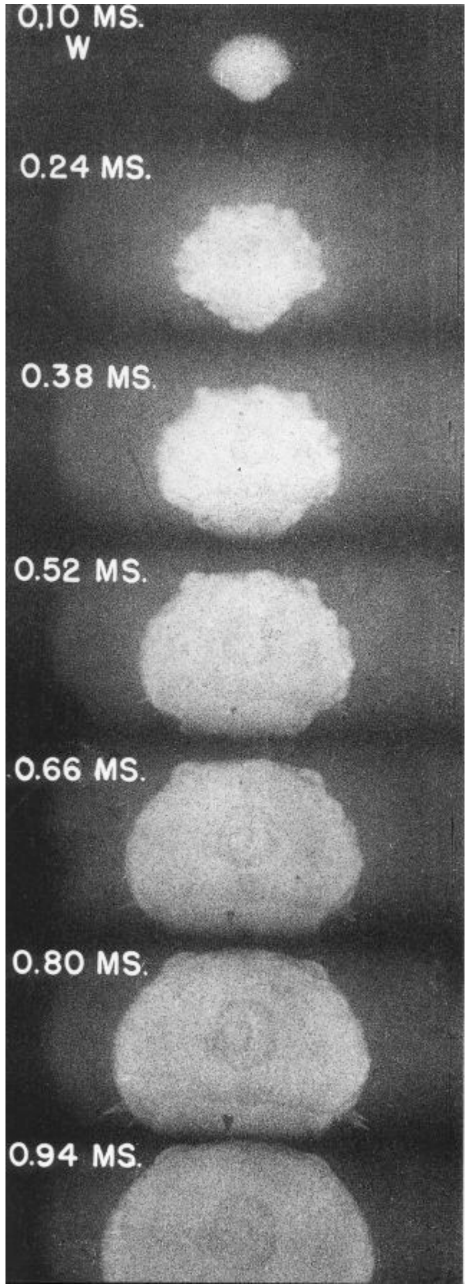
Shock Propagation in Loosely Packed Granular Media

Sudhir N. Pathak(Institute of Mathematical Sciences, Chennai)
Zahera Jabeen (Univ. Michigan, USA)
Purusattam Ray (Institute of Mathematical Sciences, Chennai)
R. Rajesh (Institute of Mathematical Sciences, Chennai)

Nuclear Explosion



How does the radius increase with time?

Dimensional Analysis

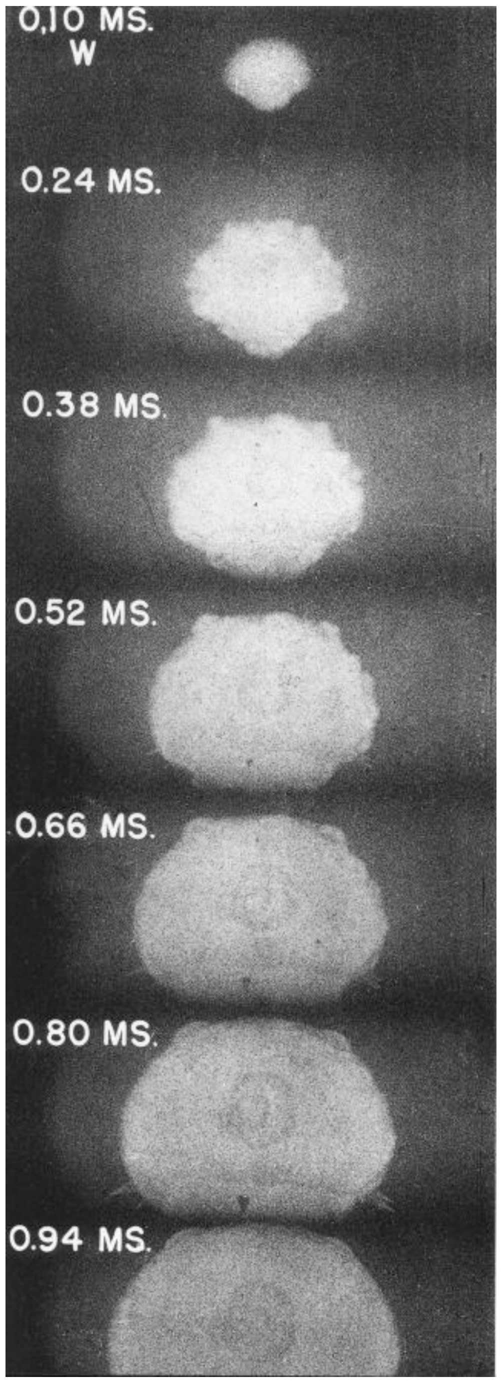
$$R(t) = f(E_0, t, \rho, \cancel{T_0})$$

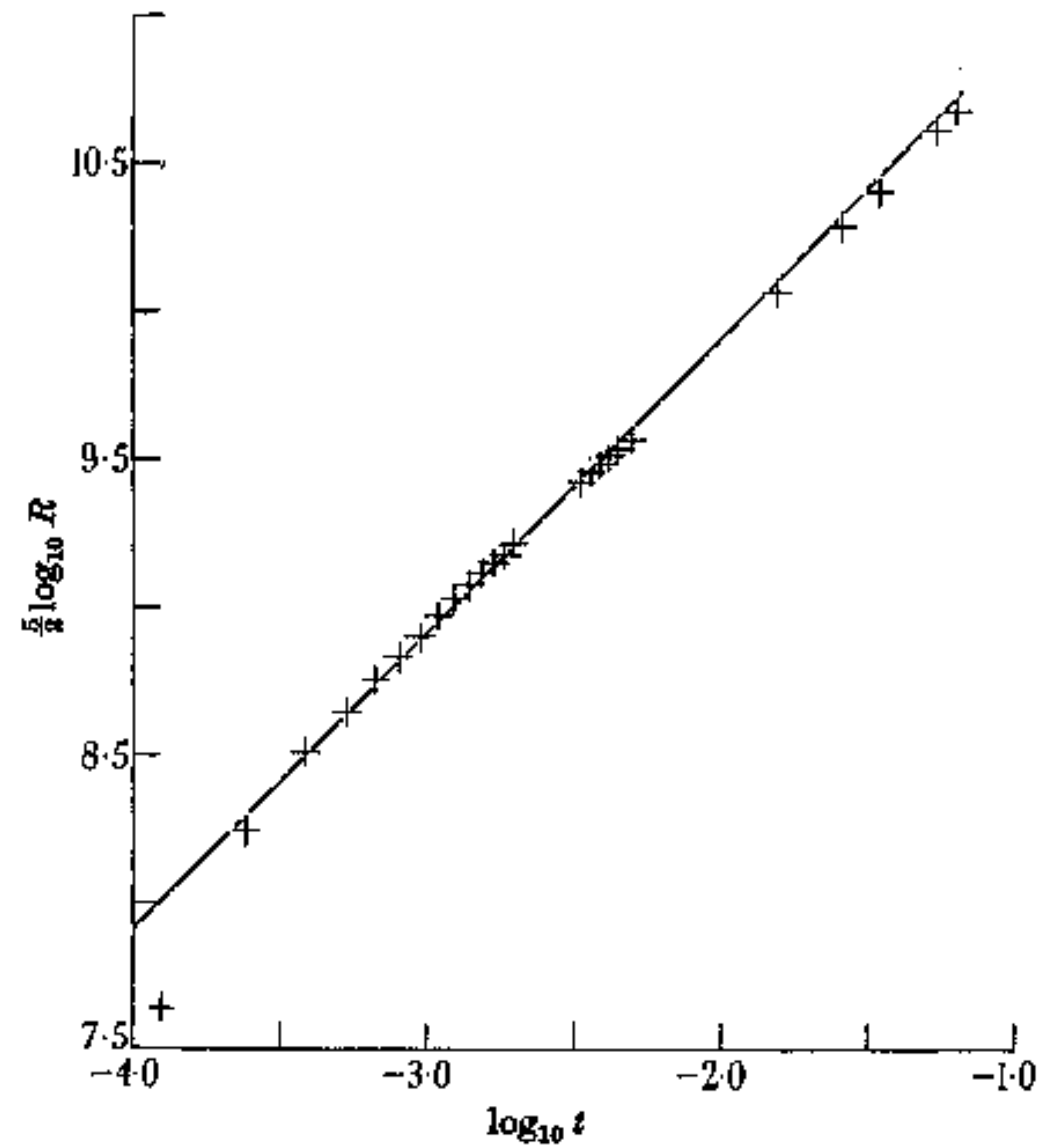
$$\begin{aligned} [E_0] &= ML^2T^{-2} \\ [\rho] &= ML^{-d} \\ [t] &= T \end{aligned}$$

$$\boxed{R(t) = c\left(\frac{E_0 t^2}{\rho}\right)^{\frac{1}{d+2}}}$$

$$d = 3 \implies R(t) \propto t^{2/5}$$

Comparison with data



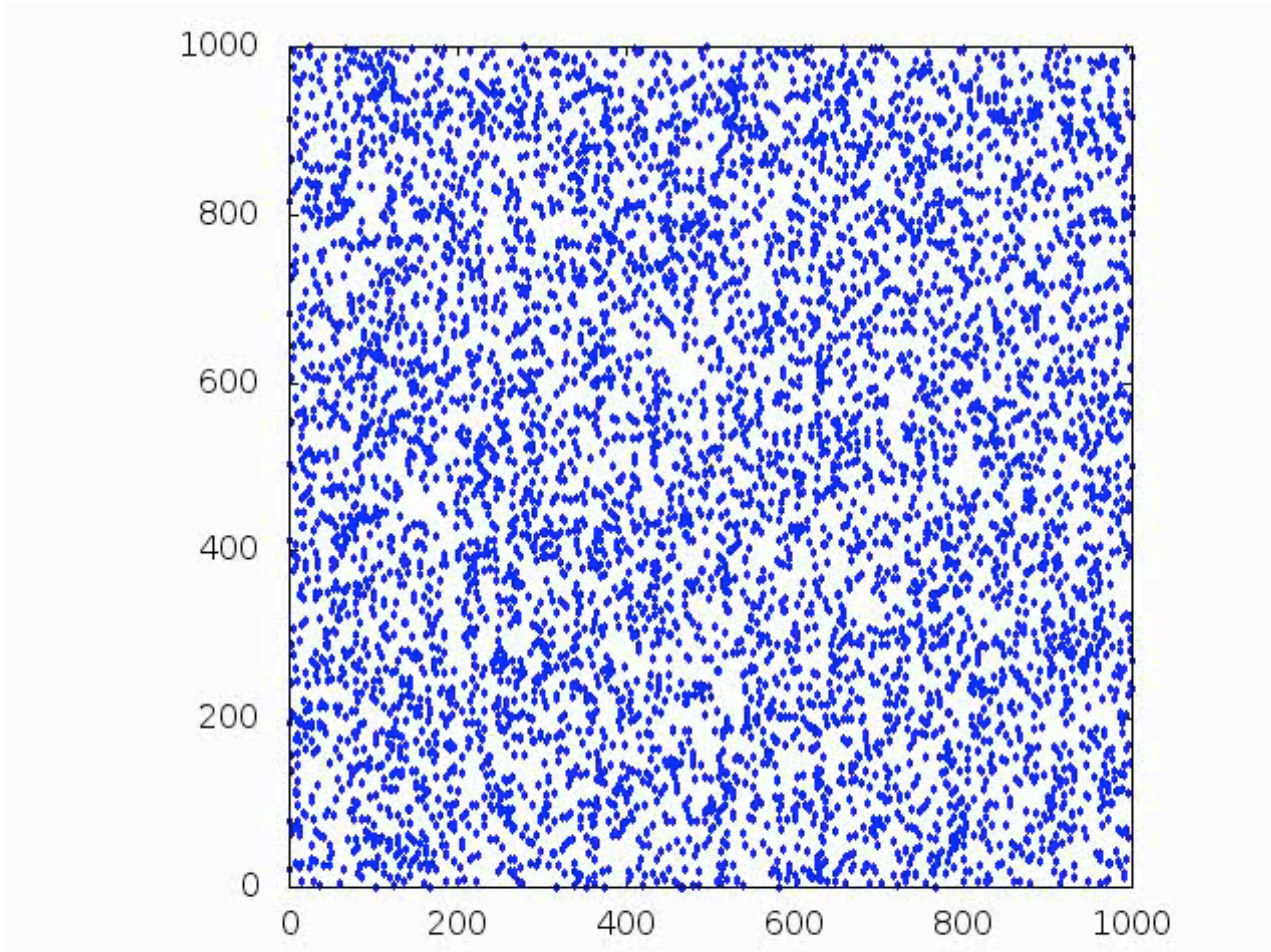


A computer model

- Ideal gas
- Particles at rest
- One particle given an impulse
- Interaction only on contact
  - ⋆ Energy conserving
  - ⋆ Momentum conserving

A computer model

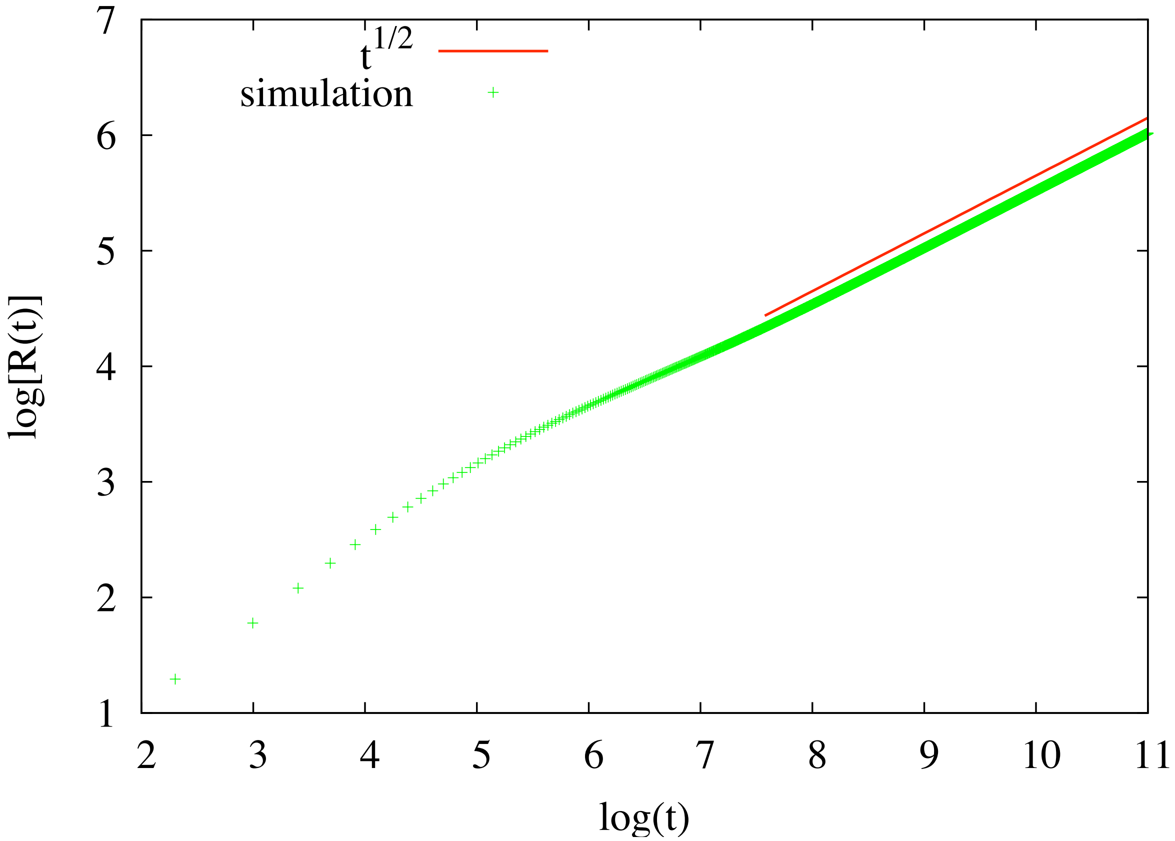
A computer model



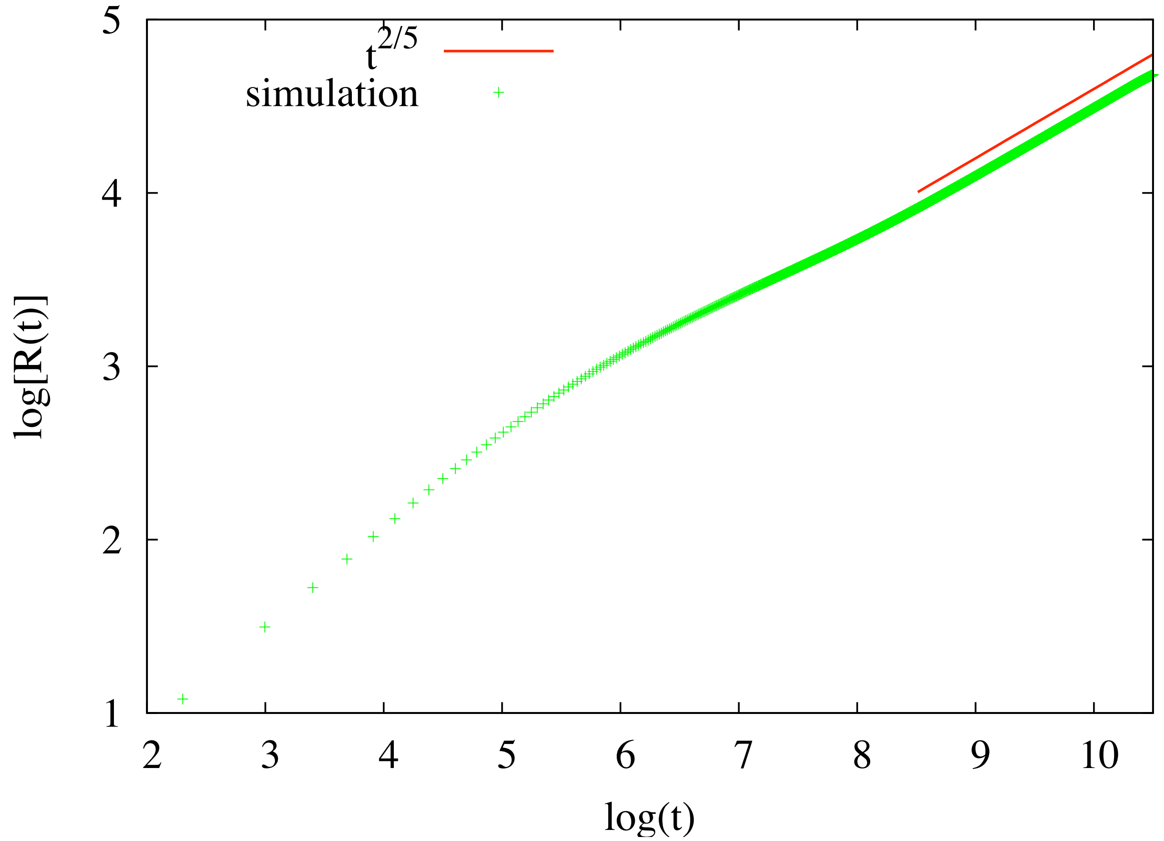
Radius vs time

$$R(t) = c\left(\frac{E_0 t^2}{\rho}\right)^{\frac{1}{d+2}}$$

2 dimensions



3 dimensions



Question

Take the above model and make the collisions inelastic.

How do the results change?

Outline of the talk

- Motivation
- Analysis
- Experiments
- Tweaked models
- Summary

Granular systems

Sand, steel balls, talcum powder

$$\text{Size} \sim 1\mu m \text{ to } 1mm$$

$$\text{Mass} \sim 1\ mg$$

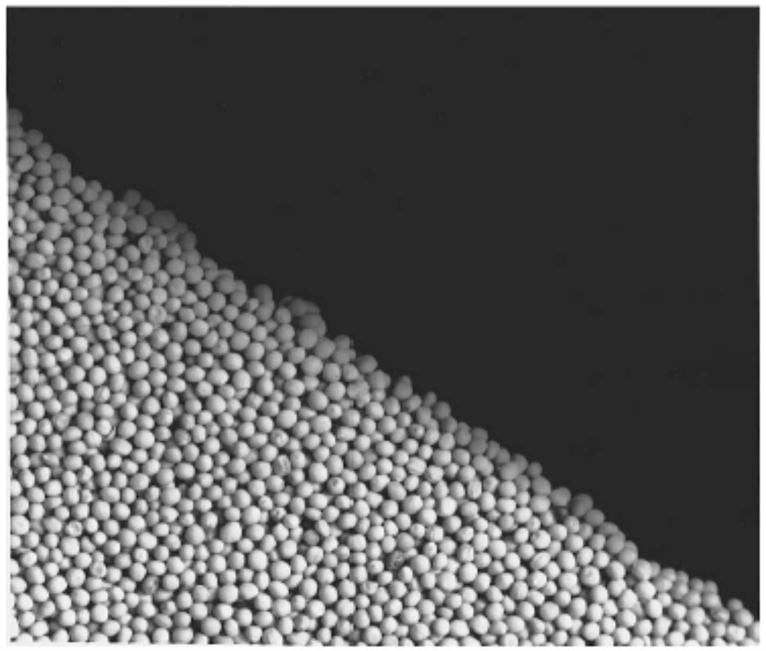
$$\text{Velocity} \sim 1cm/s$$

$$\frac{KE}{kT} = \frac{10^{-6}10^{-4}}{kT} \approx 10^{10}$$

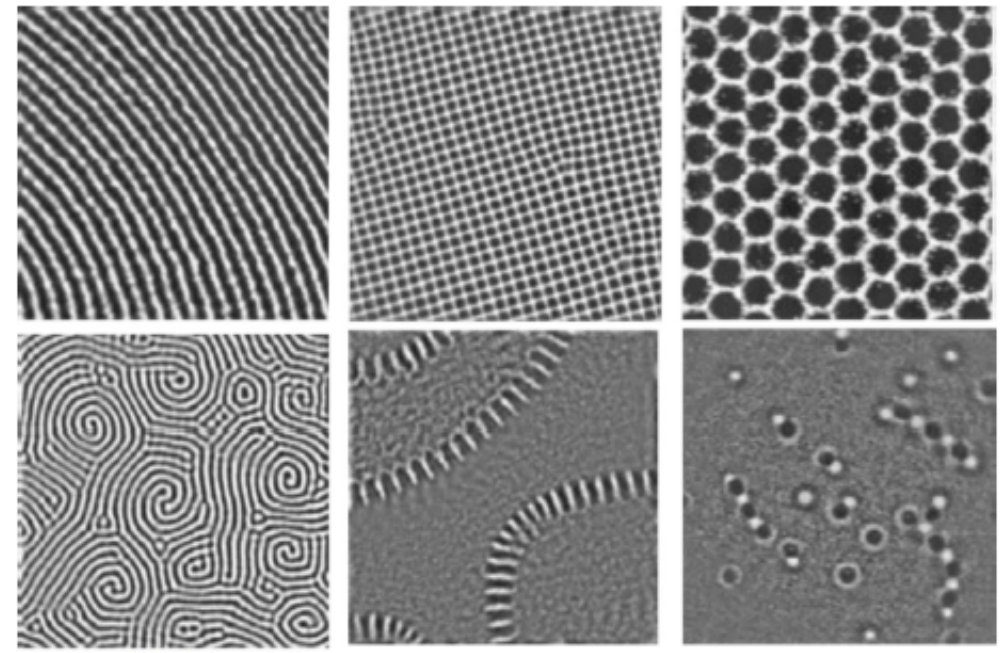
$$\frac{PE}{kT} = \frac{10^{-6}1010^{-2}}{kT} \approx 10^{13}$$

Temperature plays no role

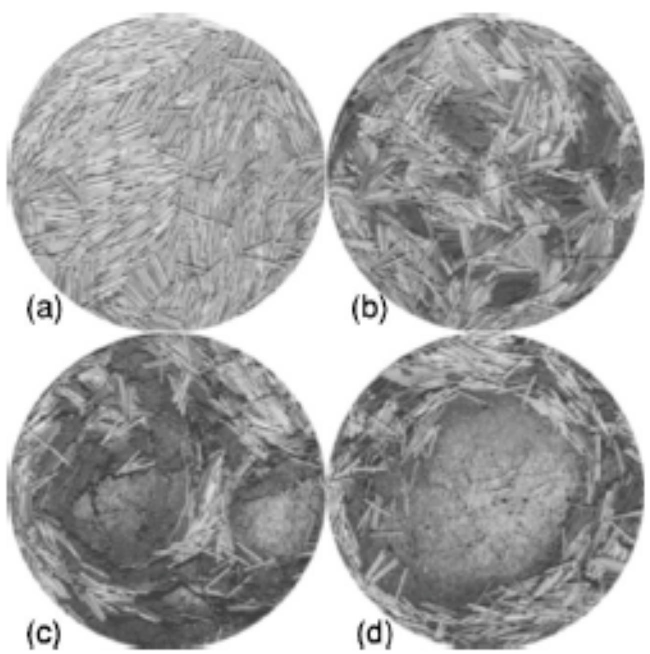
Phenomenology



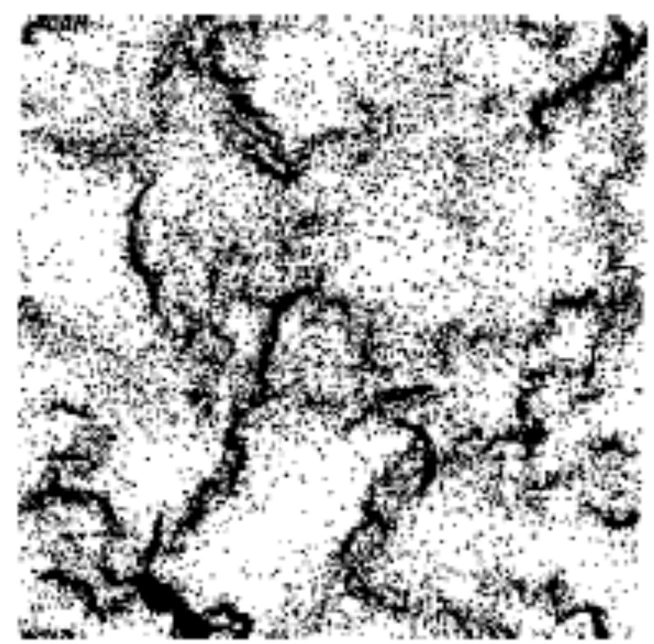
Jaeger et al, 1996



Aranson et al, 2006



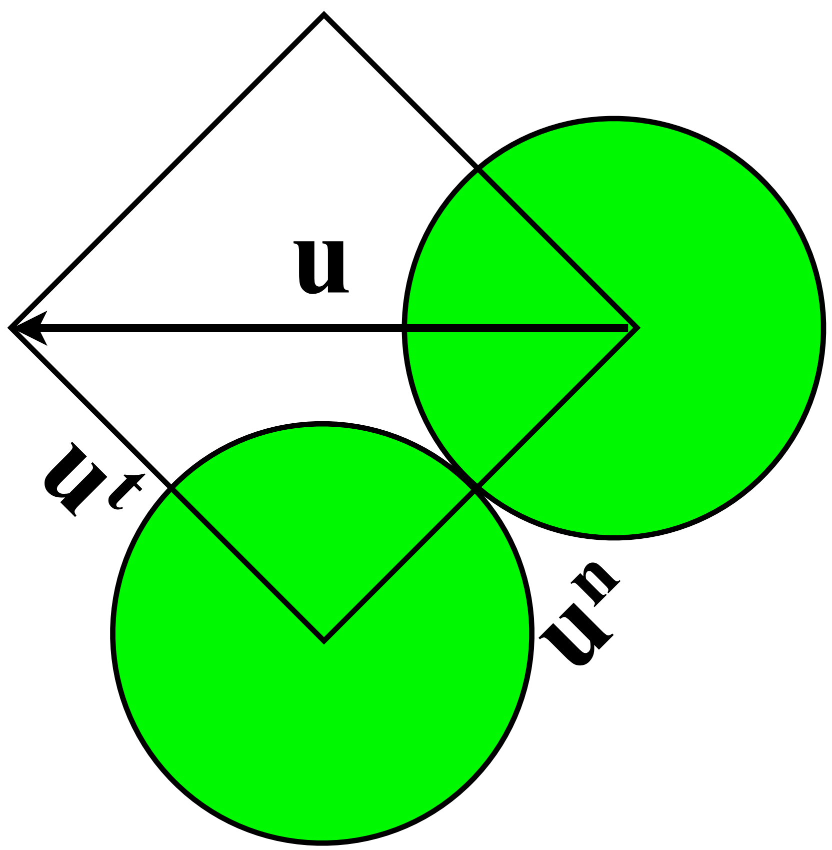
Blair et al, 2003



Goldhirsch et al, 1993

Key ingredient

Collisions are inelastic



$$\begin{aligned} v^t &= u^t \\ v^n &= -ru^n \end{aligned}$$

$$r < 1$$

Freely cooling granular gas

- Give initial energy to particles
- Isolate system
- Energy loss through collisions
- Why study?
    - ★ Isolates effects of inelastic collisions
    - ★ Direct experiments
    - ★ As parts of larger driven systems
    - ★ Interacting particle systems

Homogeneous Cooling

$$\frac{dE}{dt} = -\frac{\Delta E}{\tau}$$

$$\frac{dE}{dt} \sim \frac{(1-r^2)E}{a/\sqrt{E}}$$

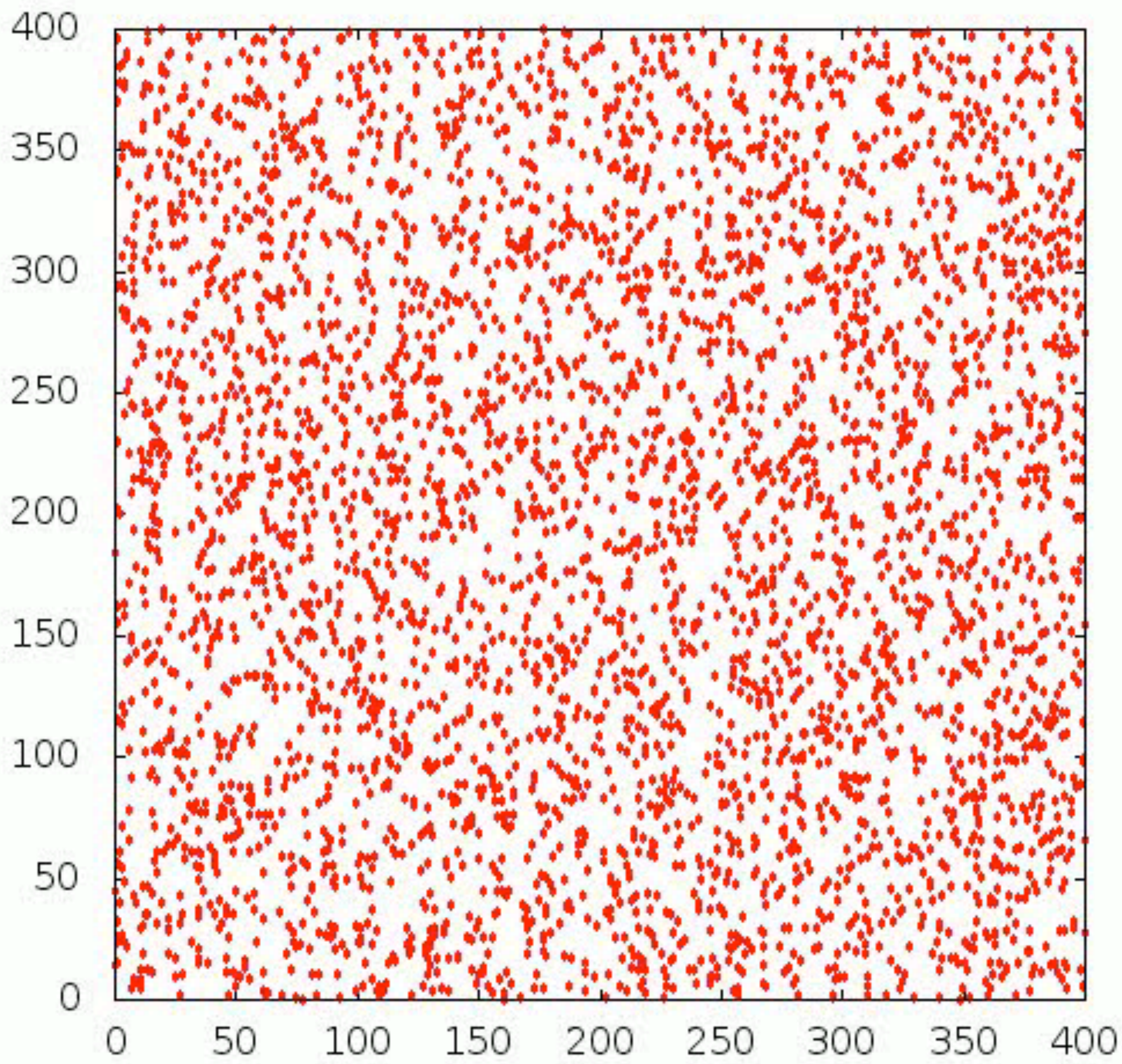
$$\boxed{E \sim \frac{1}{(1-r^2)t^2 + c^2}}$$

Haff's law Haff, 1982

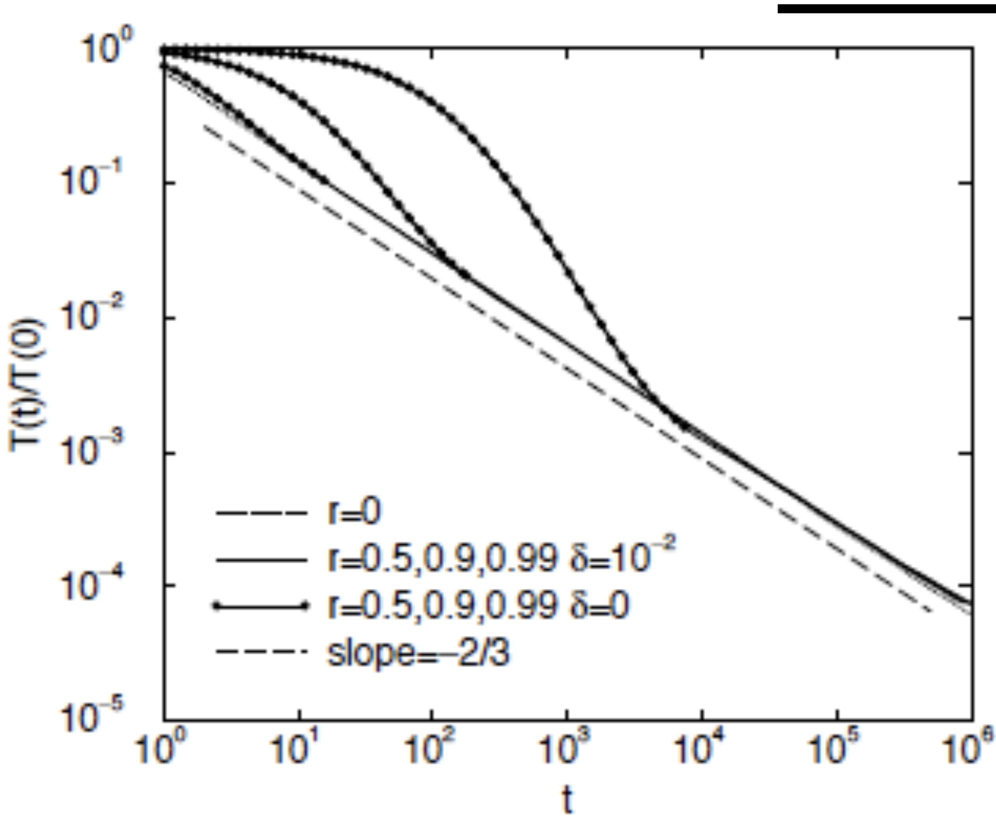
Assumption: particles are homogeneously distributed

Clustering

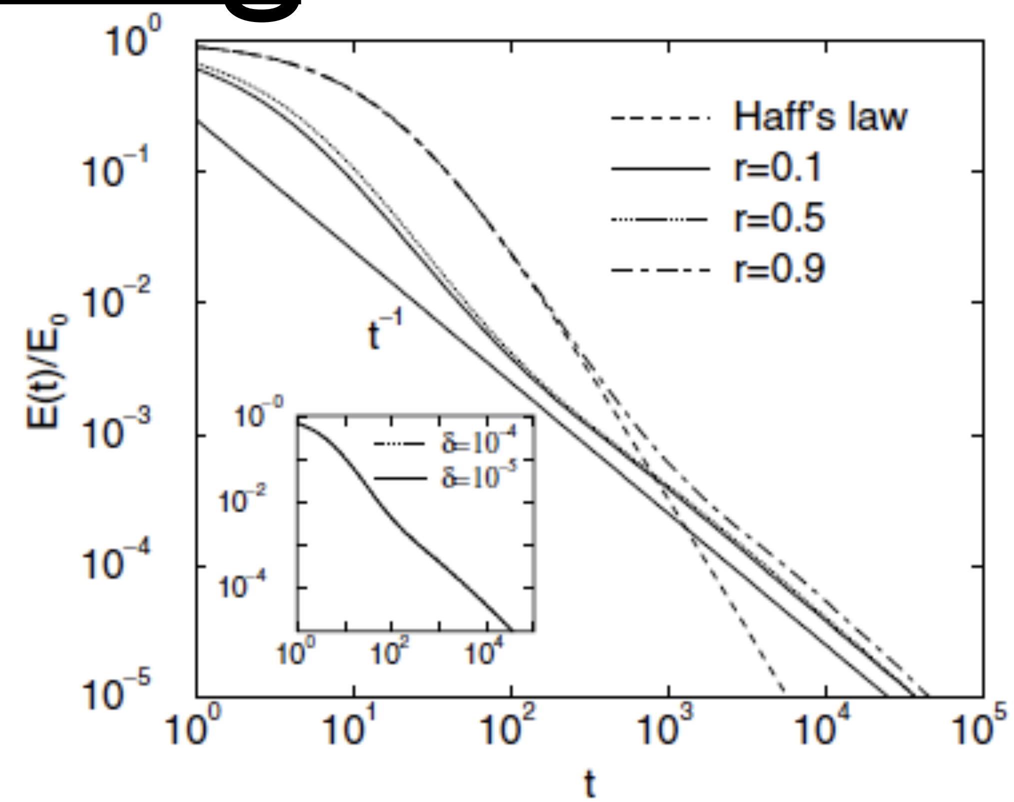
Clustering



Clustering



1d-energy



2d-energy

Ben-Naim et al, 1999, Nie et al, 2002

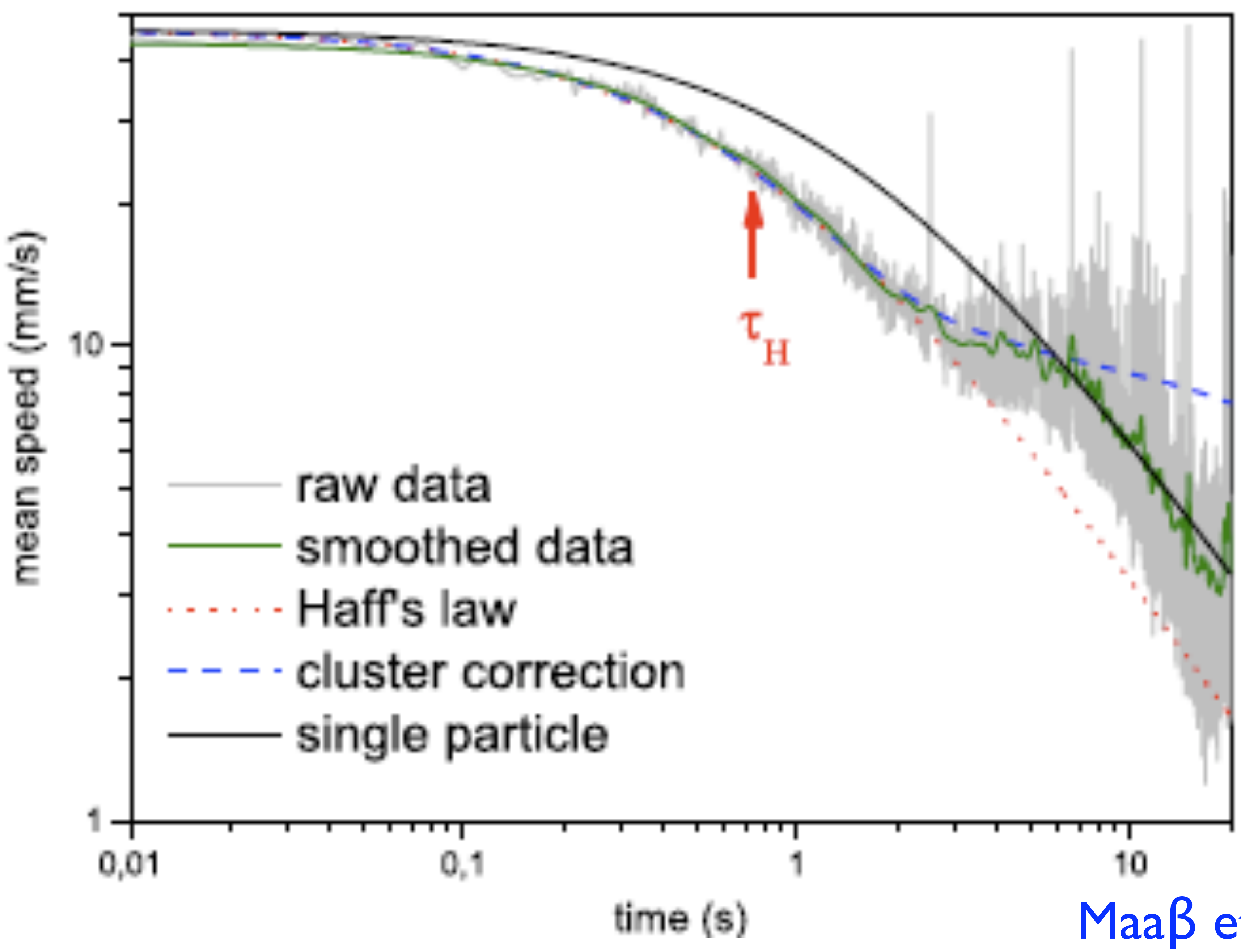
$r<1$: As $t \to \infty$, $r \to 0$

Clustering

- Breakdown of Haff's law (kinetic theory)
- New regime: inhomogeneous clustered regime

Experiments

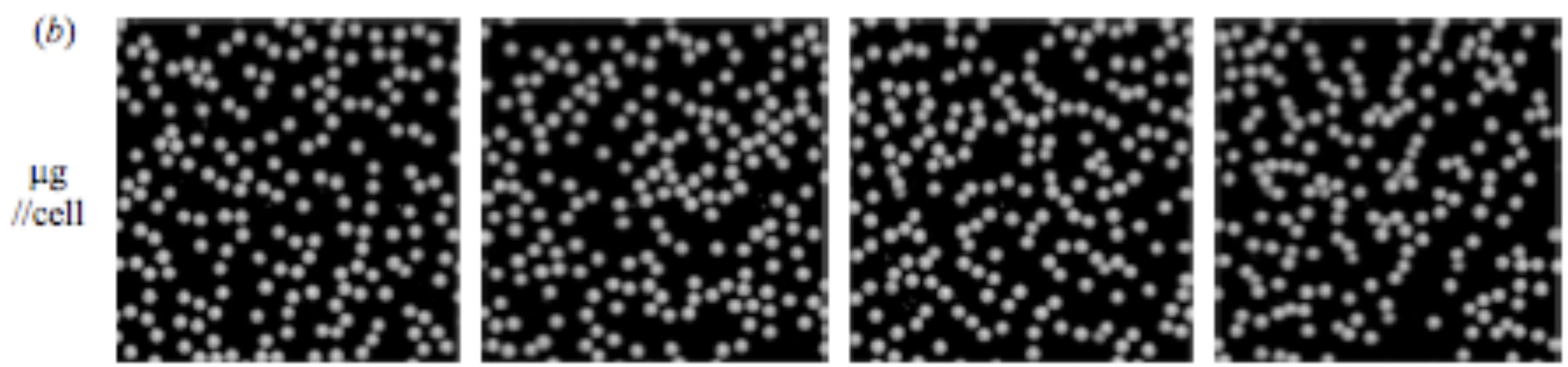
Levitation

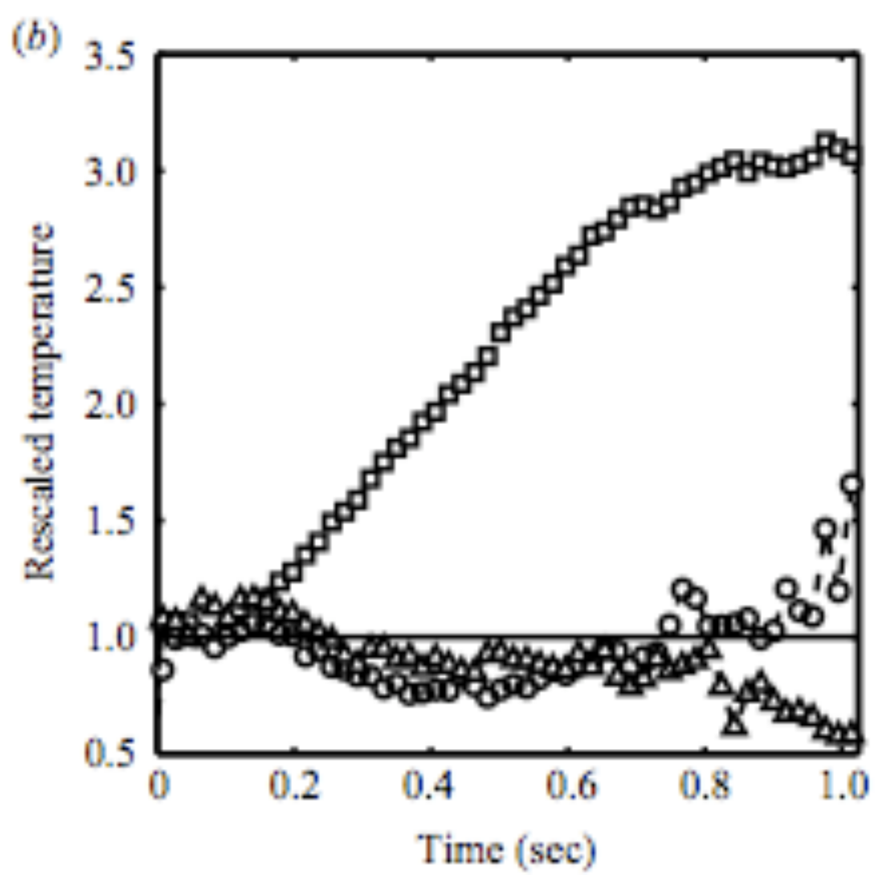


Maaβ et al, 2008

Experiments

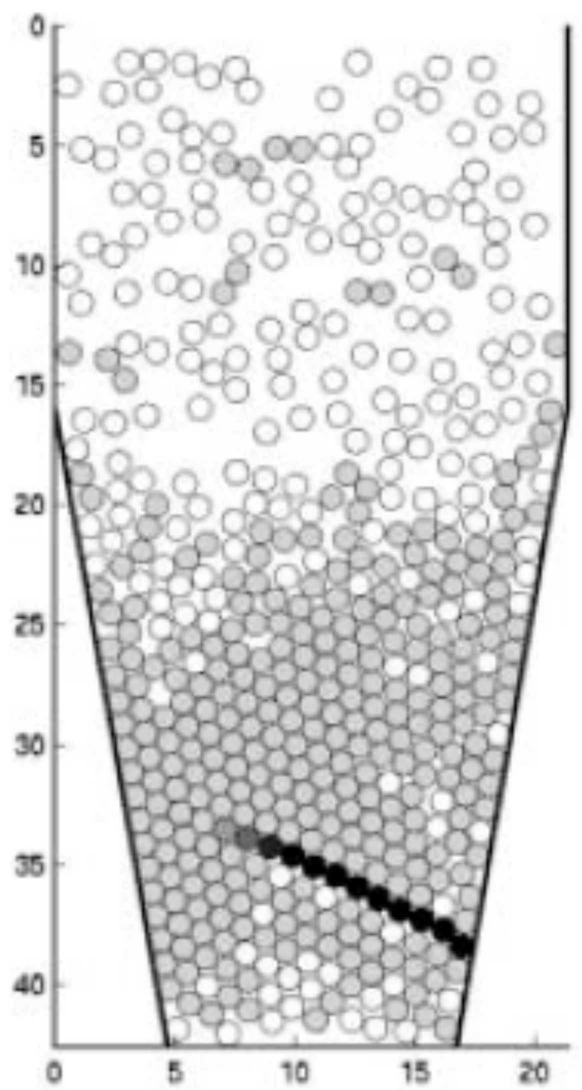
Microgravity





Tatsumi et al, 2009

Experiments (indirect)

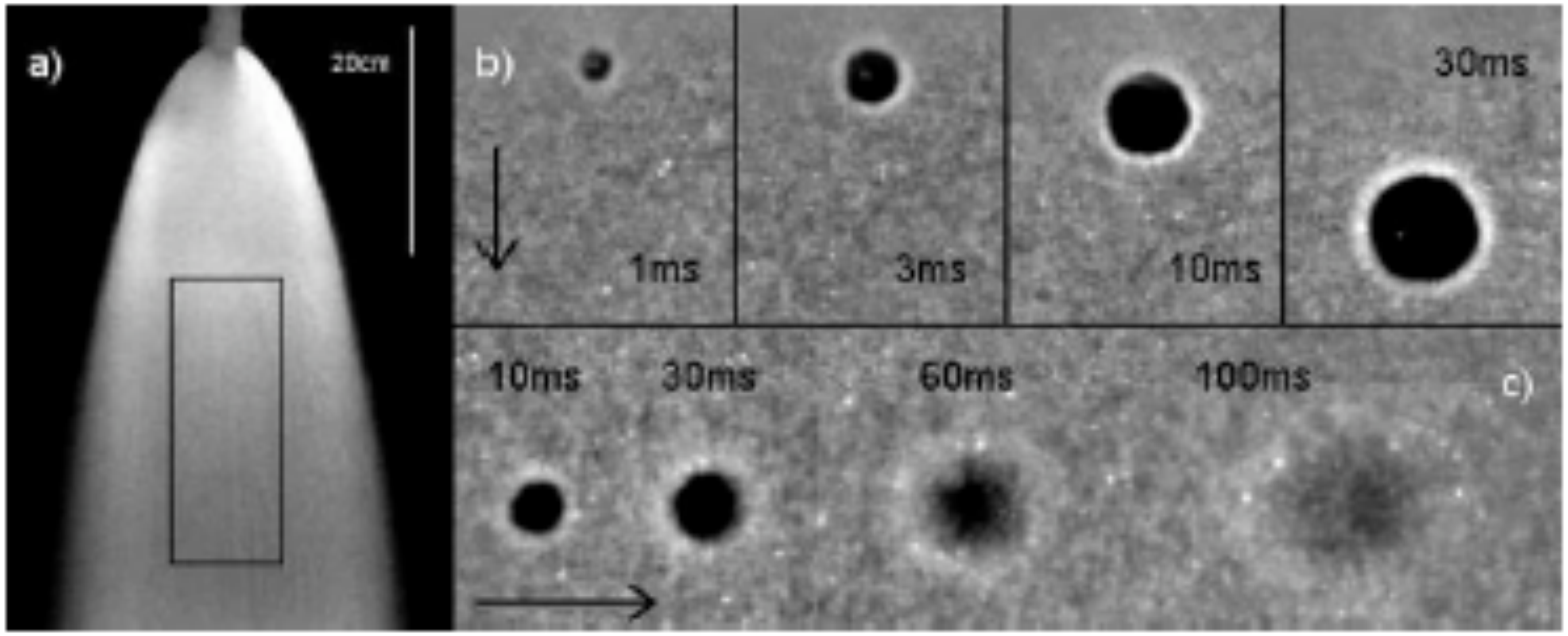


Ferguson et al, 2004

Experiments

- friction
- boundary effects
- Will argue for failure of kinetic theory for shock problem, but experimentally realizable

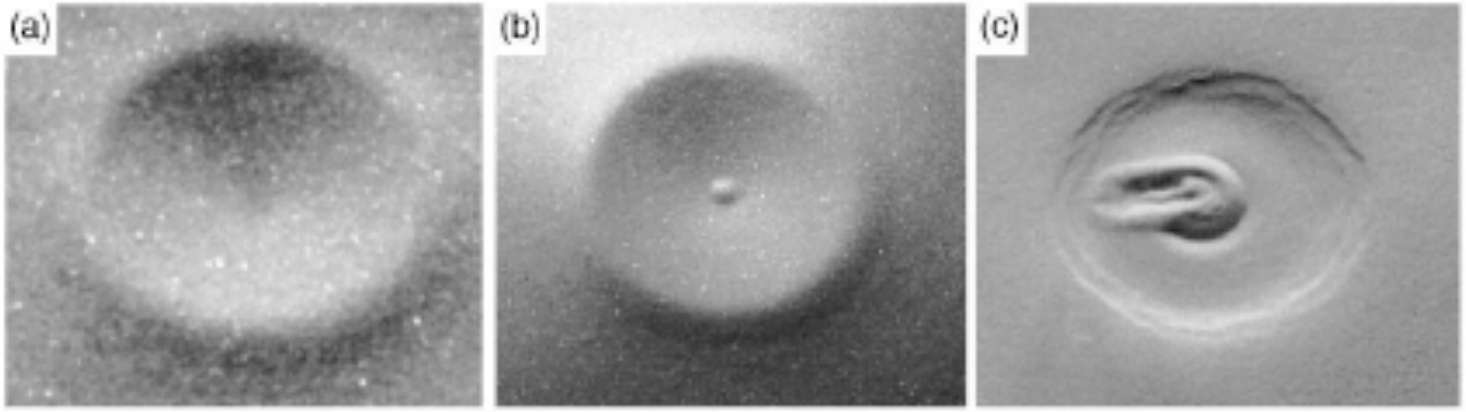
Experiments



Boudet et al, PRL 2009

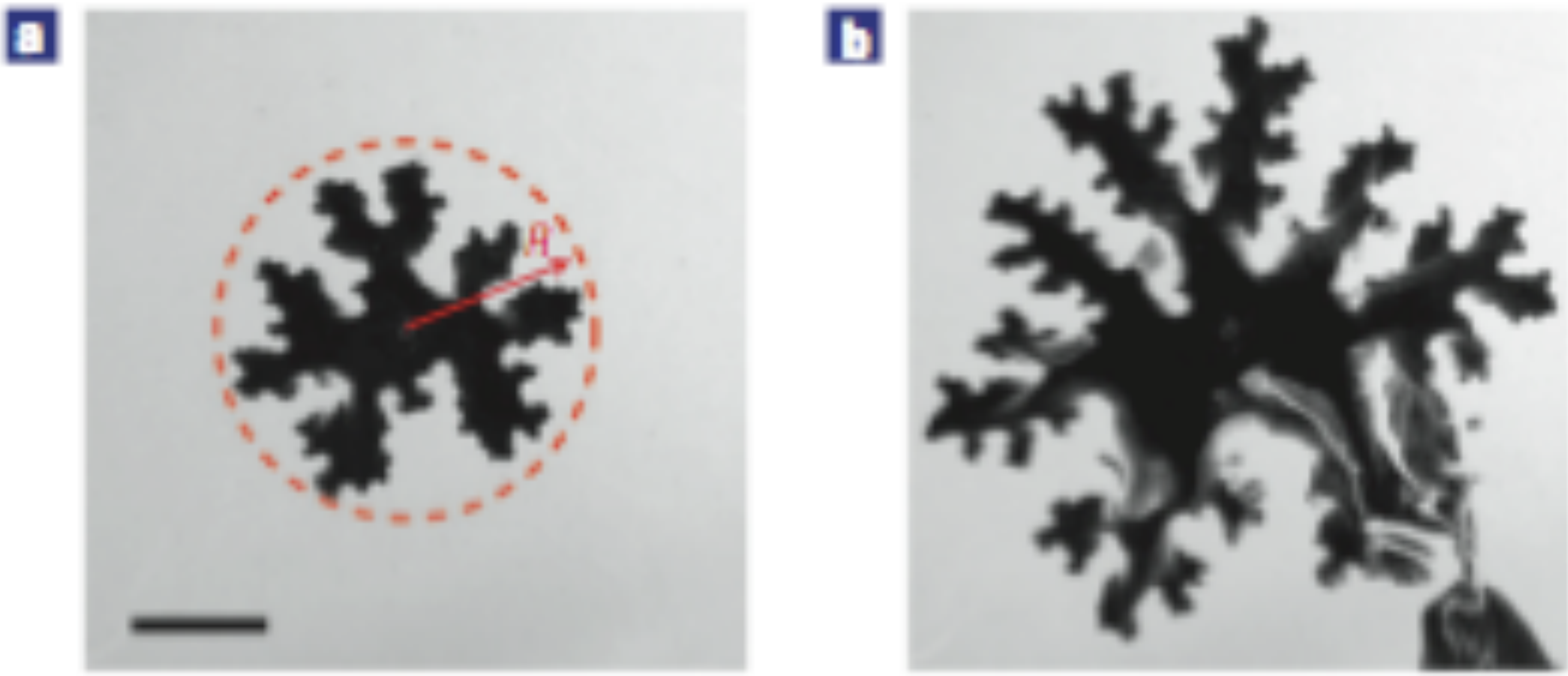
Experiments

Crater formation



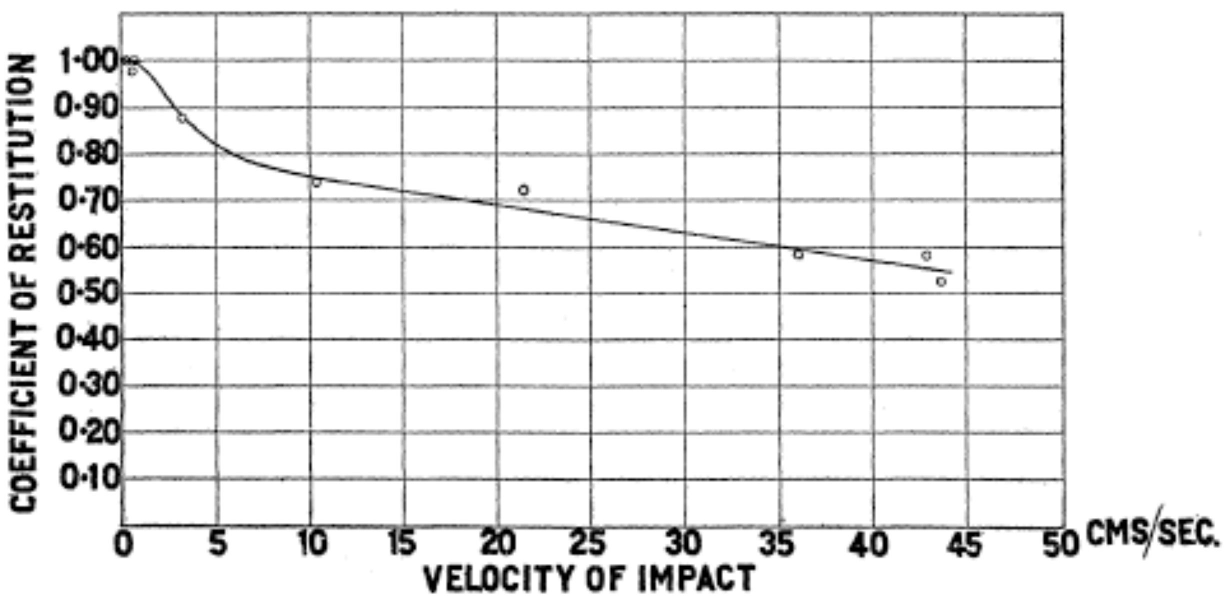
Walsh et al, PRL 2003

Experiments

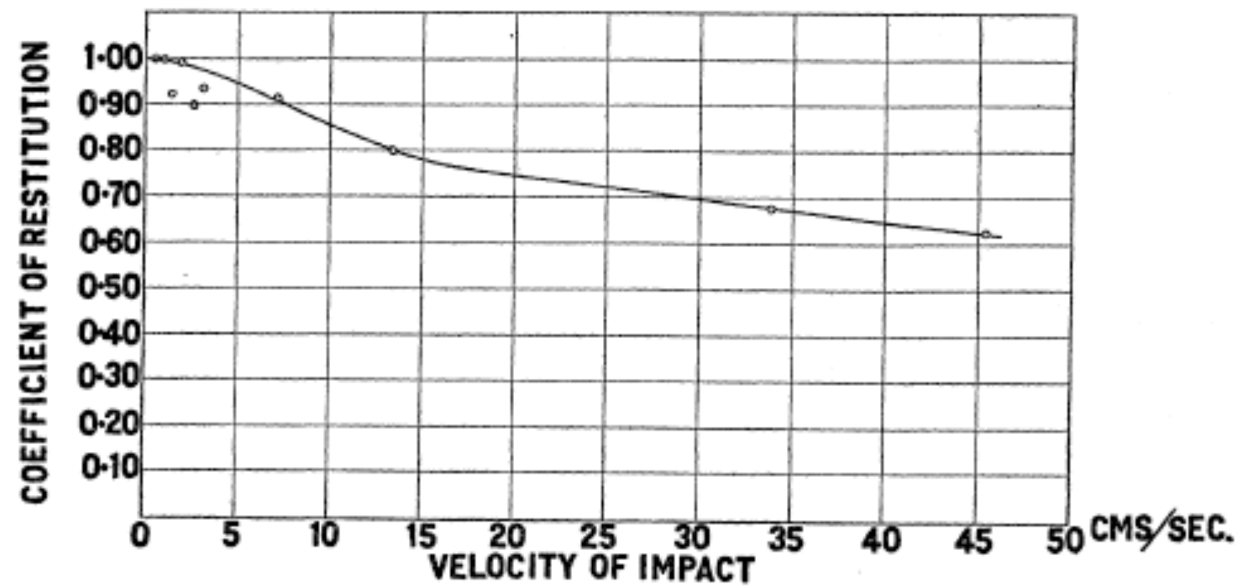


Cheng et al, Nature Phys, 2008

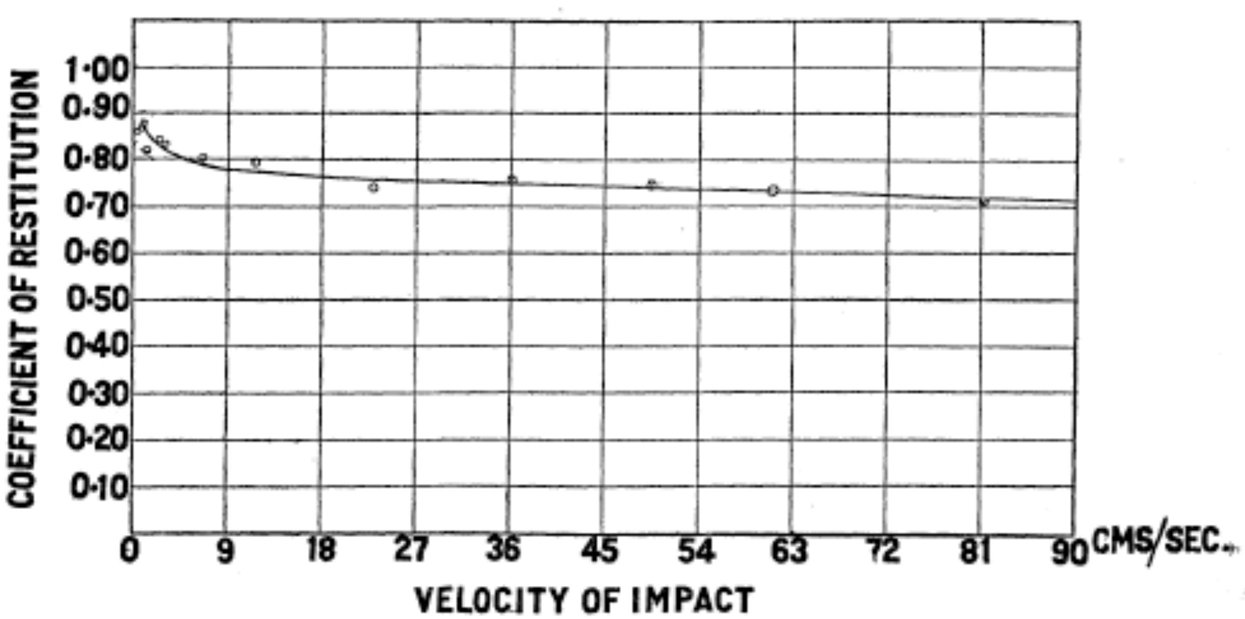
Details of simulation: a constant coefficient of restitution?



Brass



Aluminium



Marble

$$\mathrm{r} \to 1 \text{ when } v \to 0$$

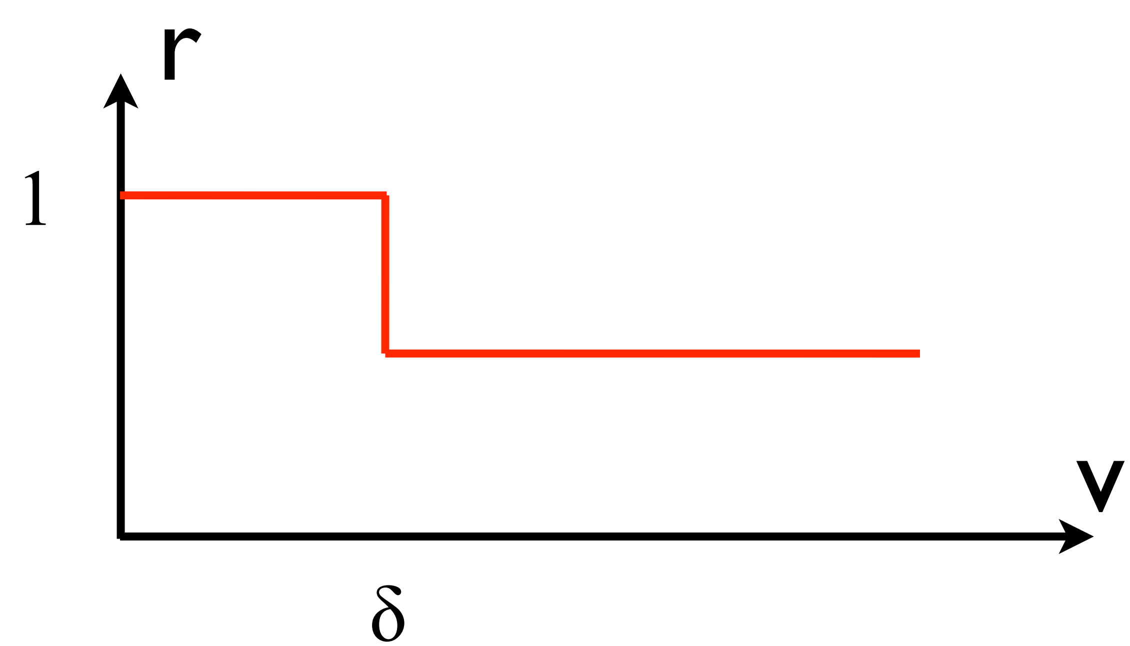
$$\mathrm{r} \to r_0 \text{ when } v \to \infty$$

C.V. Raman, 1918

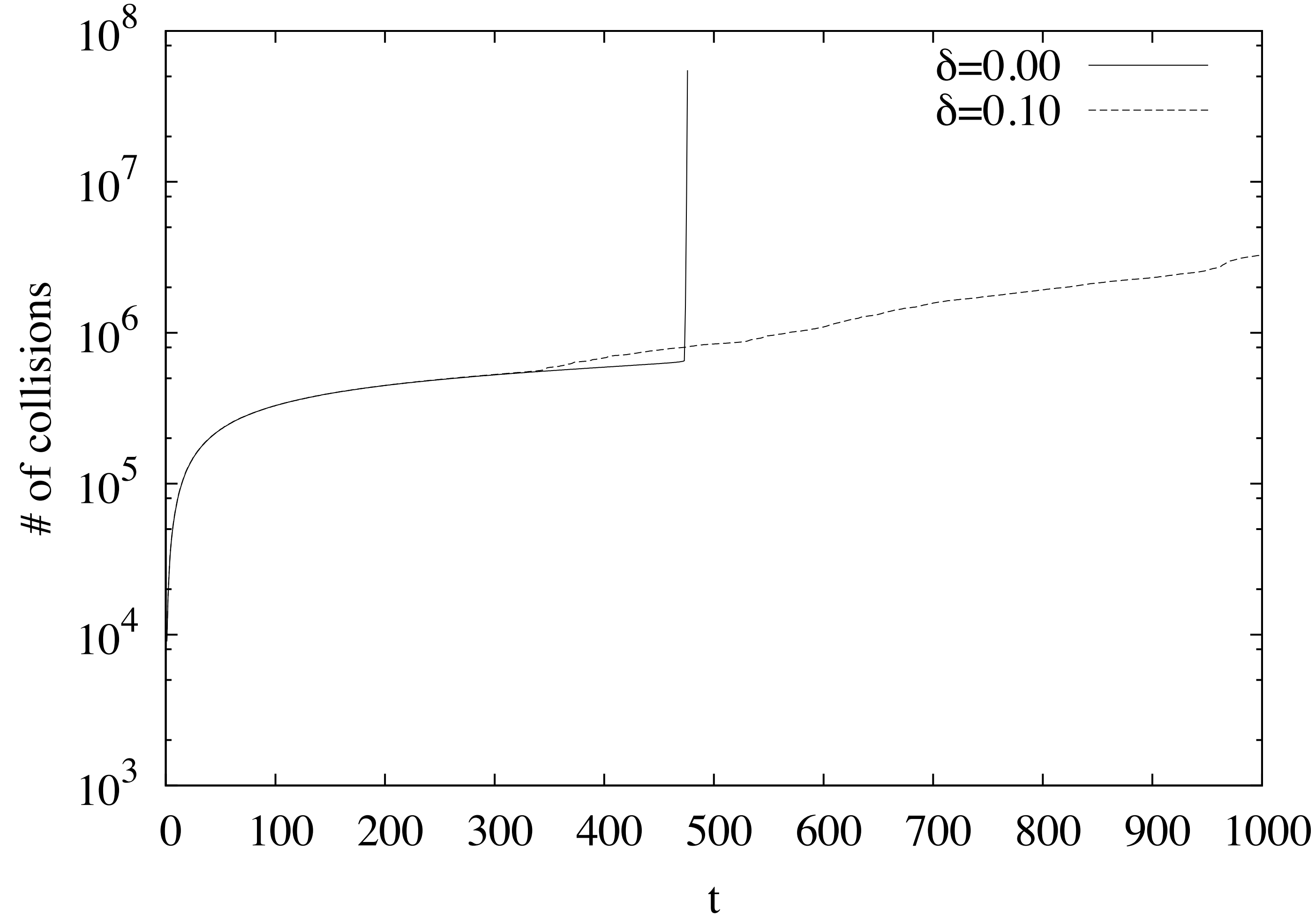
Coefficient of Restitution

$$\mathrm{r} \to 1 \text{ when } v \to 0$$

$$\mathrm{r} \to r_0 \text{ when } v \to \infty$$



Do we need δ?



Inelastic collapse

Event driven simulations

Interaction only on contact

Find out minimum of all collision times

Advance time to that collision time

For every time step $O(N^2)$ calculations

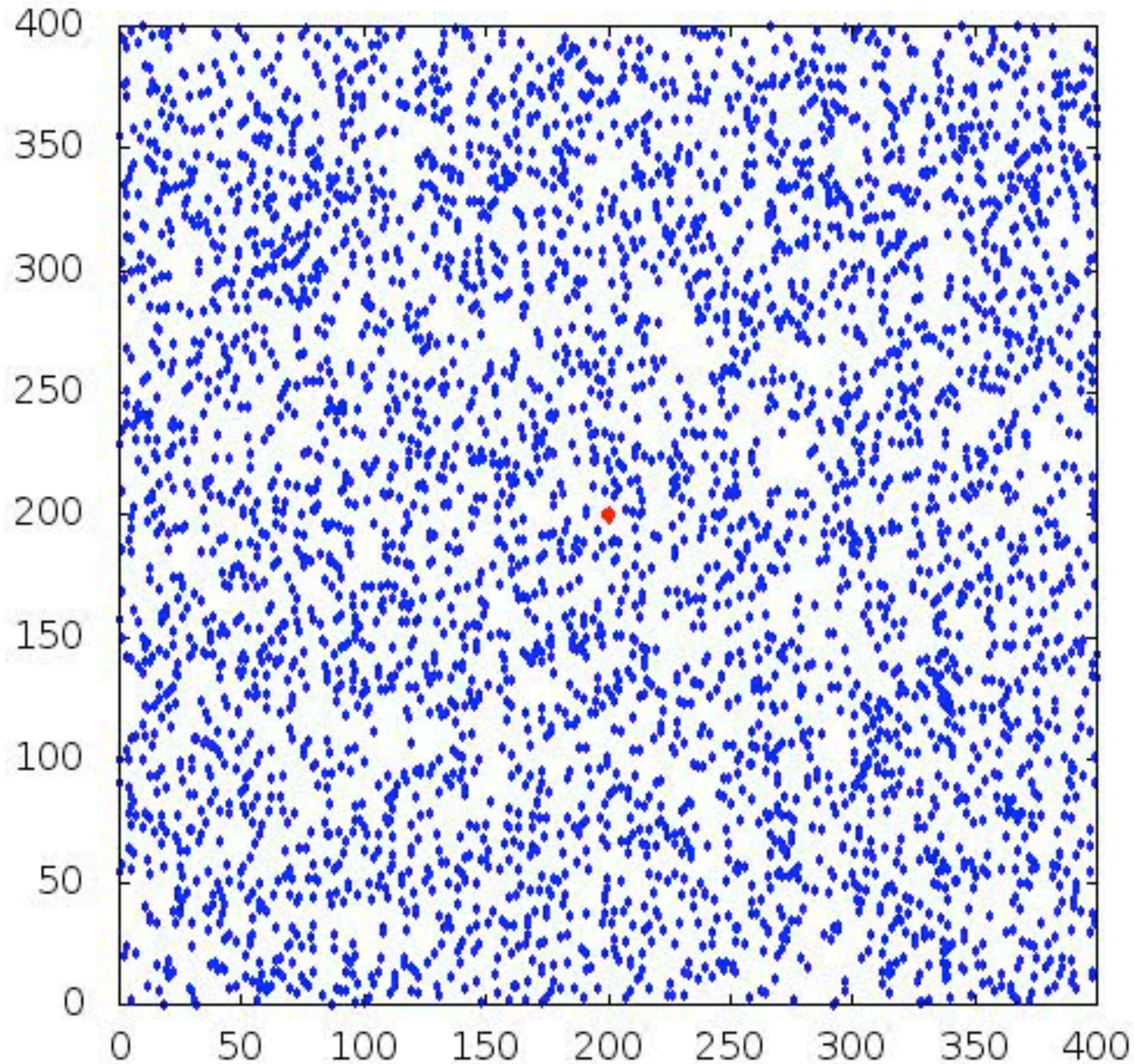
Divide space into small cells

Expand events to include collisions and cell crossing

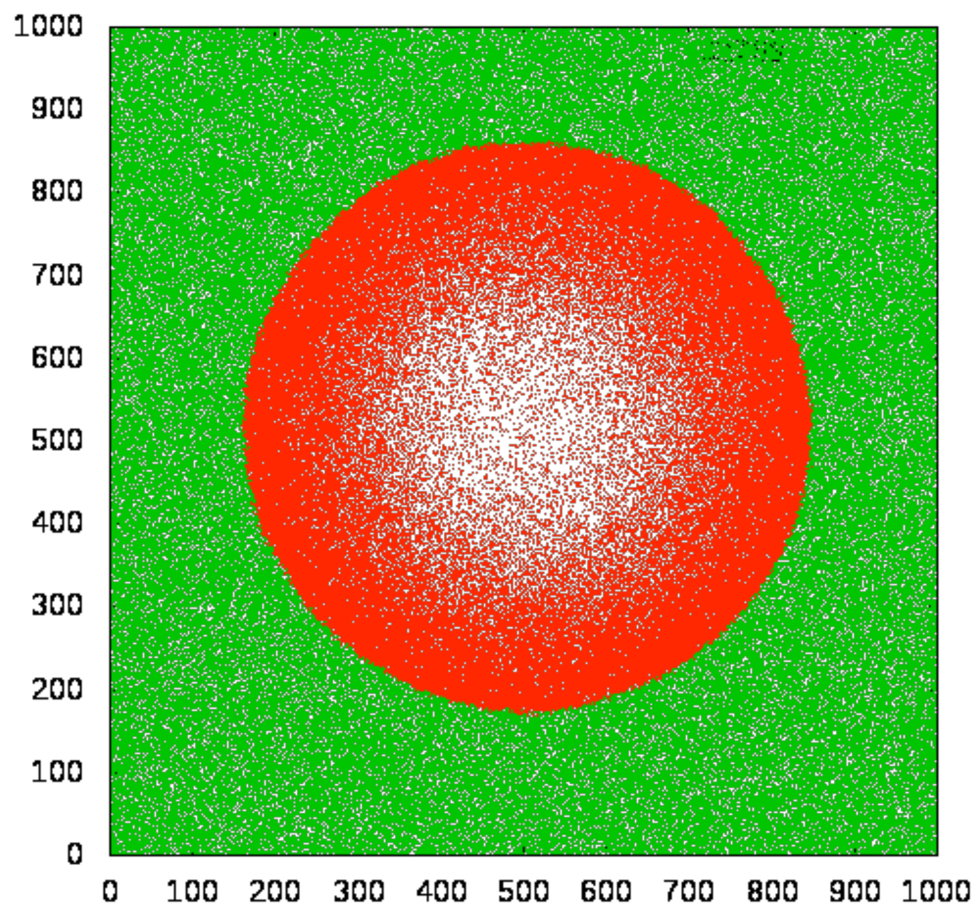
Now, all calculations are local

Computer simulation

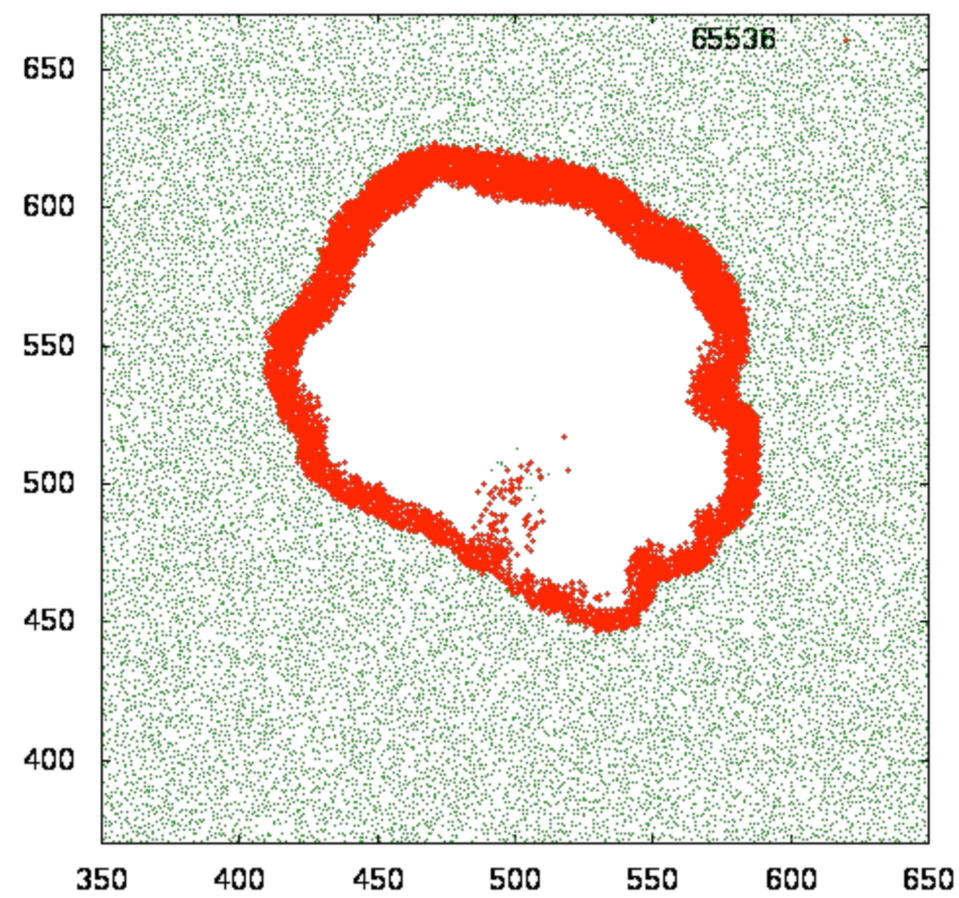
Computer simulation



Elastic vs Inelastic



Elastic particles

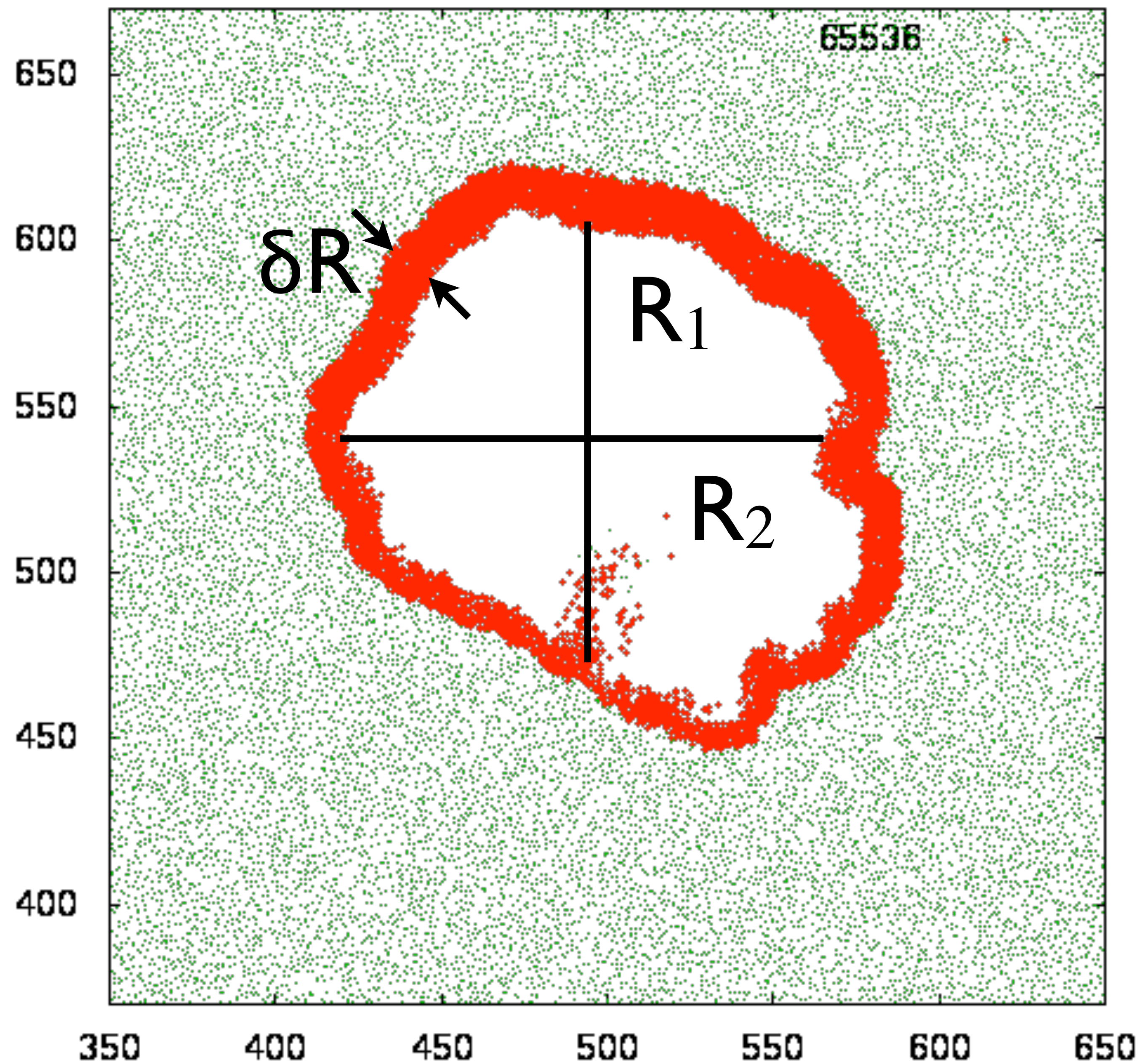


Inelastic particles

Scaling analysis

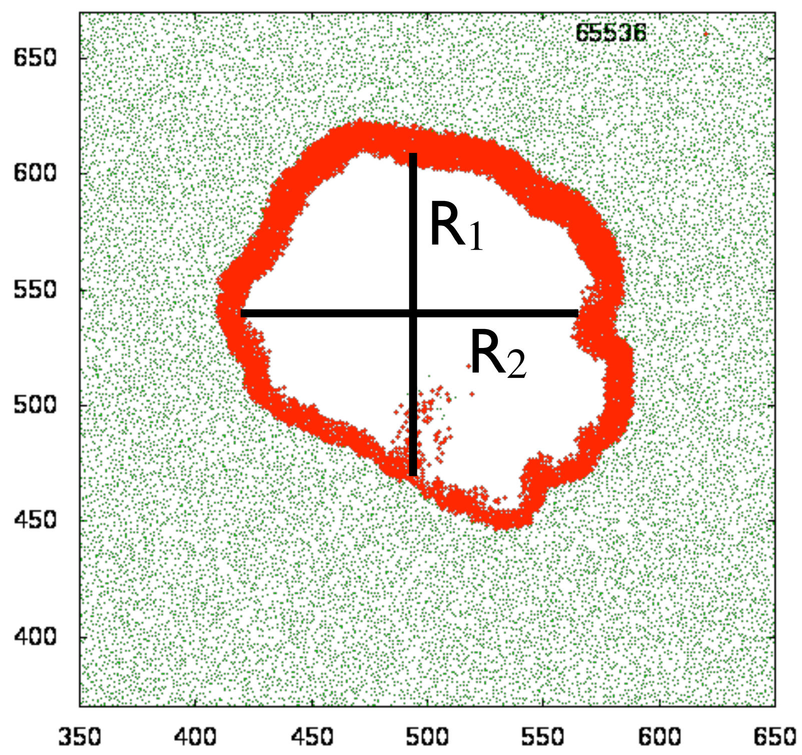
Let $R_t \sim t^\alpha$

Length scales



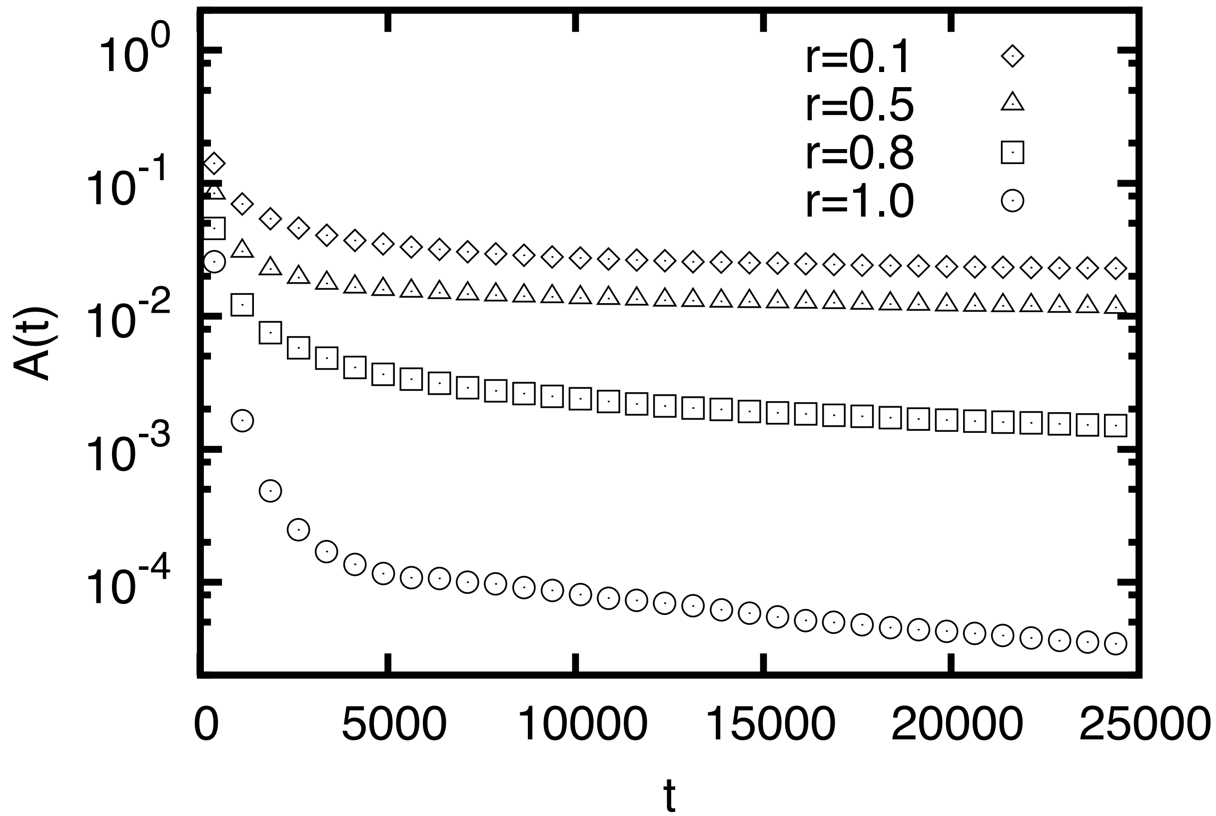
Do all these lengths scale as t^{α} ?

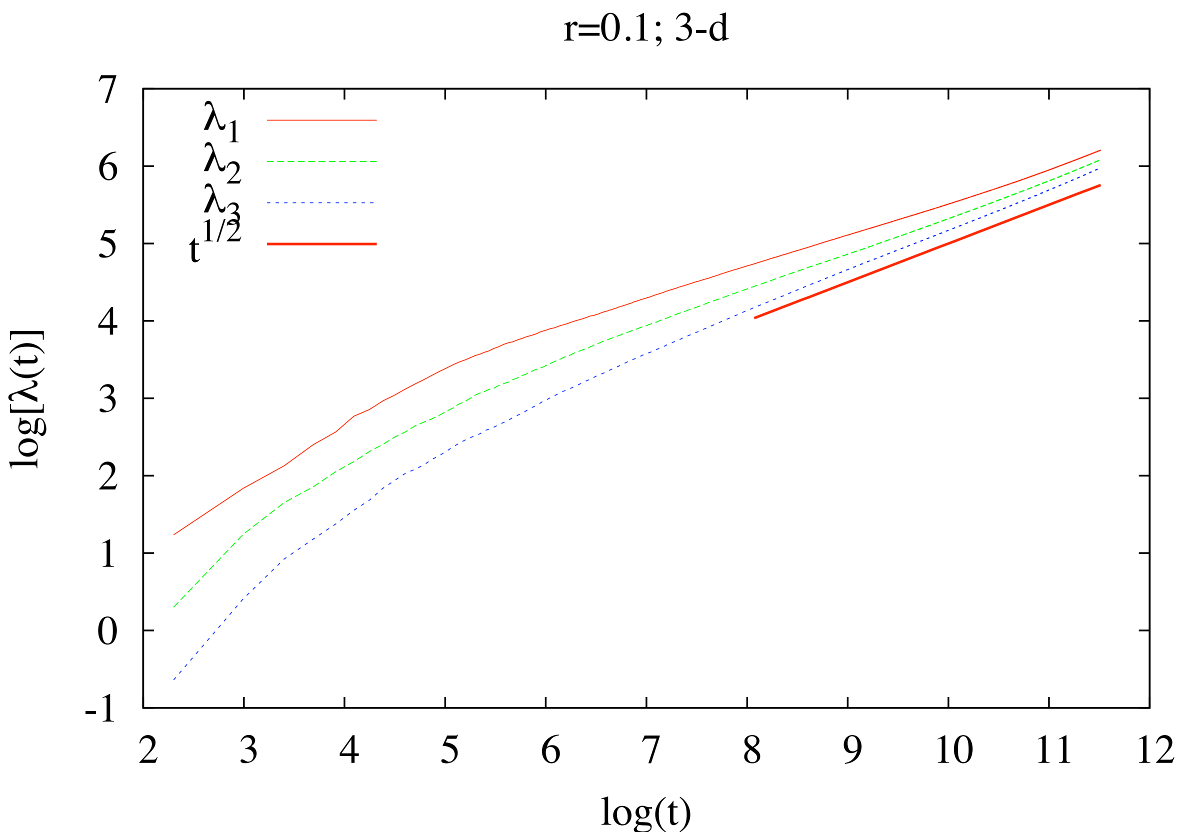
Length scales



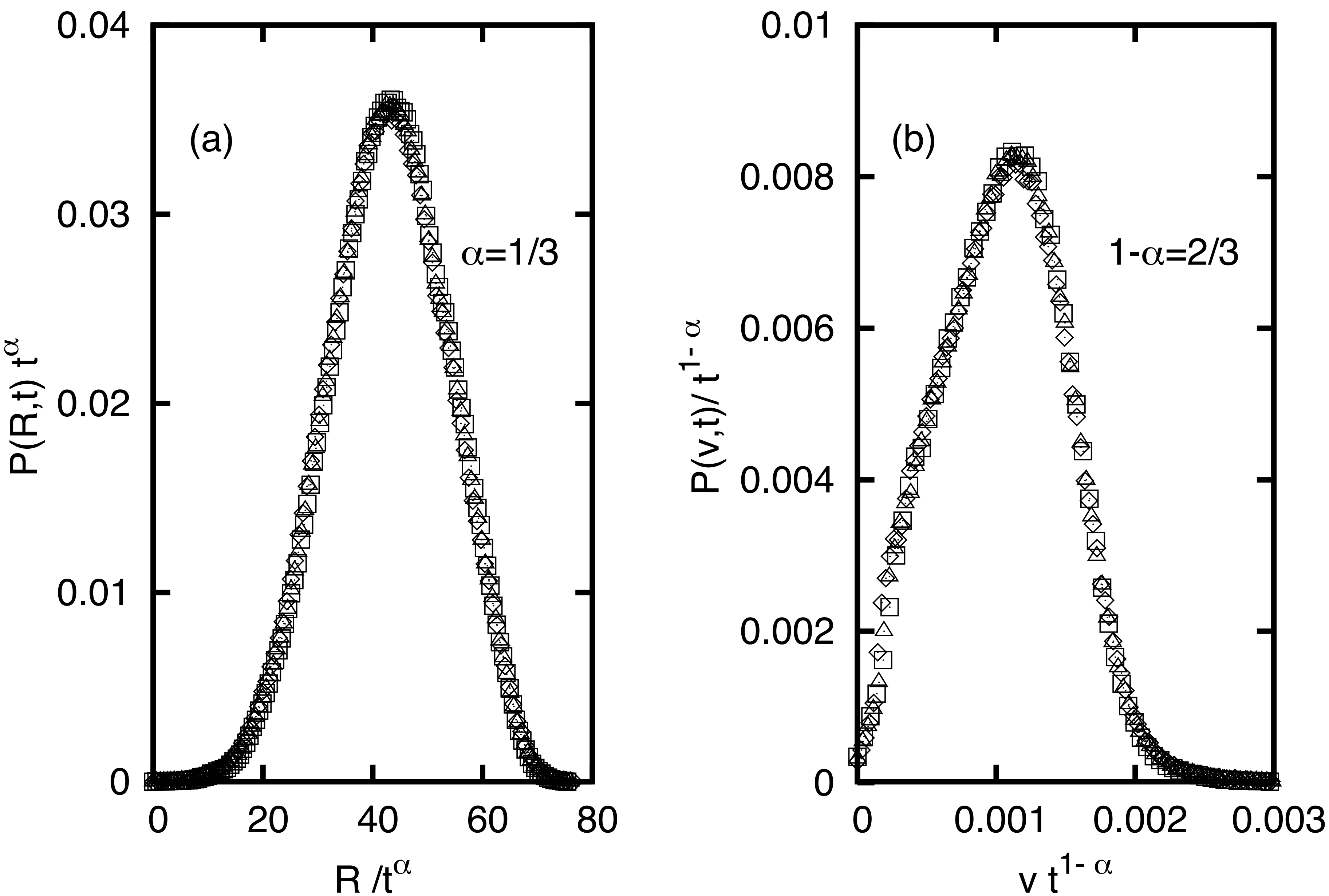
$$\begin{pmatrix} \langle R_x^2 \rangle & \langle R_x R_y \rangle \\ \langle R_x R_y \rangle & \langle R_y^2 \rangle \end{pmatrix}$$

$$A = \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \right)^2$$





Probability distribution (2-D)



Scaling analysis

$$\text{Let } R_t \sim t^{\alpha}$$

$$v_t = \frac{dR_t}{dt} \sim t^{\alpha-1}$$

$$N_t \sim R_t^d \sim t^{\alpha d}$$

$$E_t \sim N_t v_t^2 \sim t^{\alpha d+2\alpha-2}$$

Scaling (elastic limit)

$$E_t \sim N_t v_t^2 \sim t^{\alpha d + 2\alpha - 2}$$

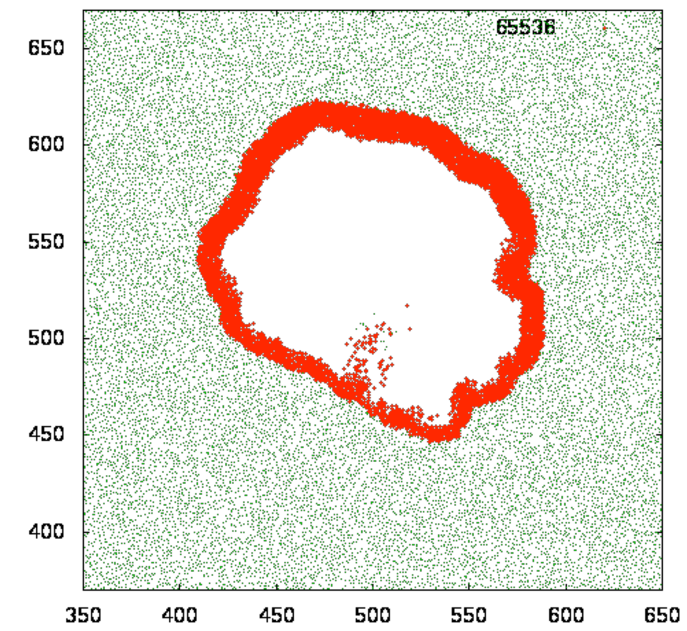
But energy is a constant

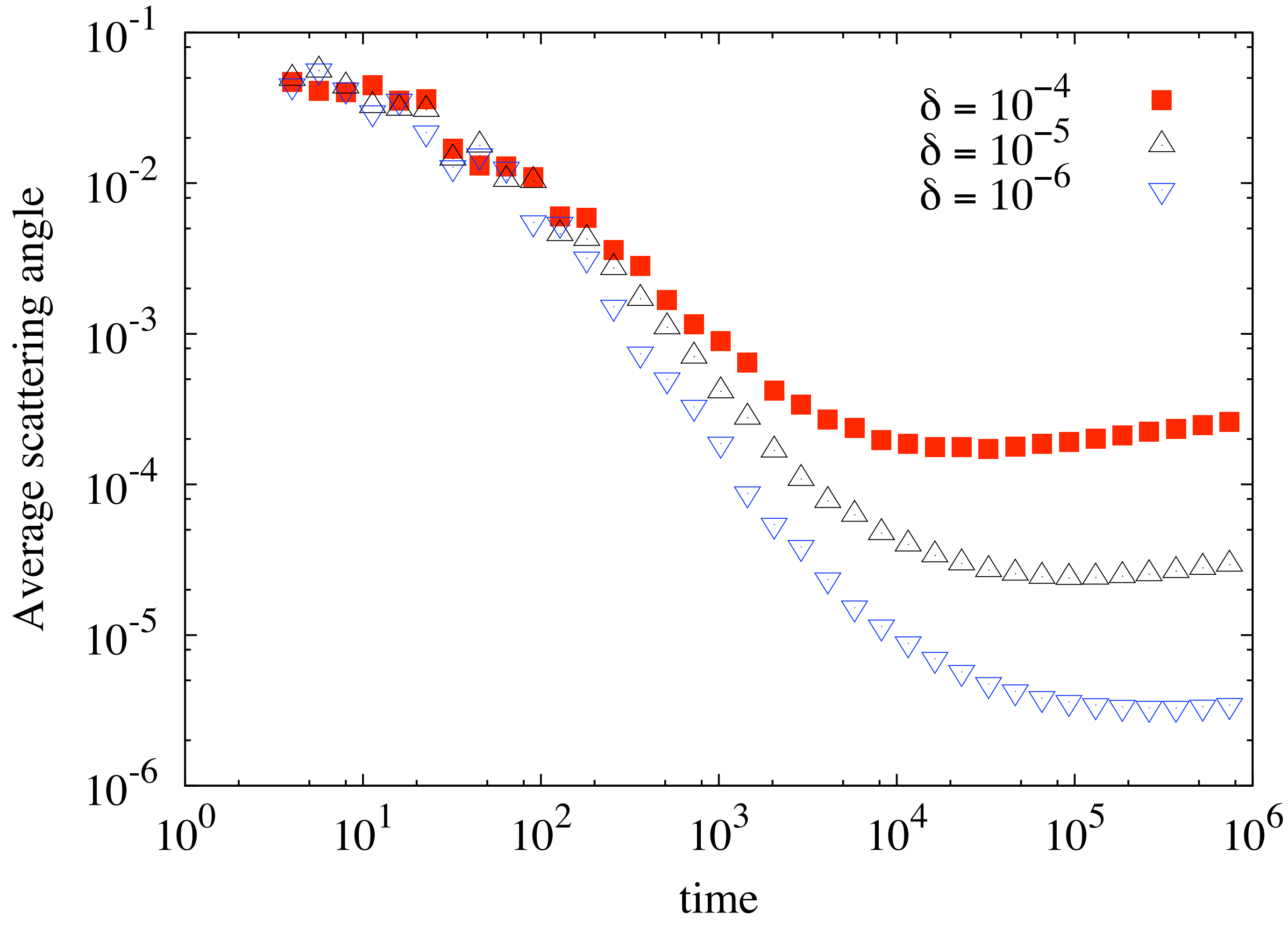
$$\alpha d + 2\alpha - 2 = 0$$

$$\alpha = \frac{2}{d+2}$$

Scaling (inelastic limit)

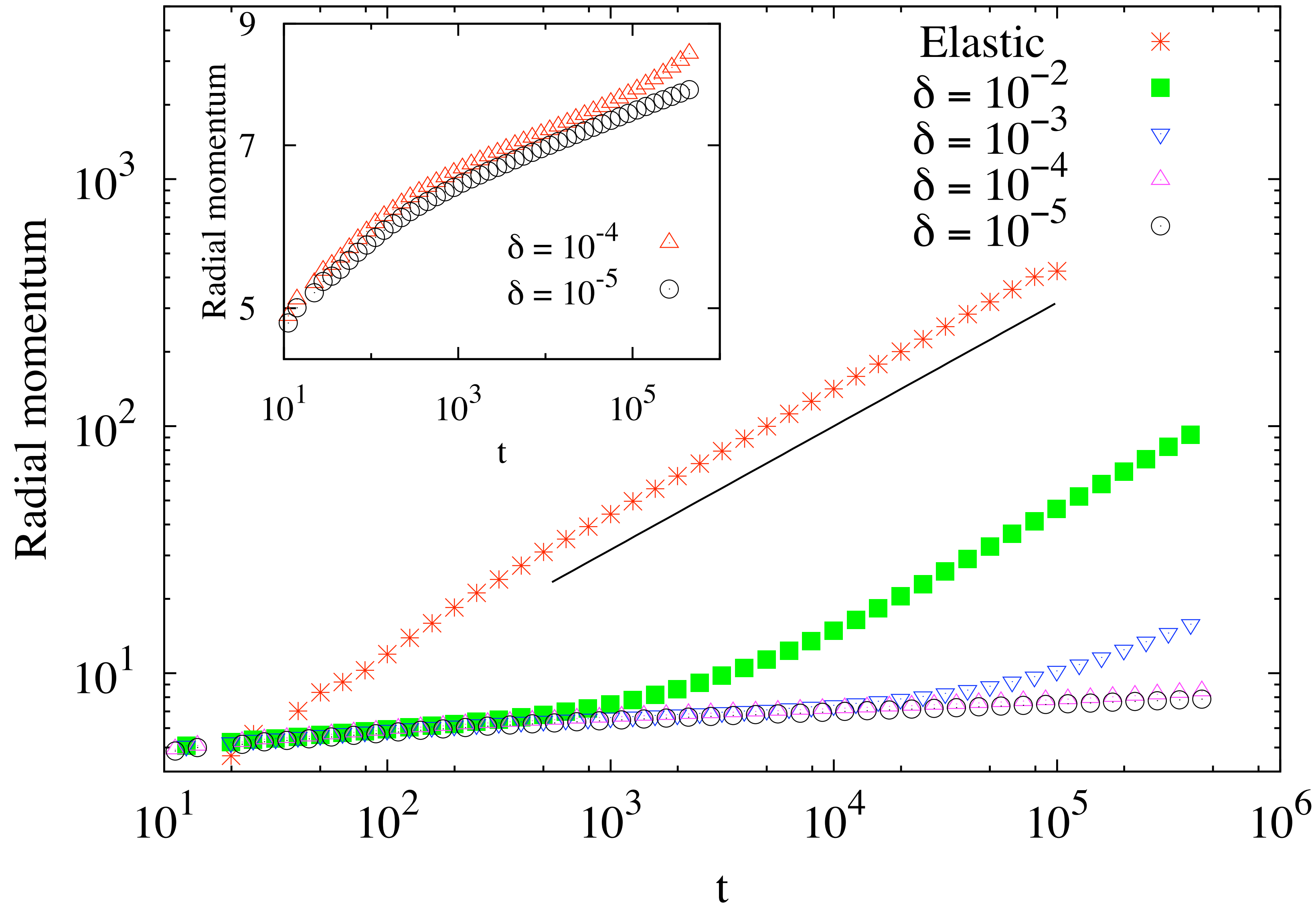
- clustering for all $r < 1$
- Particle direction remains constant





Radial Momentum

- radial momentum is conserved



Scaling (inelastic limit)

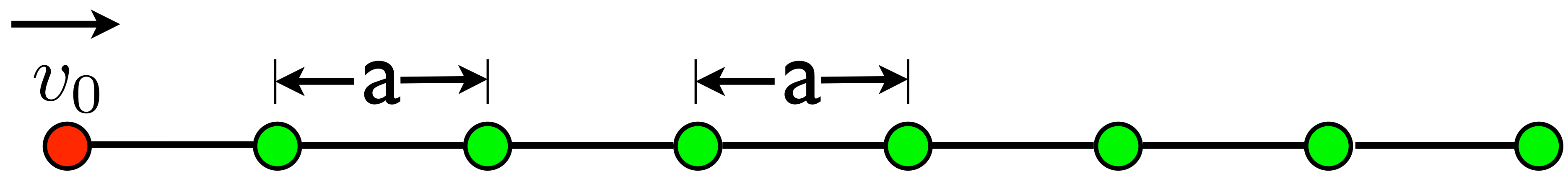
$$N_t v_t d\Omega = \text{constant}$$

$$\alpha d + \alpha - 1 = 0$$

$$\alpha = \frac{1}{d+1}$$

$$\alpha_{el} = \frac{2}{d+2}$$

A calculation in one dimension



Special case: r=0 [sticky limit]

After $m-1$ collisions, mass of particle is m

Momentum conservation $\implies v_{m-1} = \frac{v_0}{m}$

$$t_m = \frac{a}{v_0} + \frac{a}{v_1} + \ldots + \frac{a}{v_{m-1}} = a \sum_{i=1}^{m} \frac{i}{v_0} \propto m^2$$

$$m \sim N_t \sim R_t \sim \sqrt{t}$$

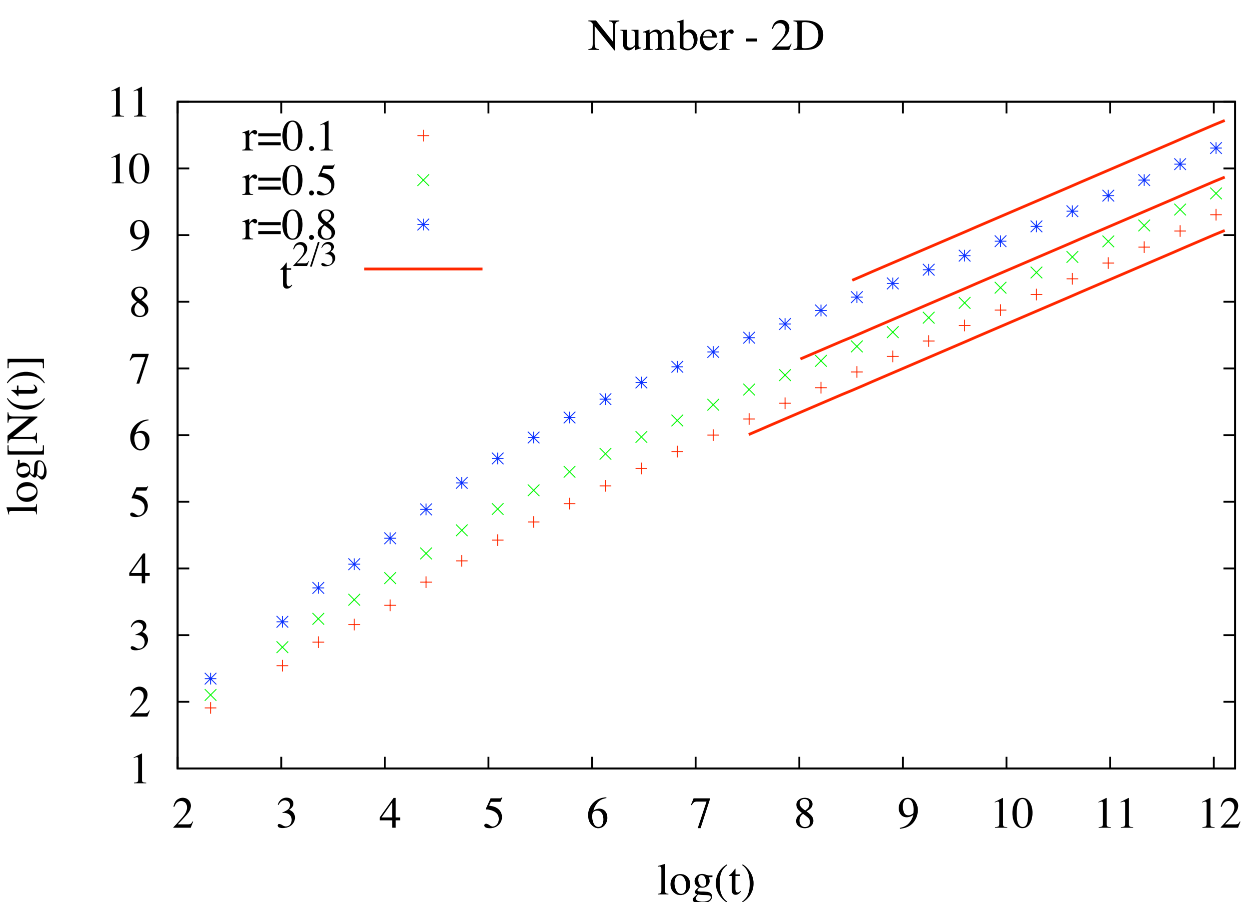
$$\alpha = \tfrac{1}{2}, \quad d = 1$$

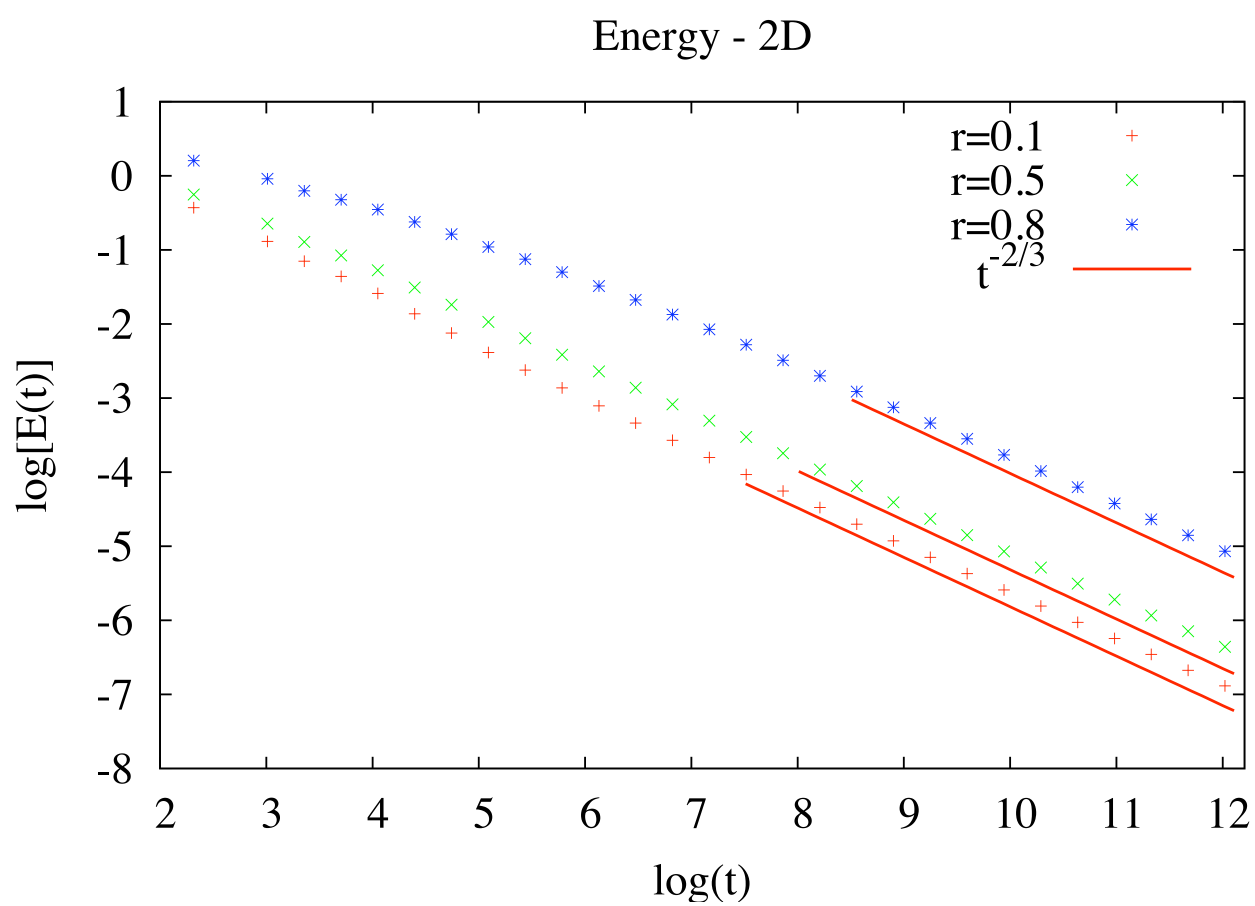
Recall

$$\alpha = \frac{1}{d+1}$$

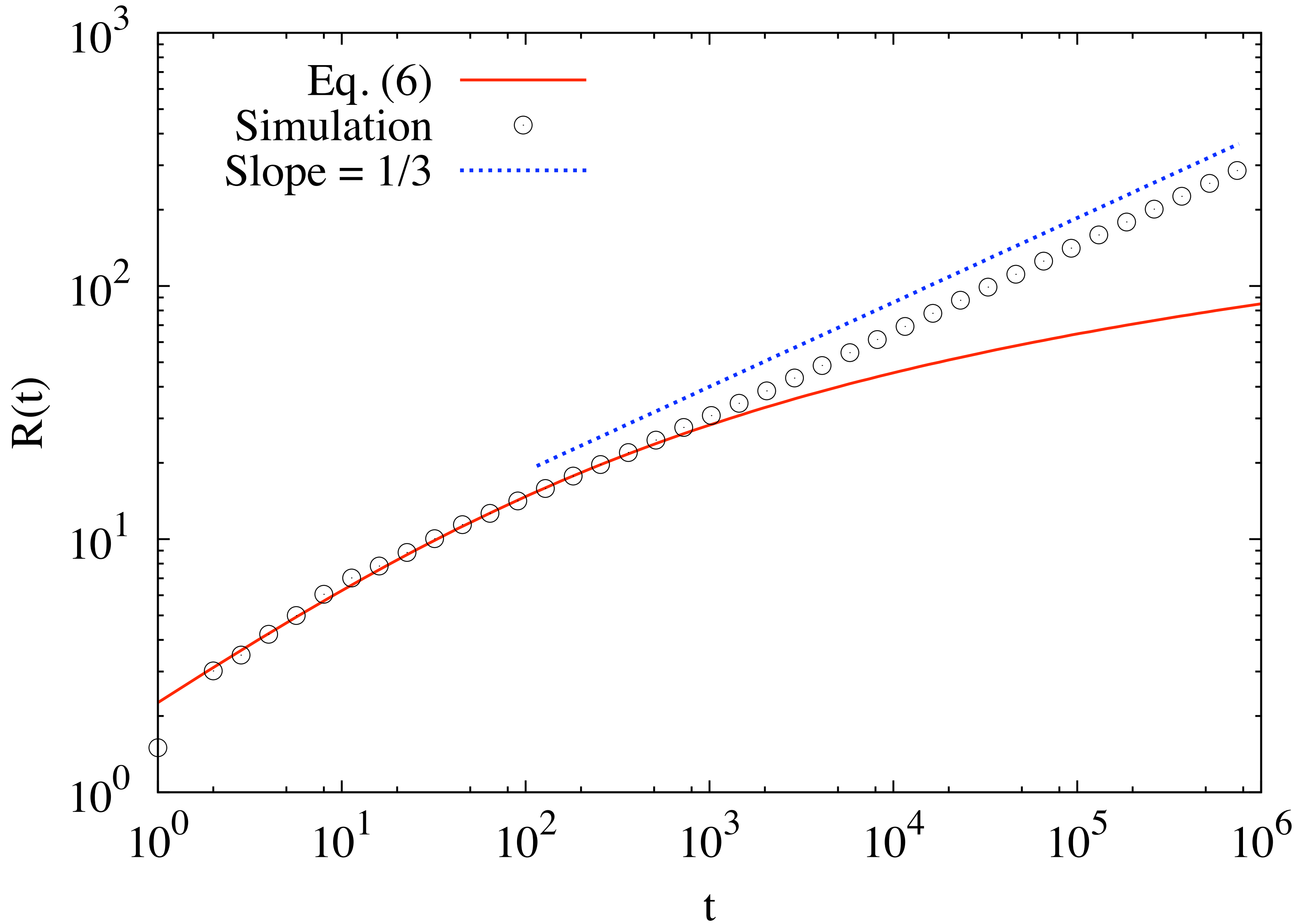
Simulation-2d

$$\langle N(t)\rangle \sim t^{2/3} \qquad \langle E(t)\rangle \sim t^{-2/3}$$





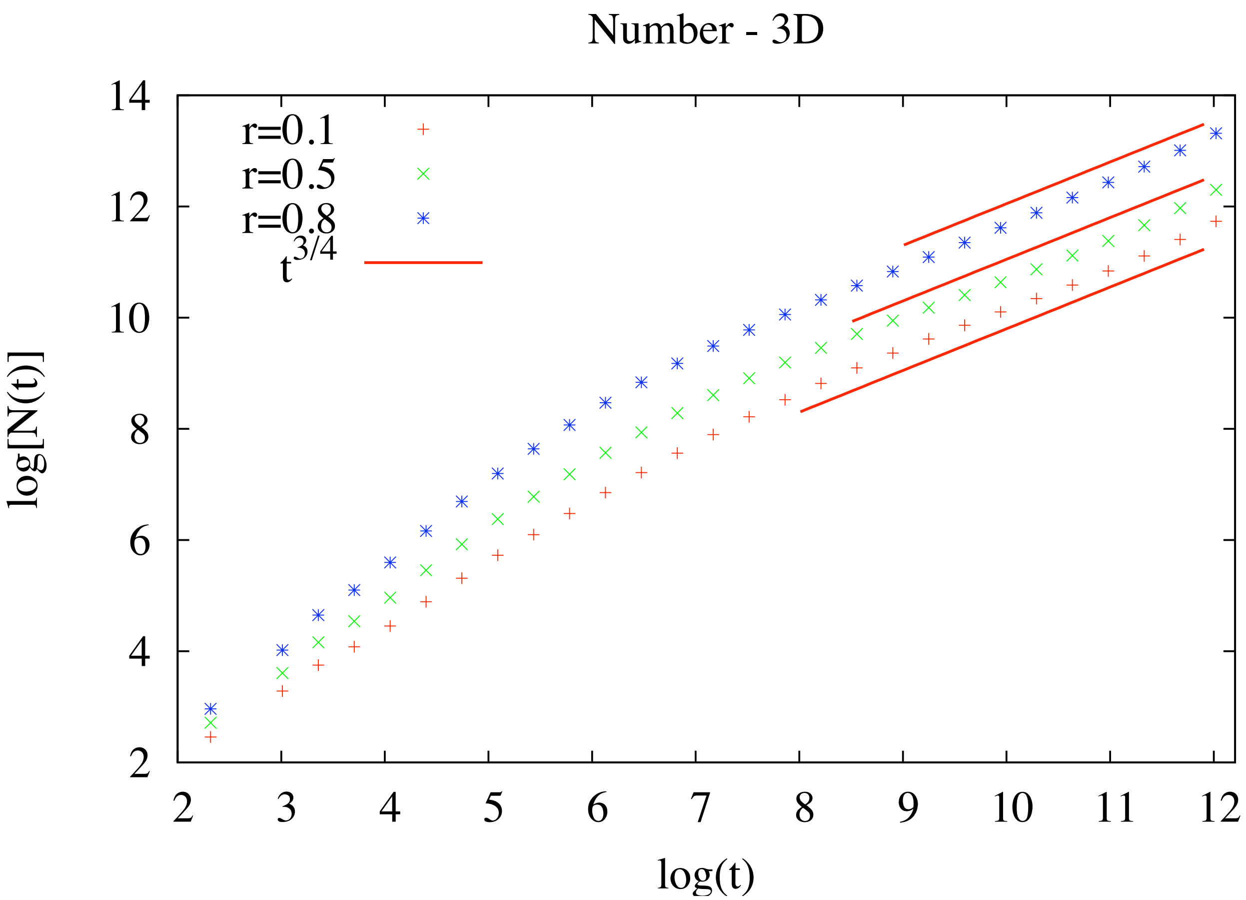
Comparison with kinetic theory

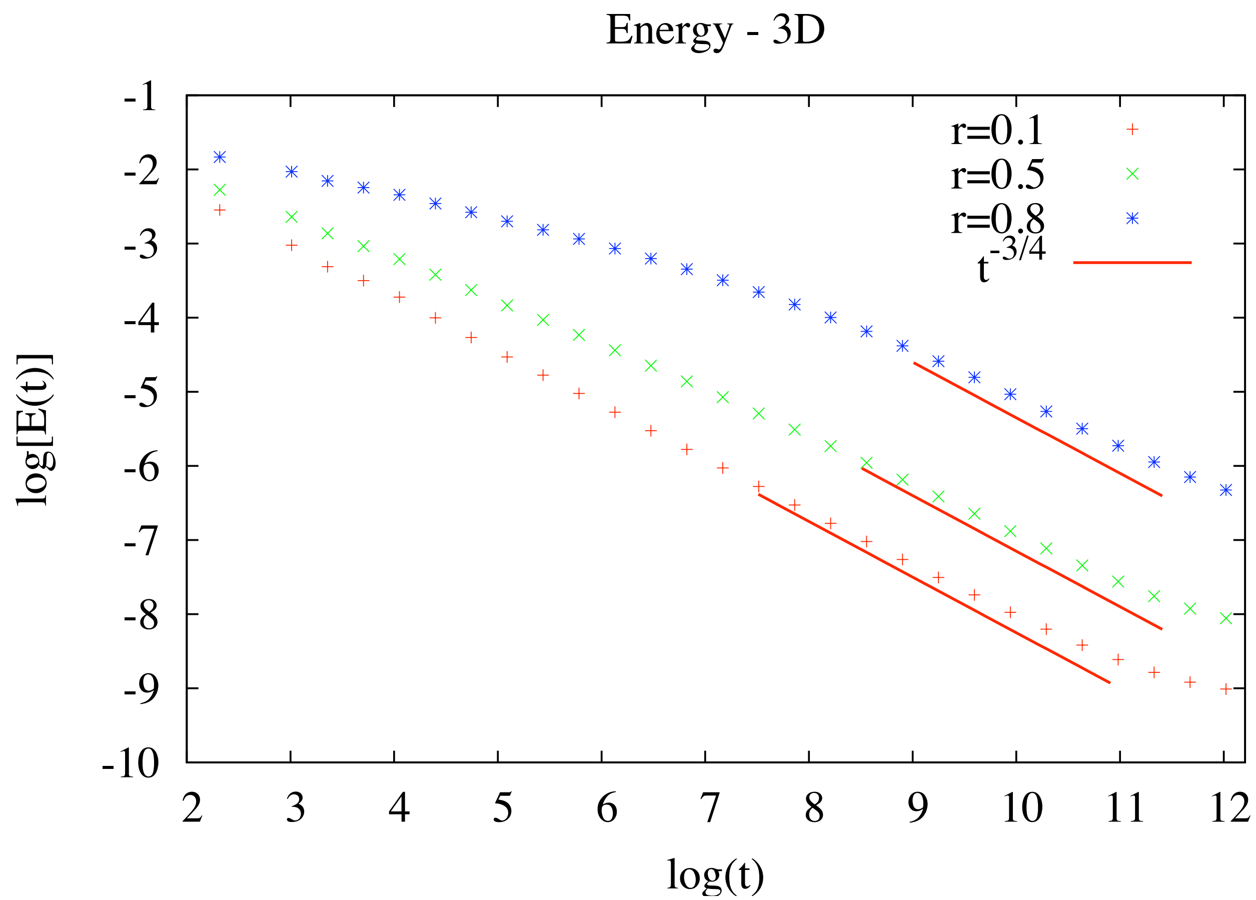


Simulation-3d

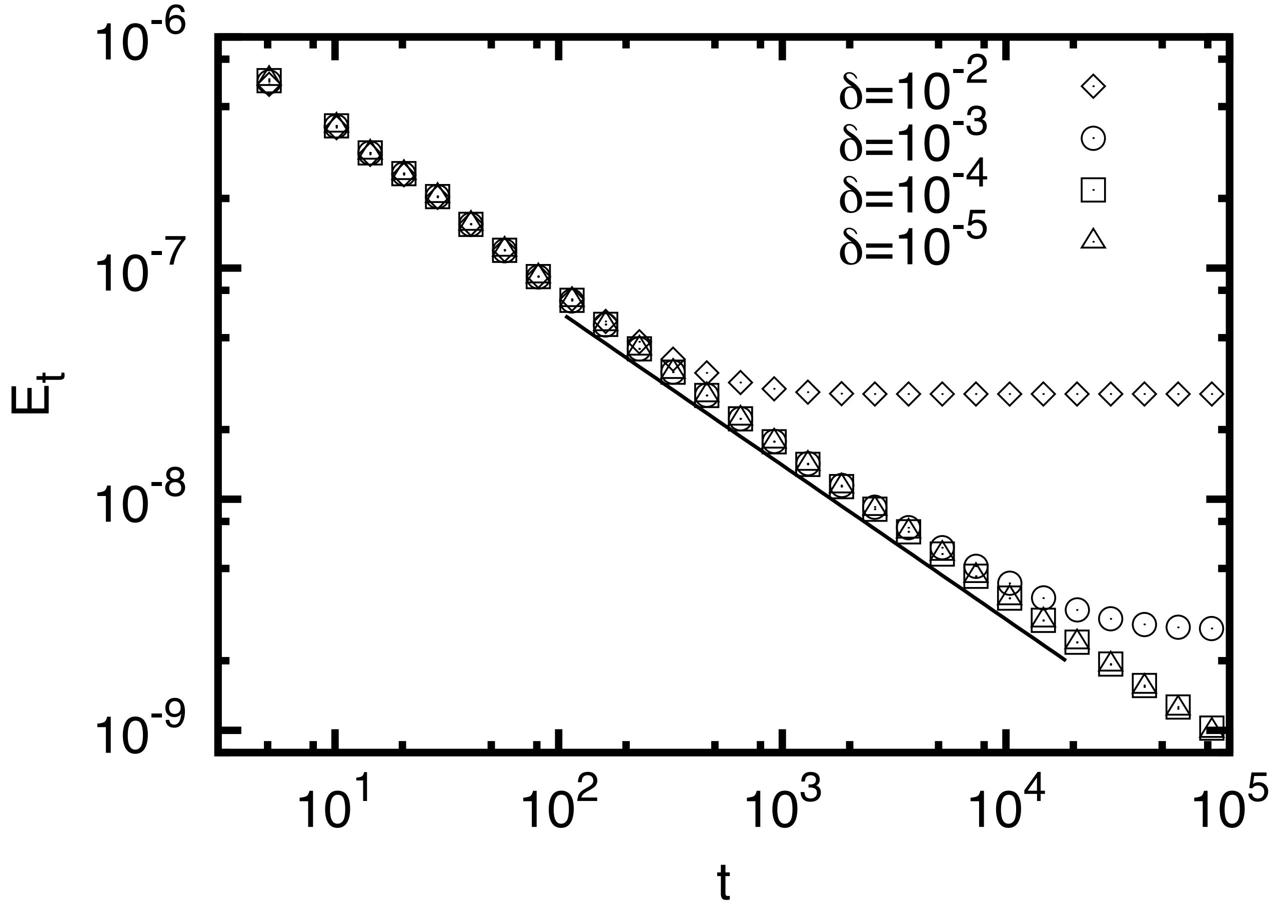
$$\langle N(t)\rangle \sim t^{3/4}$$

$$\langle E(t)\rangle \sim t^{-3/4}$$

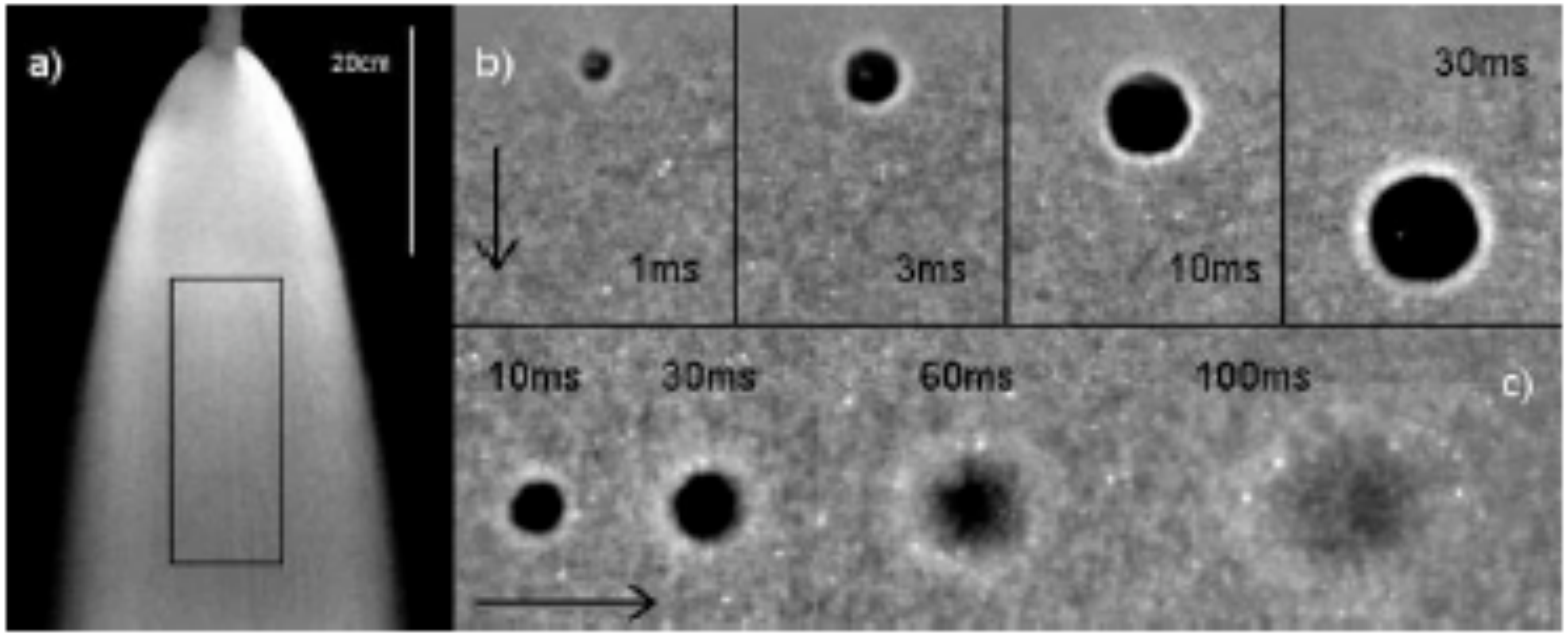




δ-dependence

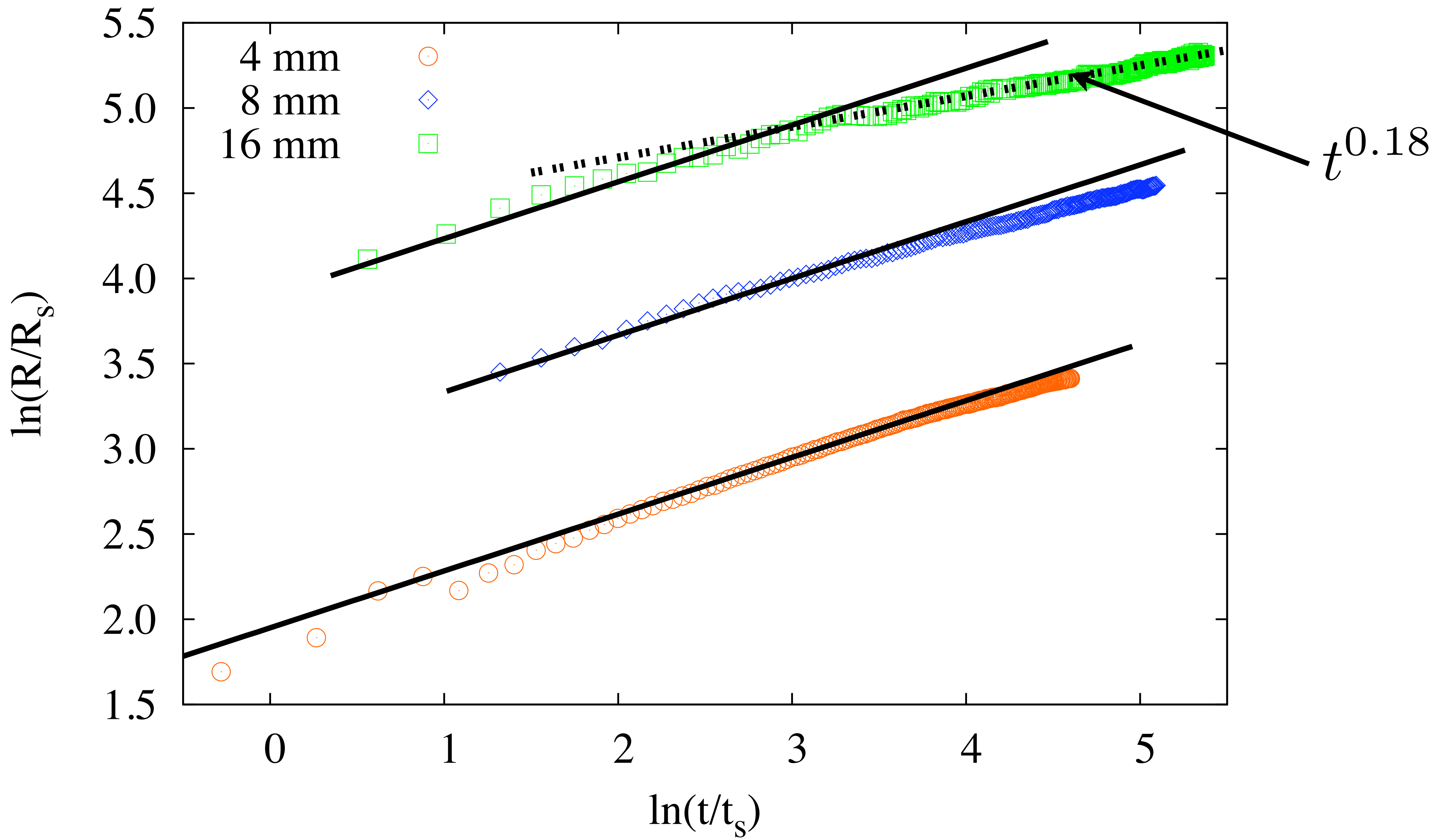


Experiments

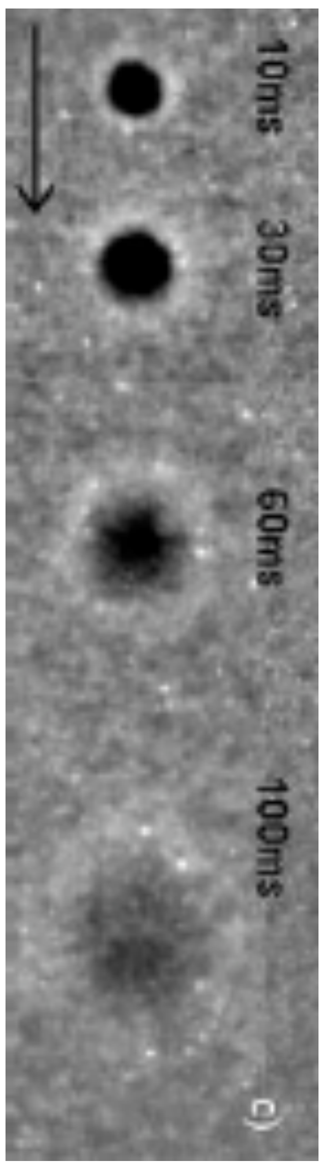


Boudet et al, PRL 2009

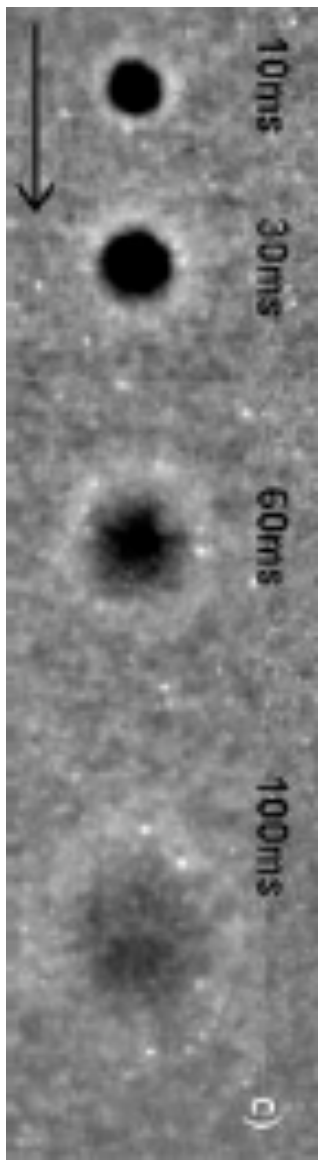
Data (Shock)

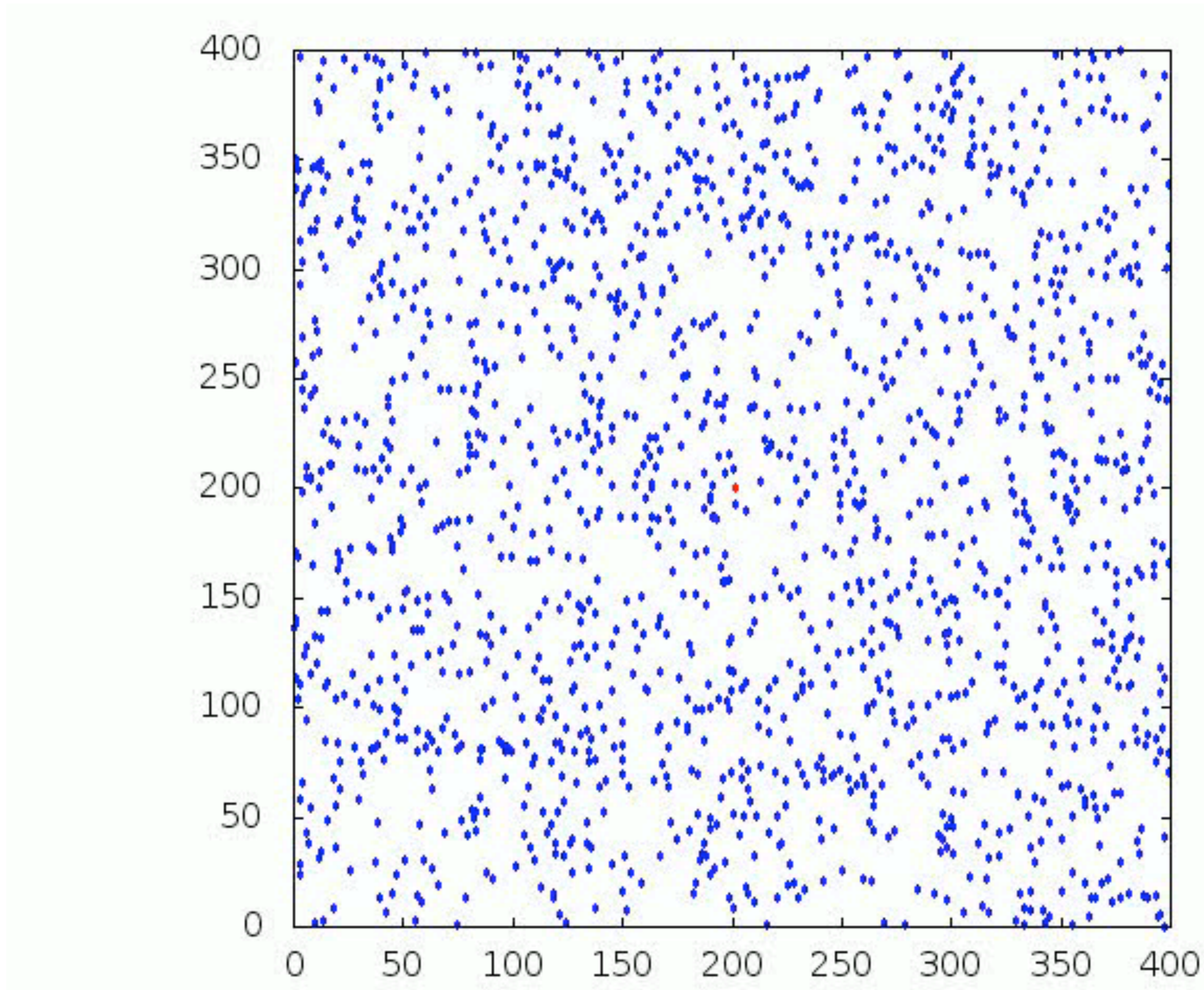


Non-zero ambient temperature

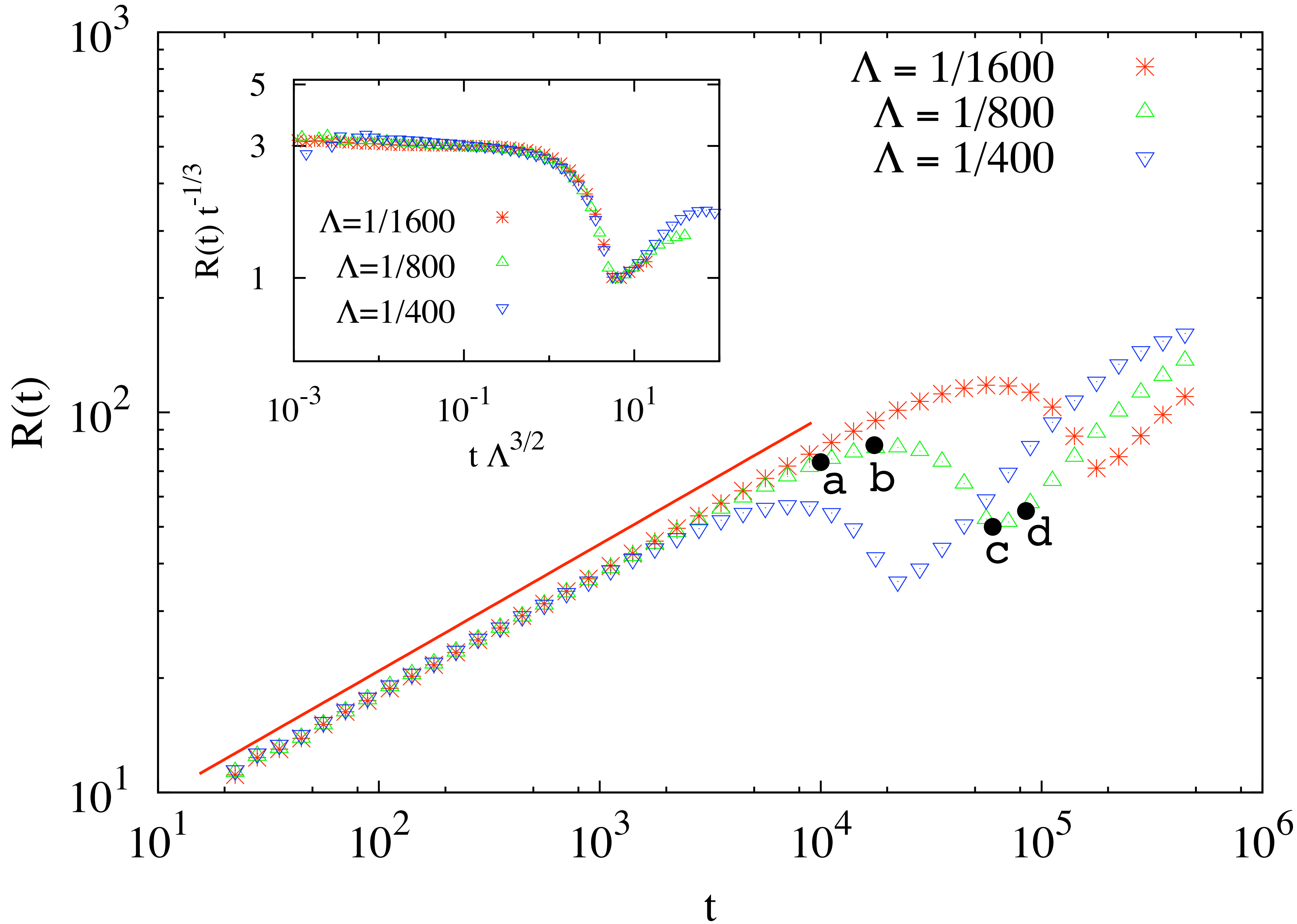


Non-zero ambient temperature

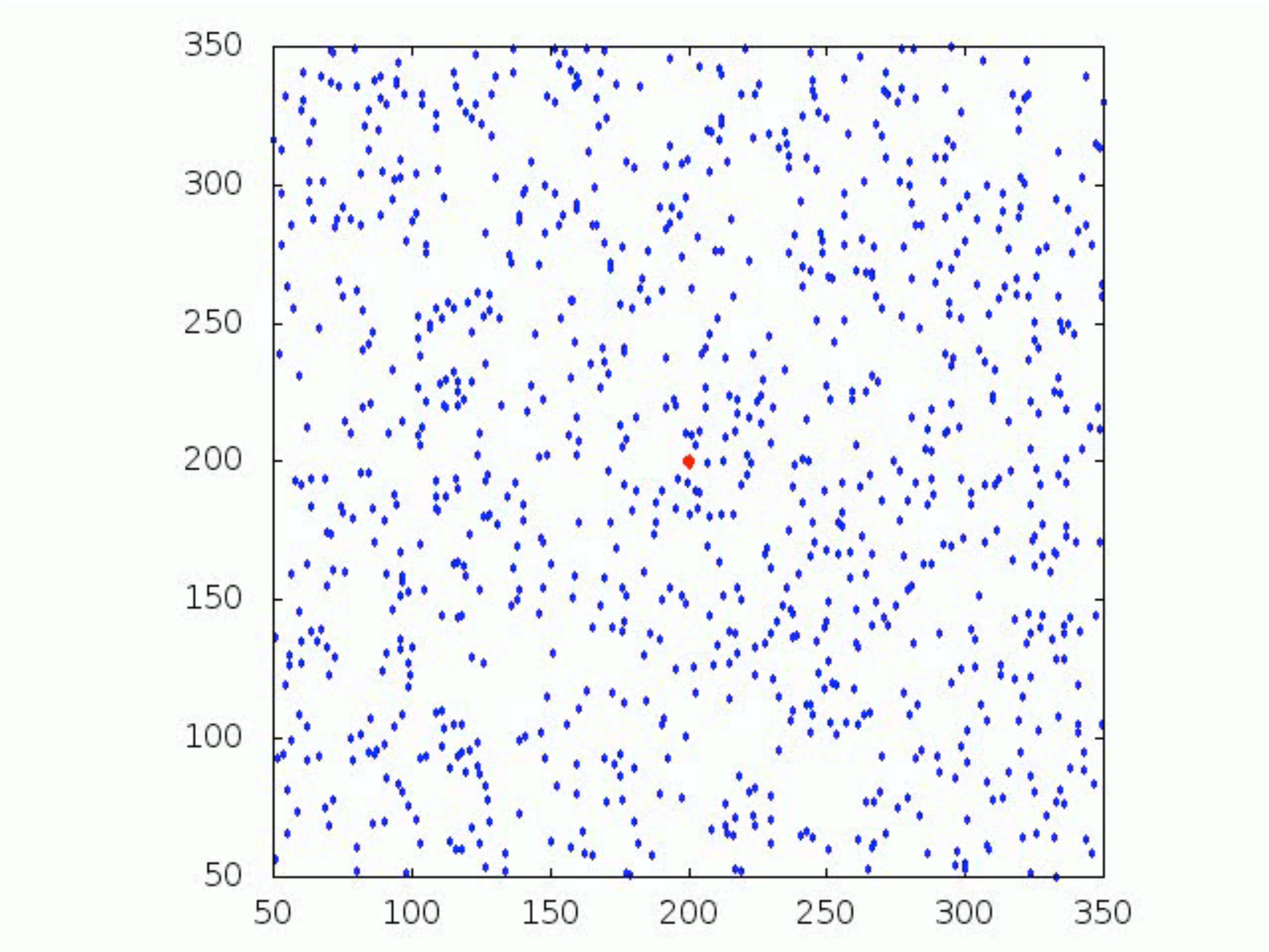




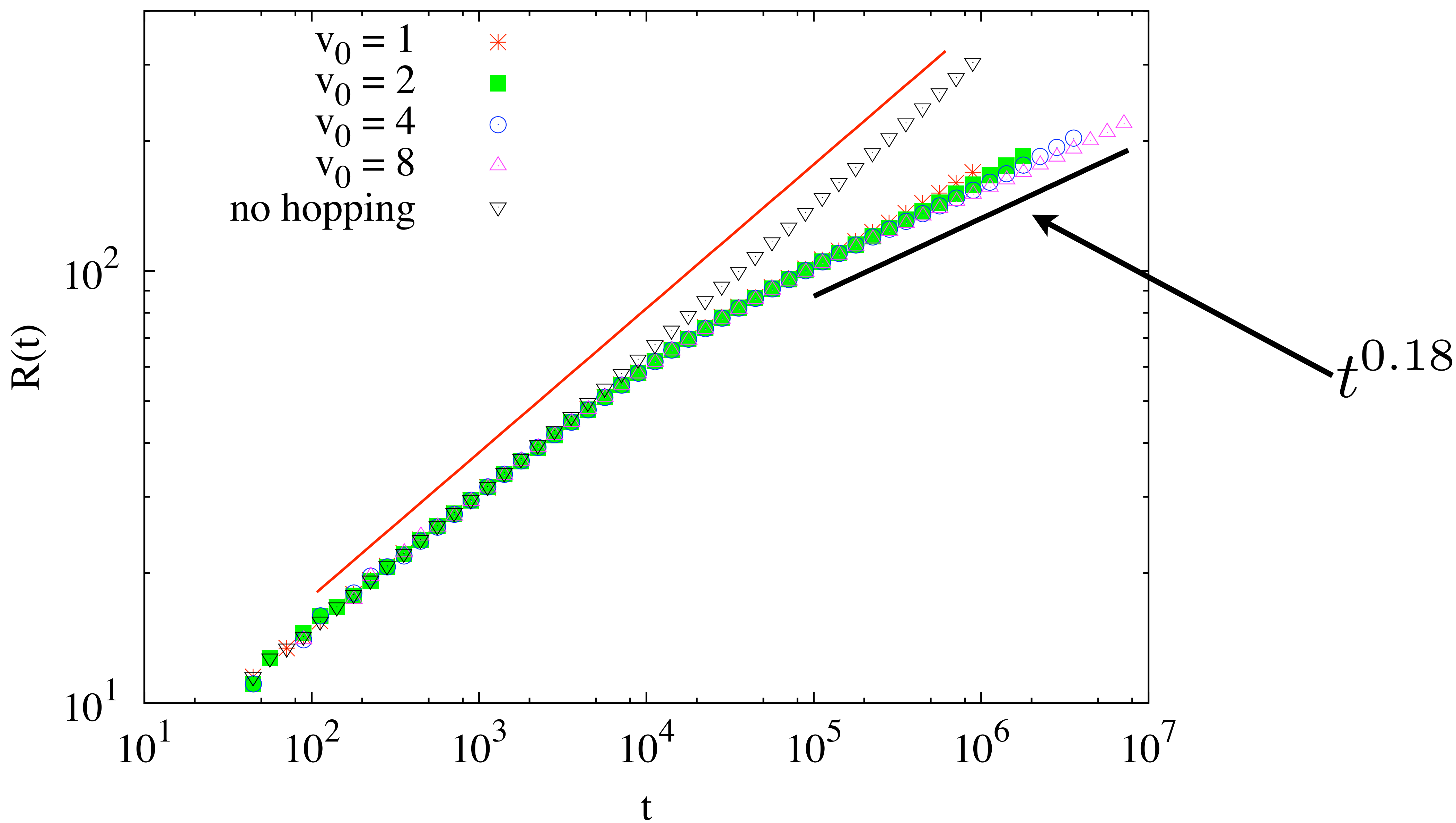
Non-zero ambient temperature



Model with escape rate

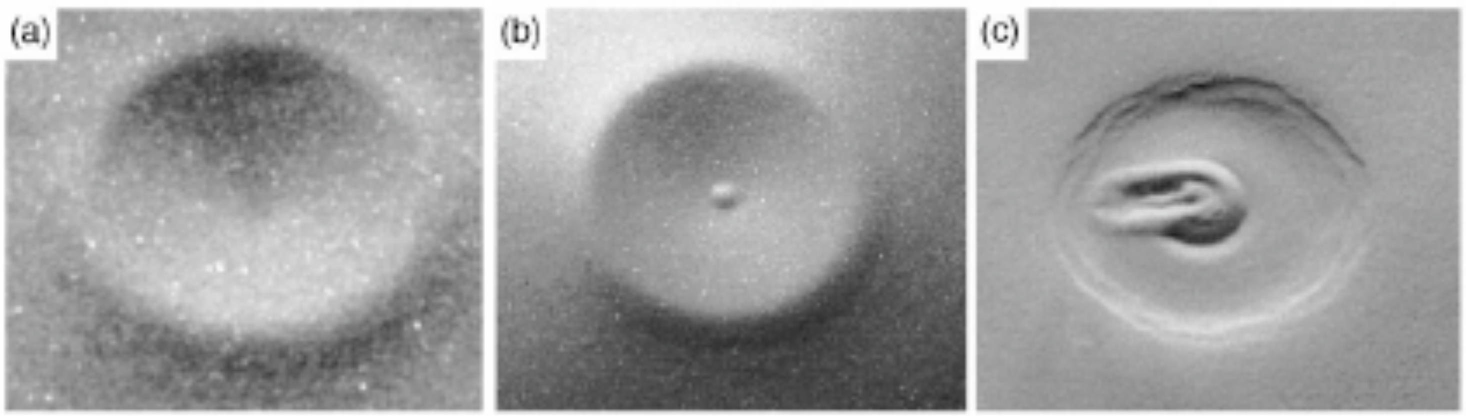


Model with escape rate



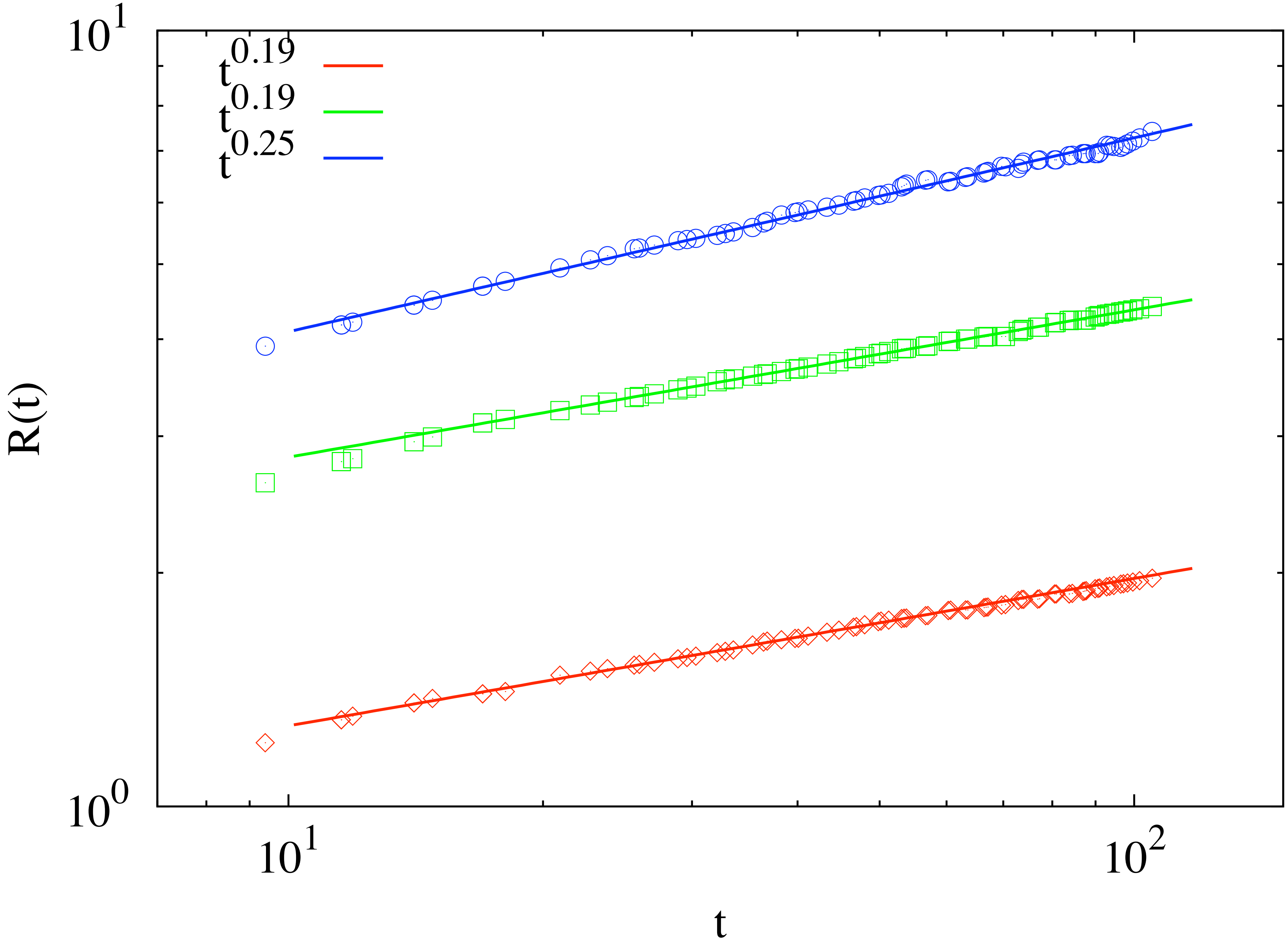
Experiment

Crater formation



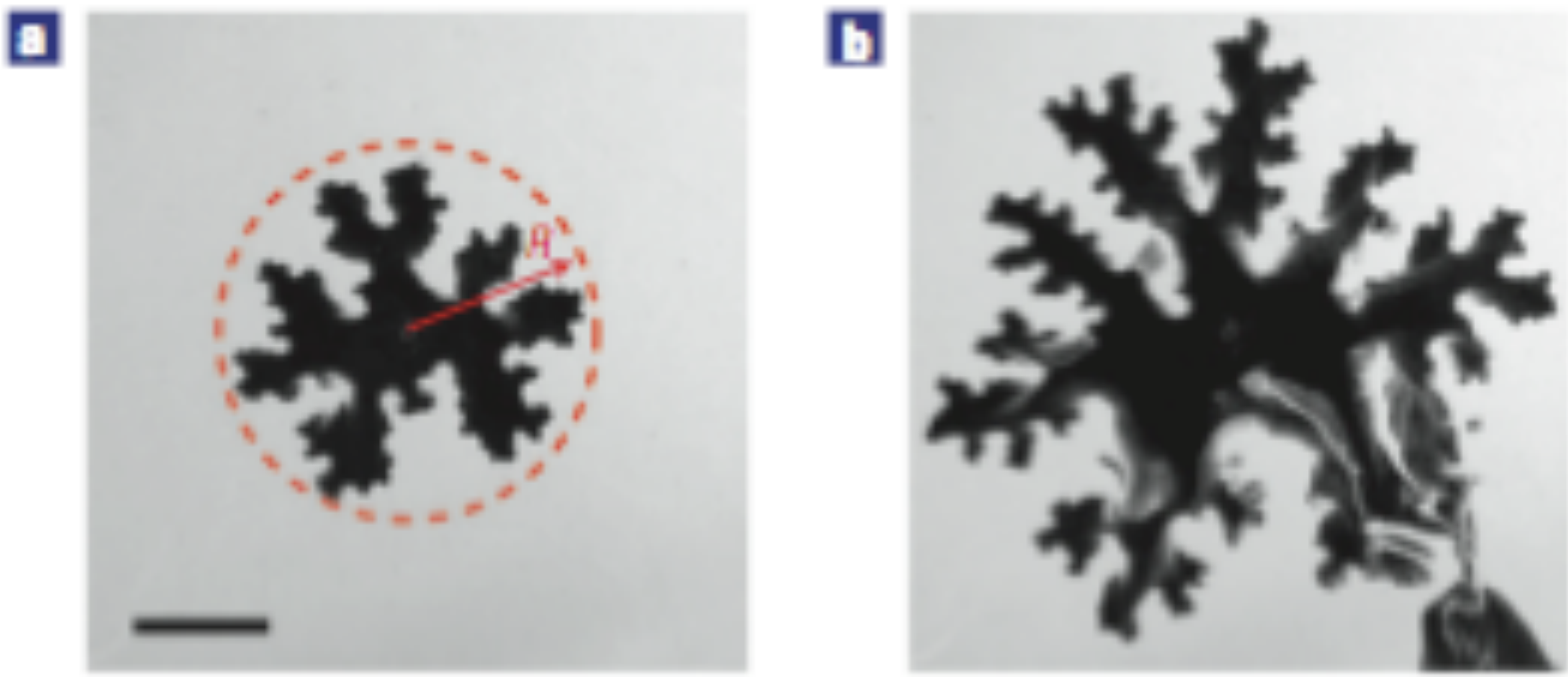
Walsh et al, PRL 2003

Data (Crater)



Experiment

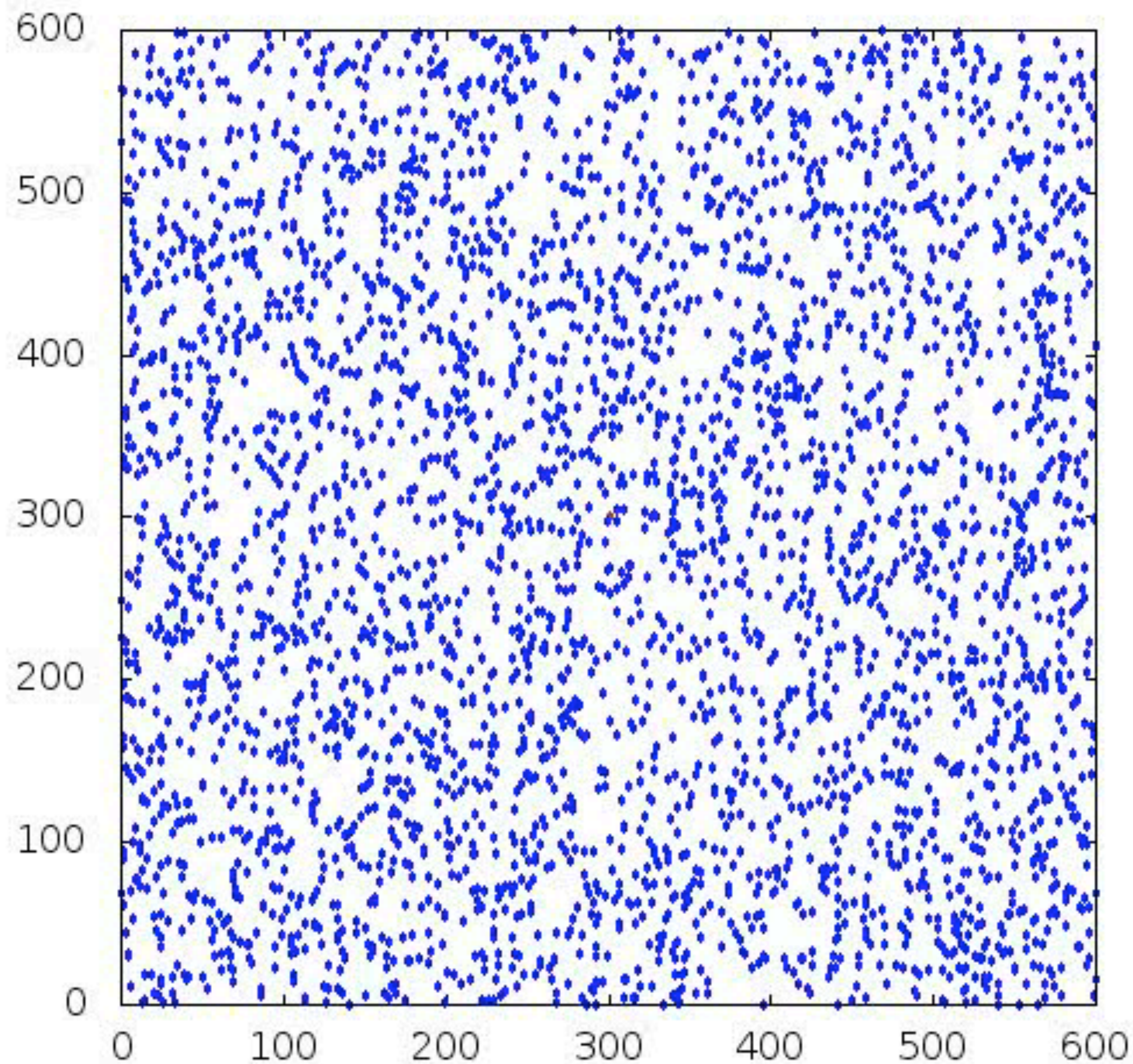
Viscous fingering



Cheng et al, Nature Phys, 2008

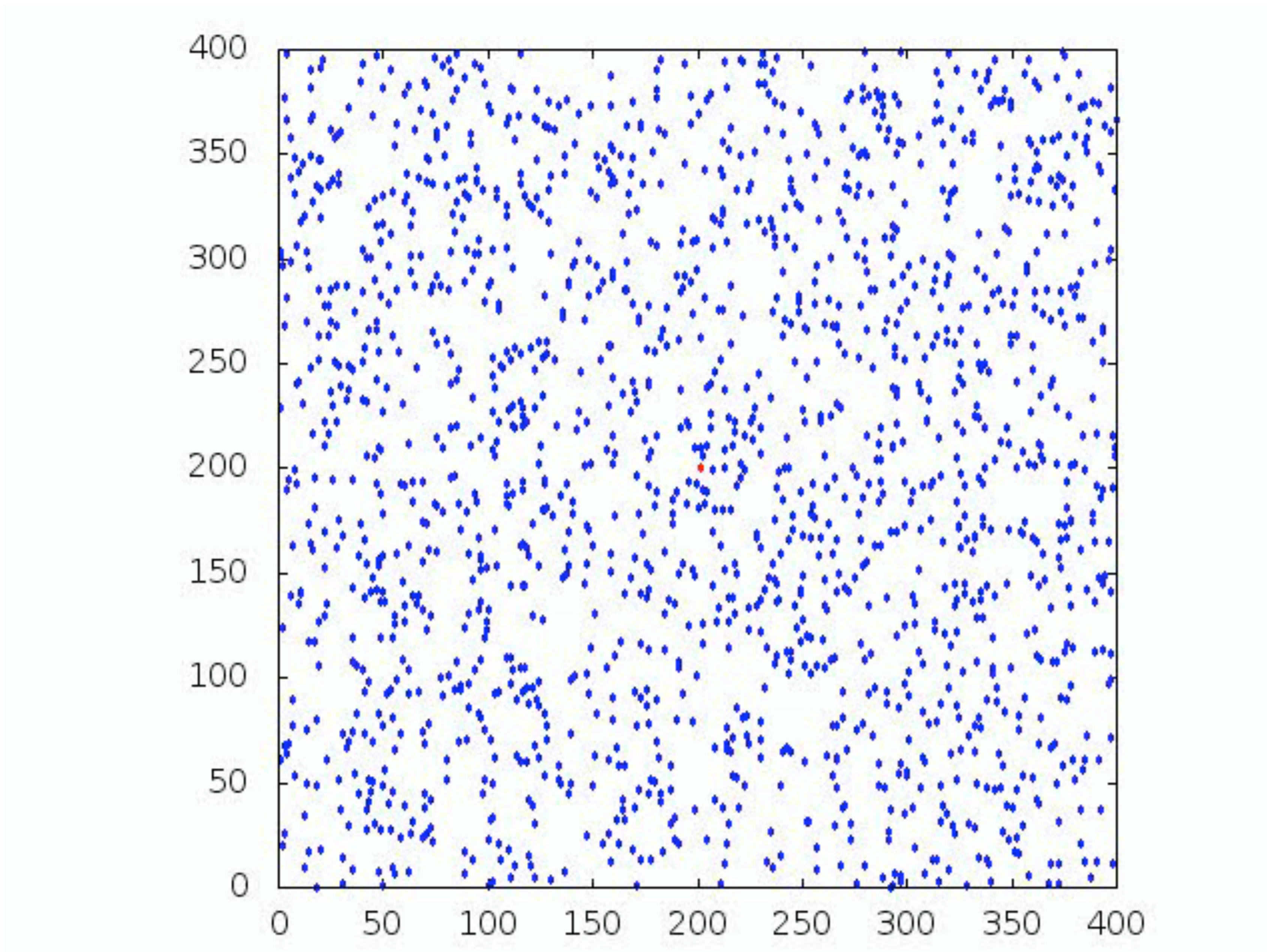
Driven gas (elastic)

Driven gas (elastic)

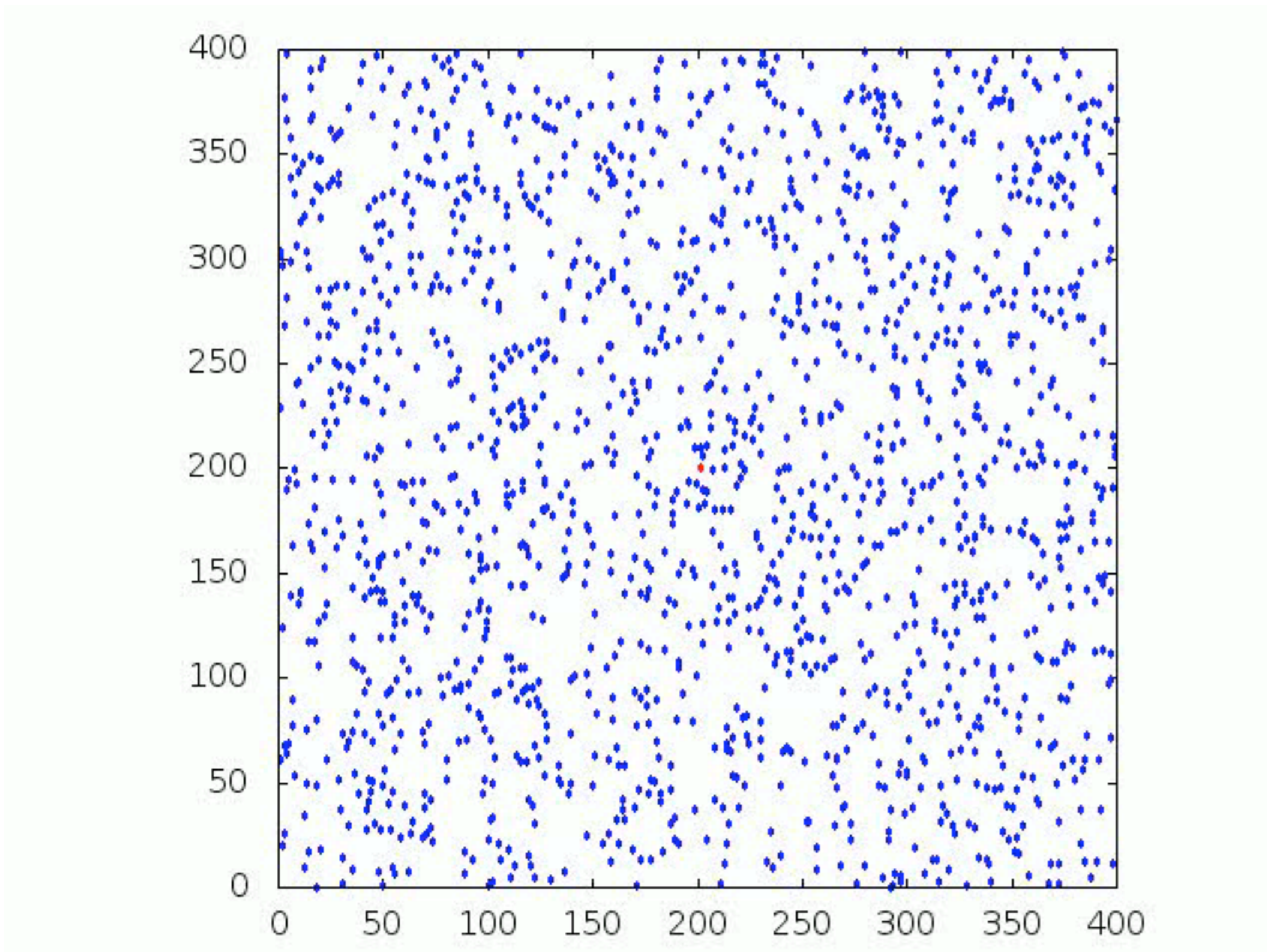


Driven gas

Driven gas



Driven gas (removal)

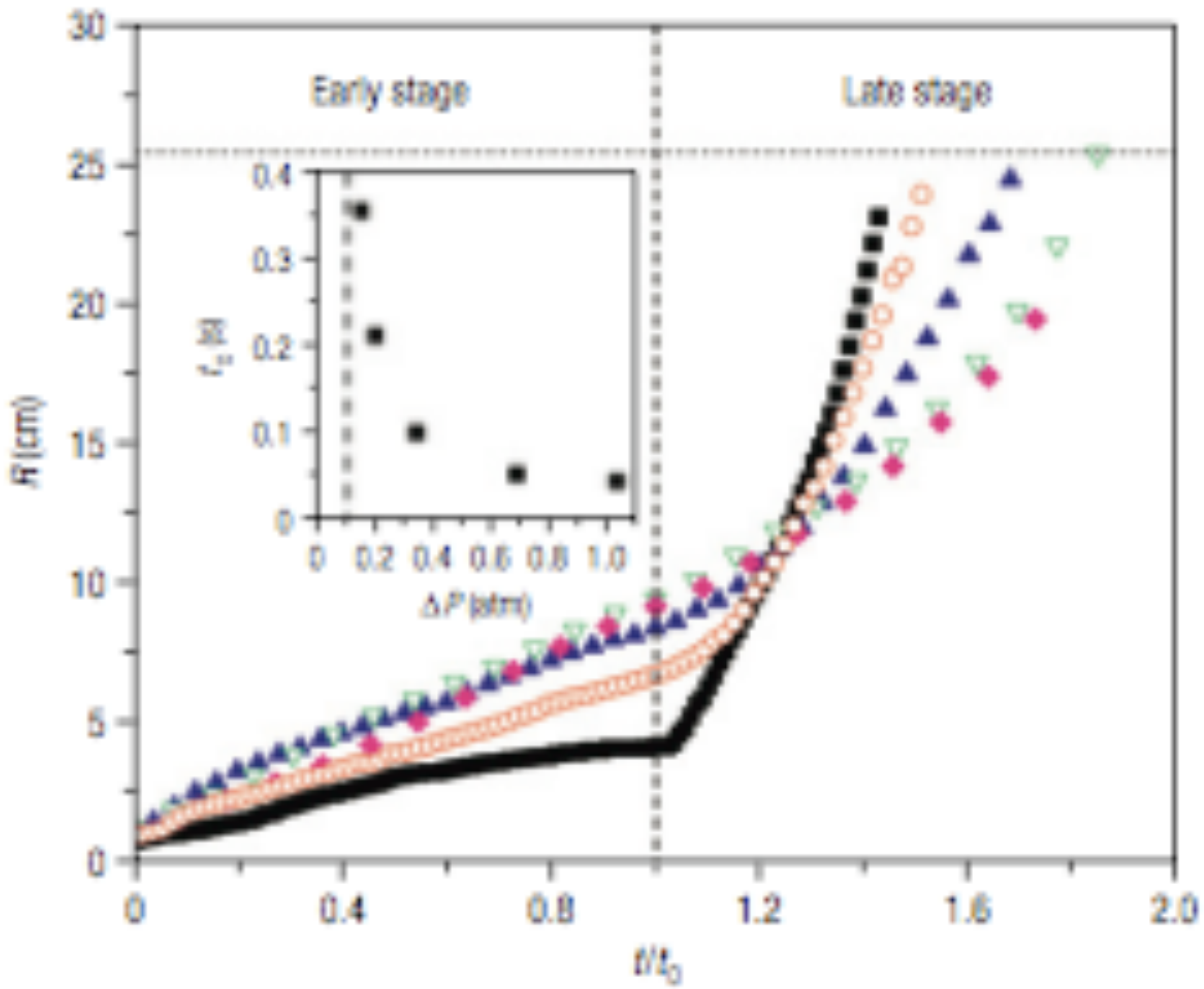


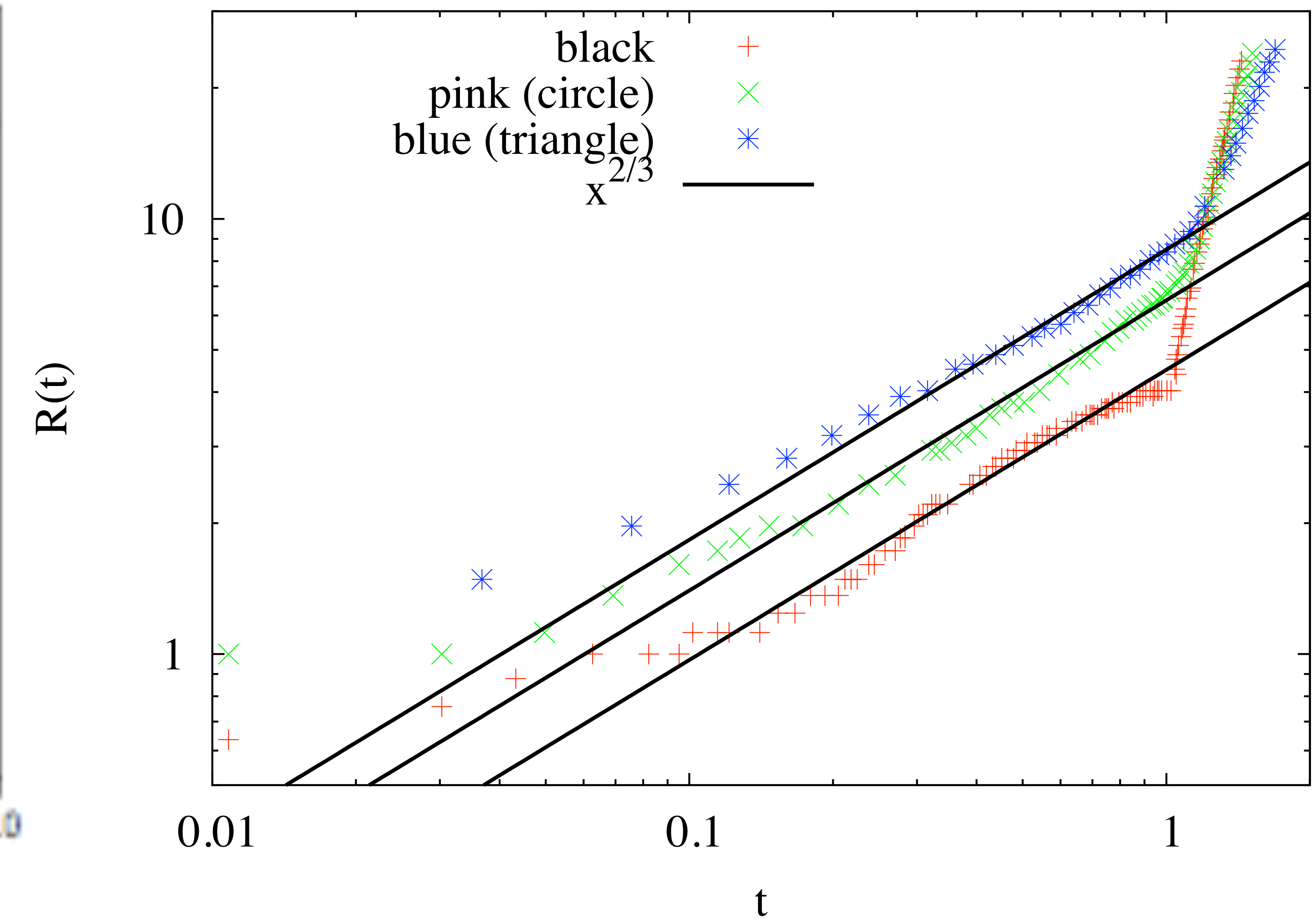
Scaling argument

Constant rate of increase of
radial momentum

$$R \sim t^{2/3}$$

Experiment


Early stage
Late stage
R (cm)
0
5
10
15
20
25
30
0
0.4
0.8
1.2
1.6
2.0
t/t0
0
0.1
0.2
0.3
0.4
0
0.2
0.4
0.6
0.8
1.0
ΔP (atm)


black
pink (circle)
blue (triangle)
x2/3
R(t)
10
1
0.01
0.1
1
t

Summary & Outlook

- A generalization of the Taylor-Sedov problem
- Inelastic $\Rightarrow$ clustering and band formation
- Conservation of radial momentum
- Exponents independent of r, form of r(v)
- Describes experimental data well

Summary & Outlook

- Understanding crossovers
- Can the freely cooling gas be understood?
- Can one solve for pressure, density distribution?
- Can experimental data be improved?