\mathbb{N}} \Gamma_n$ is trivial. We shall show that each H_n is in $\mathcal{G}$. This implies that G itself is in $\mathcal{G}$. To see this, let $\mathcal{S}_n$ be a collection of finite index subgroups of H_n satisfying condition $\mathcal{C}$. Set $\mathcal{S} = \bigcup_{n\geq 1}\{\eta_n^{-1}(N) \mid N \in \mathcal{S}_n\}$. Then each subgroup K in $\mathcal{S}$ has finite index in G and furthermore, $\bigcap_{K\in\mathcal{S}} K = \bigcap_{n\geq 1} \eta_n^{-1}(\bigcap_{N\in\mathcal{S}_n} N) = \bigcap_{n\geq 1} \Gamma_n = \{1\}$. Thus $\mathcal{S}$ satisfies condition $\mathcal{C}$ and so $G \in \mathcal{G}$.

It remains to show that $H_n = G_1 * \cdots * G_n$ is in $\mathcal{G}$ for each n. Let $\mathcal{N}_j = \{N_i^j \mid i \geq 1\}$ be a collection of finite index normal subgroups of G_j satisfying condition $\mathcal{C}$ for each $1 \leq j \leq n$. For $\mathbf{i} = (i_1, \ldots, i_n) \in \mathbb{N}^n$, denote by $H_{n,\mathbf{i}}$ the group $G_1/N_{i_1}^1 * \cdots * G_n/N_{i_n}^n$ and by $\eta_{n,\mathbf{i}} \colon H_n \longrightarrow H_{n,\mathbf{i}}$ the canonical quotient map. Let $\Gamma_{n,\mathbf{i}}$ be the kernel of $\eta_{n,\mathbf{i}}$. It is clear that $\bigcap_{\mathbf{i}\in\mathbb{N}^n} \Gamma_{n,\mathbf{i}} = \{1\}$.

By Lemma 6(ii), $H_{n,\mathbf{i}} \in \mathcal{G}$. Let $\mathcal{S}_\mathbf{i}$ be a collection of subgroups of $H_{n,\mathbf{i}}$ satisfying condition $\mathcal{C}$. Set $\mathcal{S}_n := \{\eta_{n,\mathbf{i}}^{-1}(N) \mid N \in \mathcal{S}_\mathbf{i}, \mathbf{i} \in \mathbb{N}^n\}$. Then $\bigcap_{K\in\mathcal{S}_n} K = \bigcap_{\mathbf{i}\in\mathbb{N}^n} \bigcap_{N\in\mathcal{S}_\mathbf{i}} \eta_{n,\mathbf{i}}^{-1}(N) = \bigcap_{\mathbf{i}\in\mathbb{N}} \eta_{n,\mathbf{i}}^{-1}(\bigcap_{N\in\mathcal{S}_\mathbf{i}} N) = \bigcap_{\mathbf{i}\in\mathbb{N}^n} \Gamma_{n,\mathbf{i}} = \{1\}$. Thus $\mathcal{S}_n$ satisfies condition $\mathcal{C}$ and so $H_n \in \mathcal{G}$.

(ii) Since $F_c \in \mathcal{G}$ by Lemma 6(iv), we see that $G * F_c \in \mathcal{G}$ for any $G \in \mathcal{G}$. Choose a basis $x_\alpha, \alpha \in \mathbb{R}$, for F_c and set $G_\alpha := x_\alpha G x_\alpha^{-1} \cong G$.

Then the subgroup of $G * F_c$ generated by $G_\alpha, \alpha \in \mathbb{R}$, which is the free product $*_{\alpha\in\mathbb{R}} G_\alpha$, is also in $\mathcal{G}$. □

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