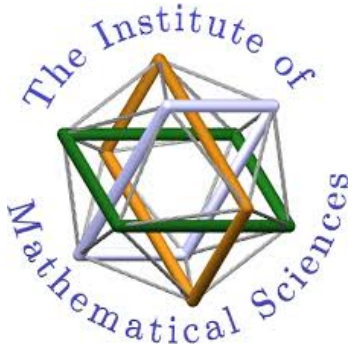
"Applied" String Theory



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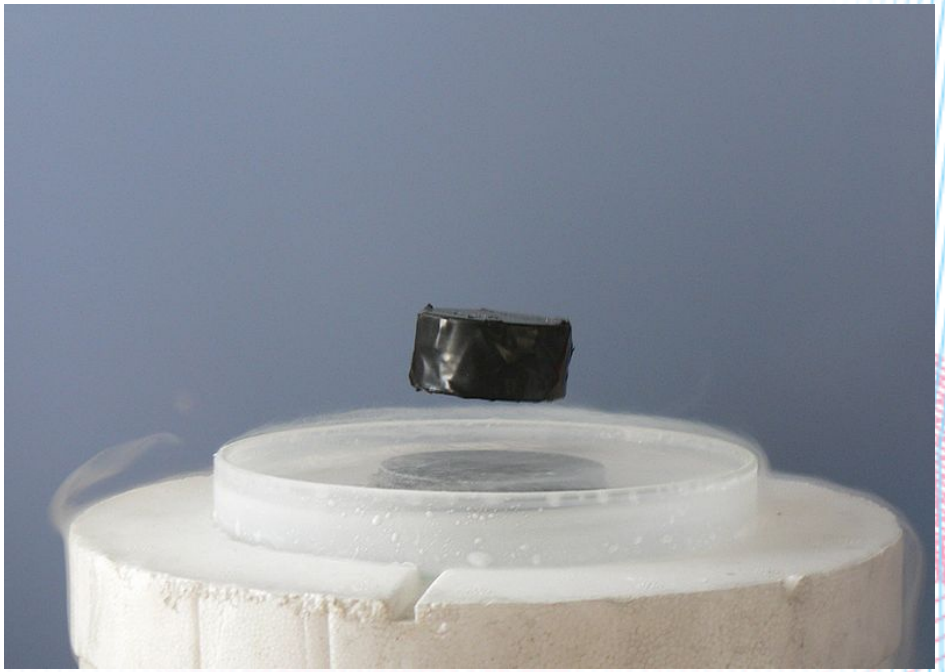
12th July, 2013

Department of Physics, Visva Bharati

Disclaimer!!

I am *not* an expert in this subject!

What is it all about?



The Fundamental Forces

- Electromagnetic force
- Weak force
- Strong force
- Gravity

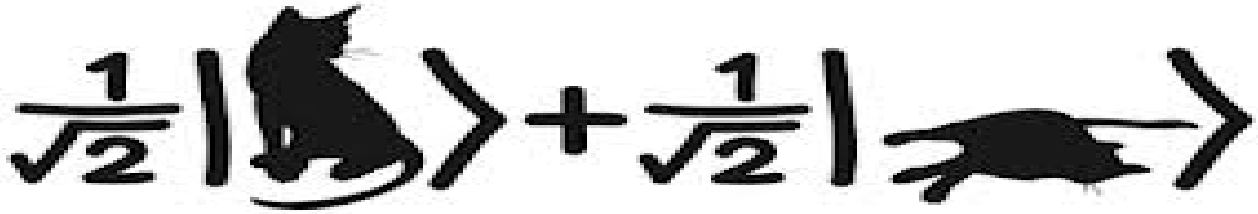
What do we know so far ?

- Quantum Mechanics

  elementary particles, atoms, molecules, ..

- General Relativity

  stars, galaxies, ...



The Quantum World

- Uncertainty : $\Delta p . \Delta x \geq \hbar/2$
- Observables : Hermitian operators in $\mathcal{H}$.
- States : Vectors (rays) in $\mathcal{H}$.
- Superposition
- Unitary evolution of states
- Entanglement

Quantum Field Theories

- Action-at-a-distance. X
- Classical fields must be quantized. √
- Particles can be born, they can die! √

Symmetries are Crucial

- Space-time symmetry

1. Lotrentz symmetry : 3 Boosts , 3 rotations
2. 4 Space-time symmetries

1 & 2 ➡ Poincare group

- Internal symmetry

1. Global symmetry
2. Local / Gauge symmetry

Beauty of Gauge Theories

Poincare + Gauge symmetries

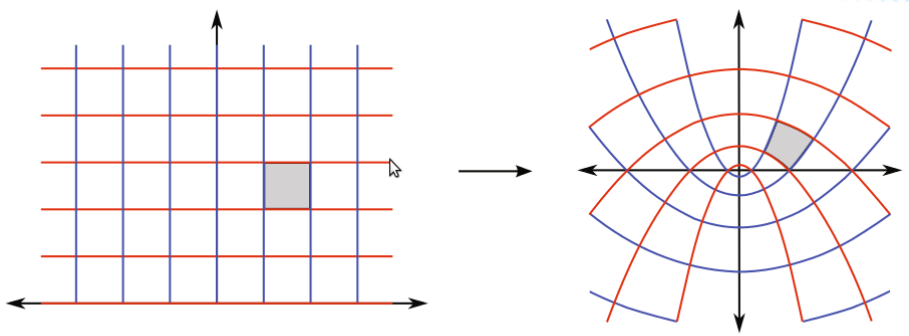
- Electromagnetic U(1)
- Weak SU(2)
- Strong SU(3)

}

- Gravity ??

What is a CFT?

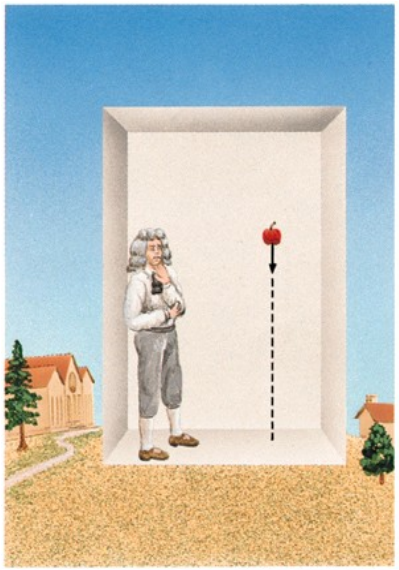
- Naively, a QFT which is scale invarient.
- Imposes constraints on Green functions.
- Symmetry group SO(d,2)
- Of extreme theoretical interest.



The Relativistic world

- Speed limit

- Equivalence principle



Gravity is different ...

- Newton's 2nd law : $F = m_I a$
- The law of Gravity: $F_G = \frac{G M_G m_G}{r^2}$

$$F_G = m_G g$$

- Inertial mass (m_I) = Gravitational mass (m_G)

$$F = F_G \Rightarrow m_I a = m_G g$$

Einstein equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{4\pi G}{c^2} T_{\mu\nu}$$

Some solutions

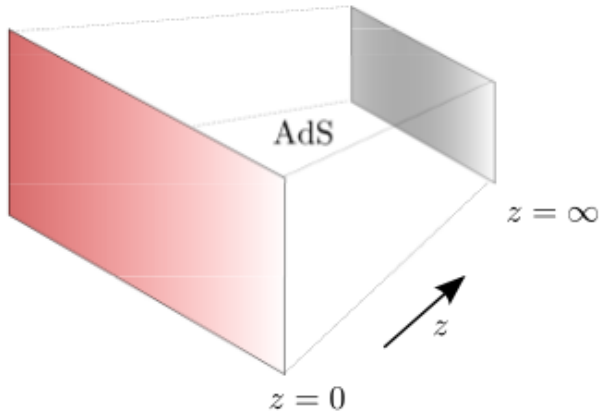
- Minkowski space $\Lambda = 0$
- De Sitter space $\Lambda > 0$
- Anti de Sitter space $\Lambda < 0$

What is AdS space?

- AdS_{d+1} is the maximally symmetric spacetime in d+1 dimensions.
- It has constant negative curvature and is solution to Einstein eqn with negative cosmological constant.
- Its isometry group is SO(d,2). ** *Same as* CFT_d !!
- Its metric has different representations.

Geometry of AdS

Poincare patch : $$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$



- Infinite number of Minkowski slices which are warped along z .

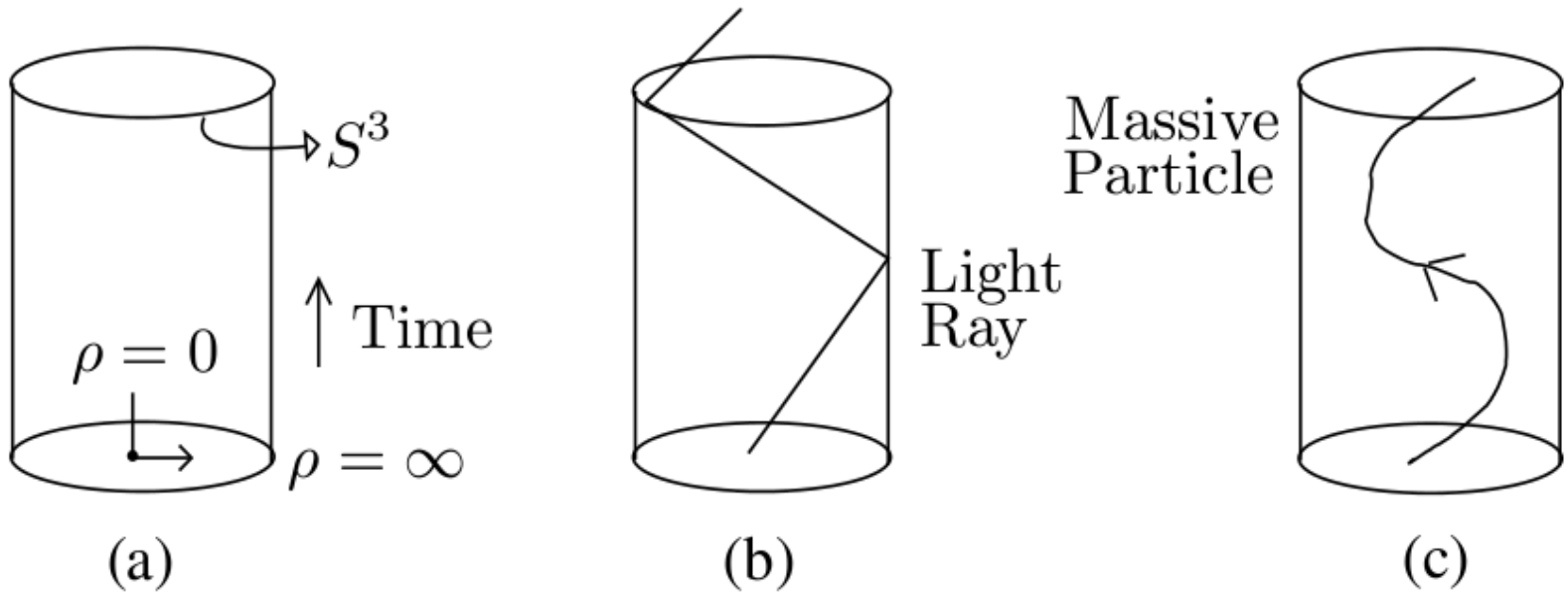
- Poincare invariance along the "slice" coordinates .

- Scaling is evident.

$$\begin{aligned} t &\to \lambda t \\ x &\to \lambda x \\ z &\to \lambda z \end{aligned} \quad \Rightarrow \quad ds^2 \to ds^2$$

Geometry of AdS

- Global AdS : $ds^2 = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega_{d-1}^2$



- Massless particles reach boundary in finite time
- Massive particles never reach !

Quantum Field Theory

+

General Relativity

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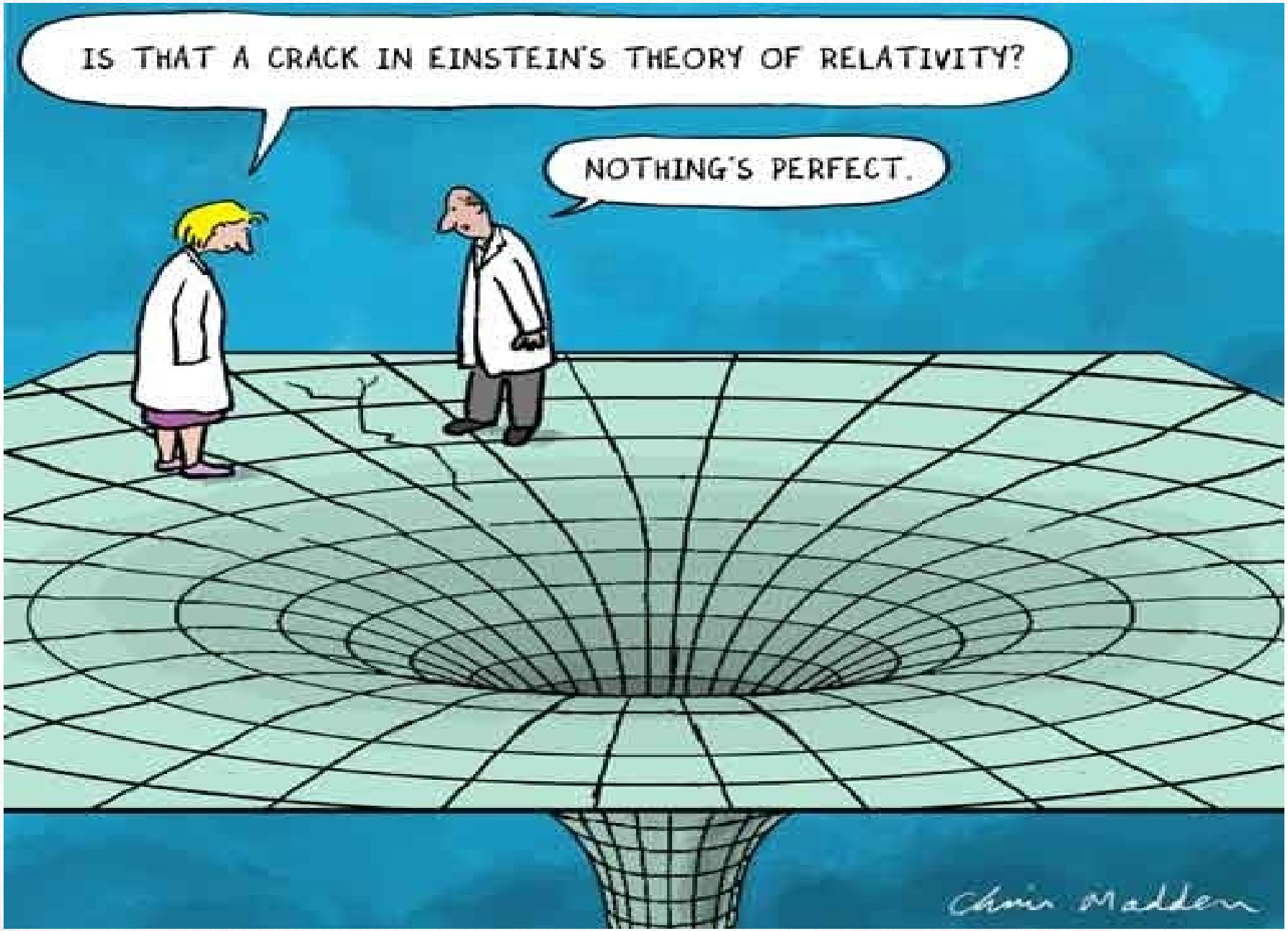
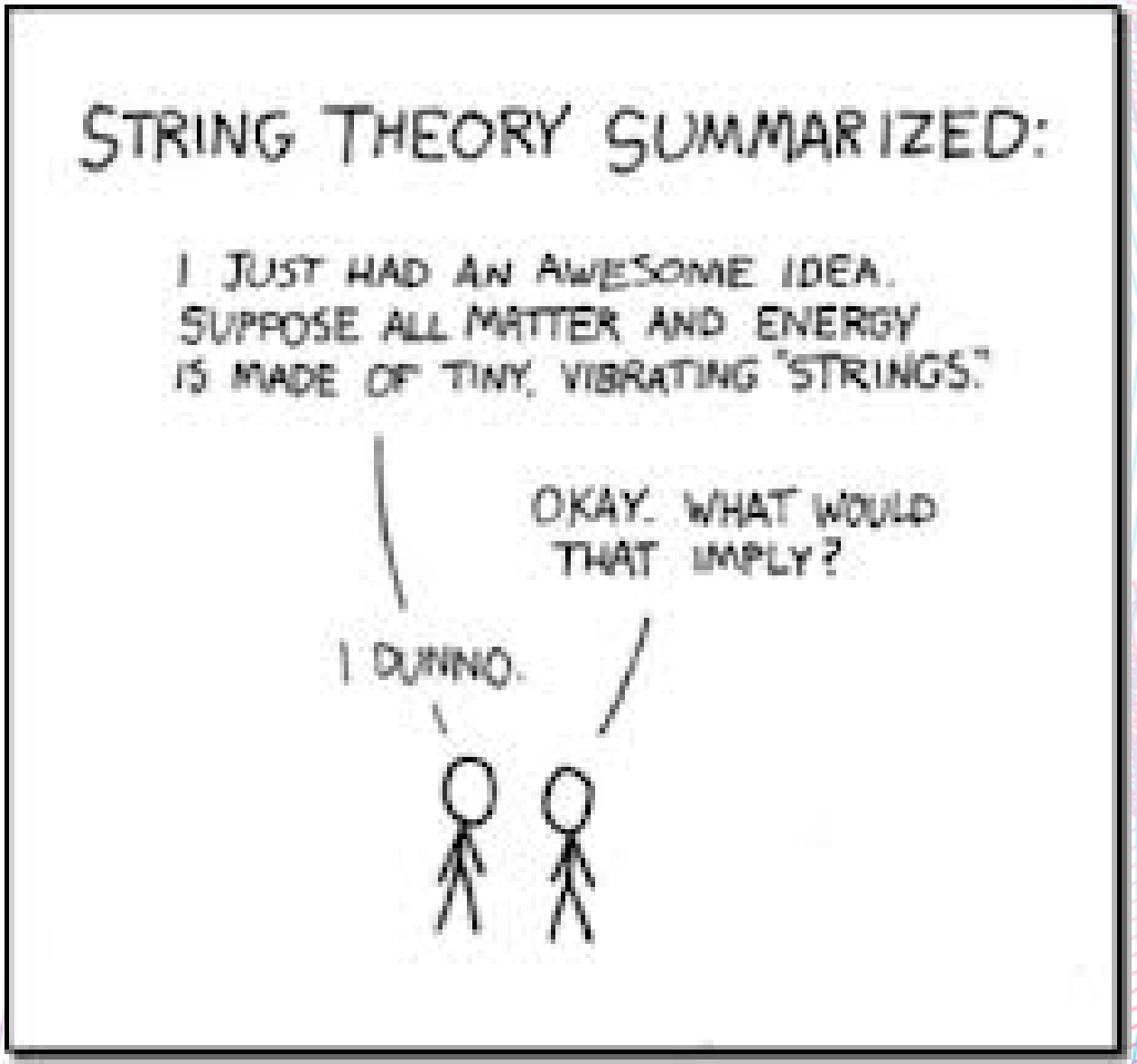
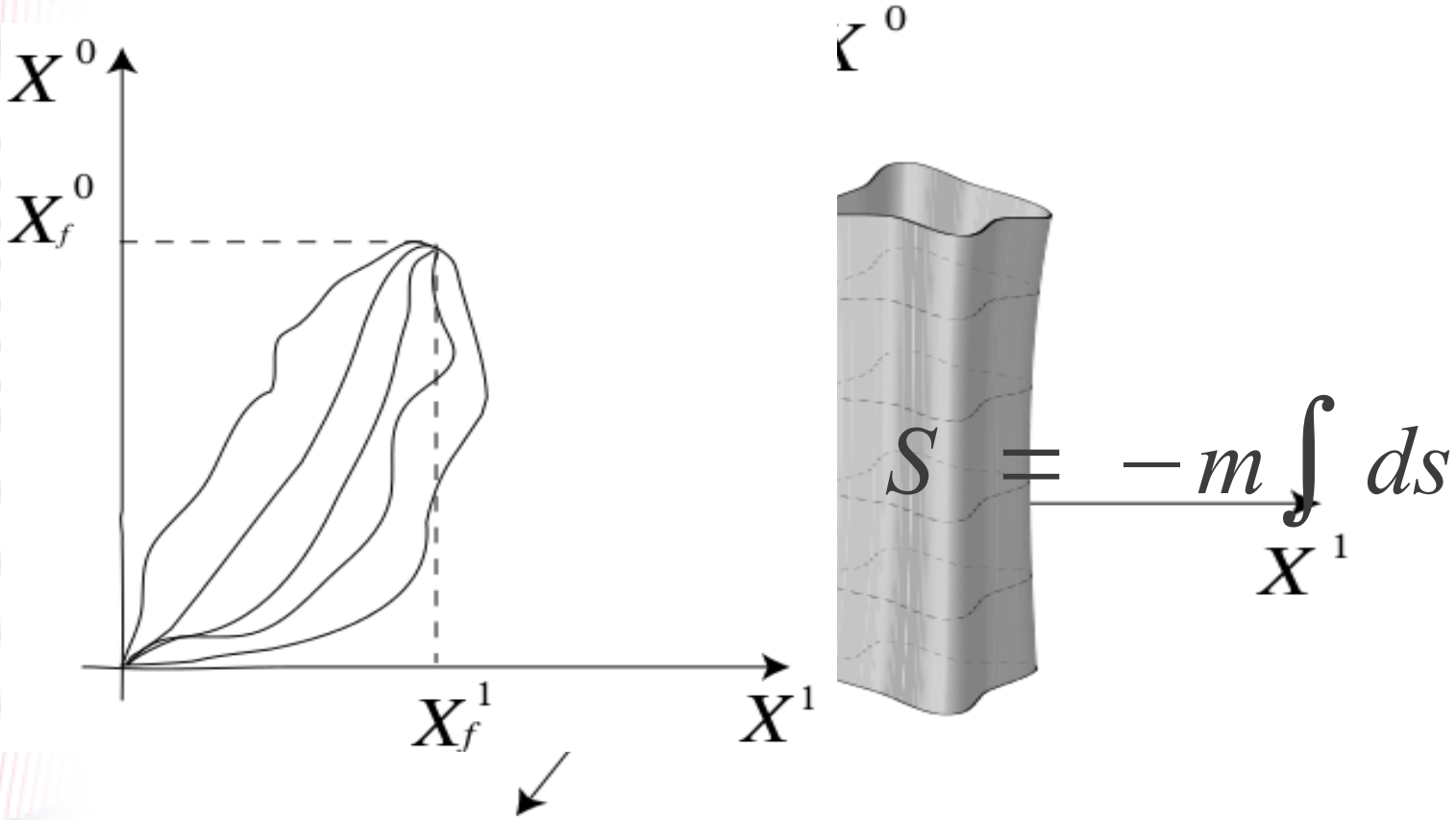

IS THAT A CRACK IN EINSTEIN'S THEORY OF RELATIVITY?
NOTHING'S PERFECT.

Change the Paradigm!!



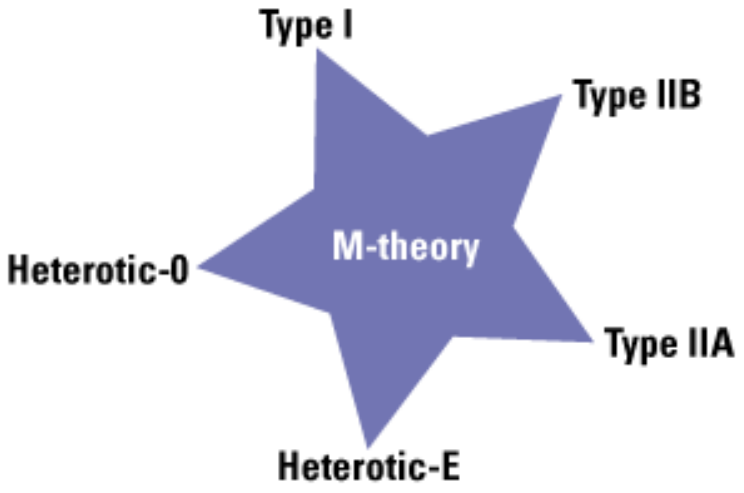
String Theory



- Nambu-Goto action : $S_{NG} = -T_0 \int d\tau\, d\sigma \sqrt{\gamma}$

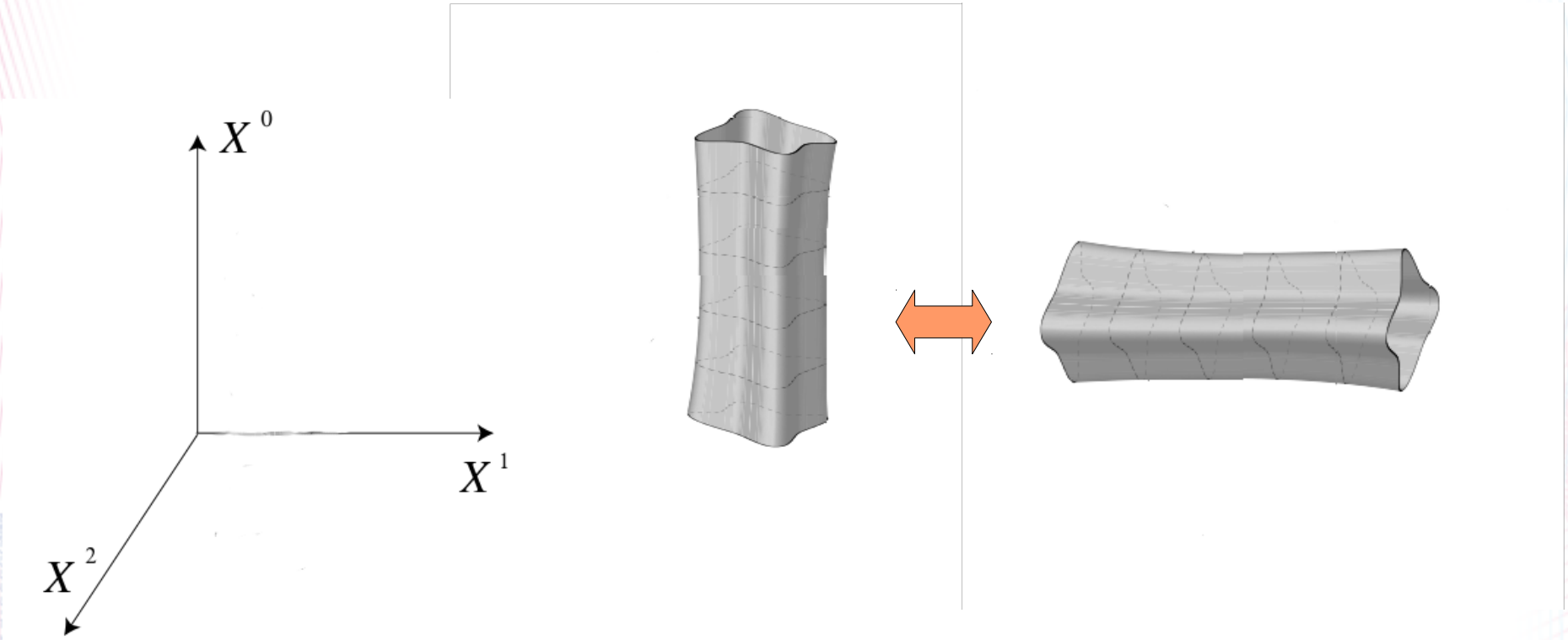
String Theory

- Bosonic string needs 26 spacetime dimensions!
- Has tachyon.
- No Fermions in its spectrum.
- Superstring lives in 10 dimensions.



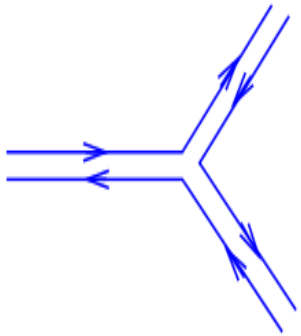
Motivating AdS/CFT

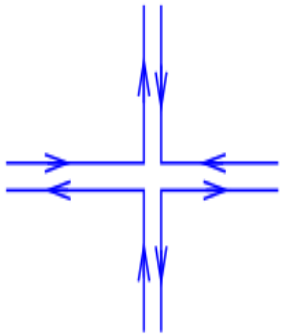
1. Open string-closed string duality

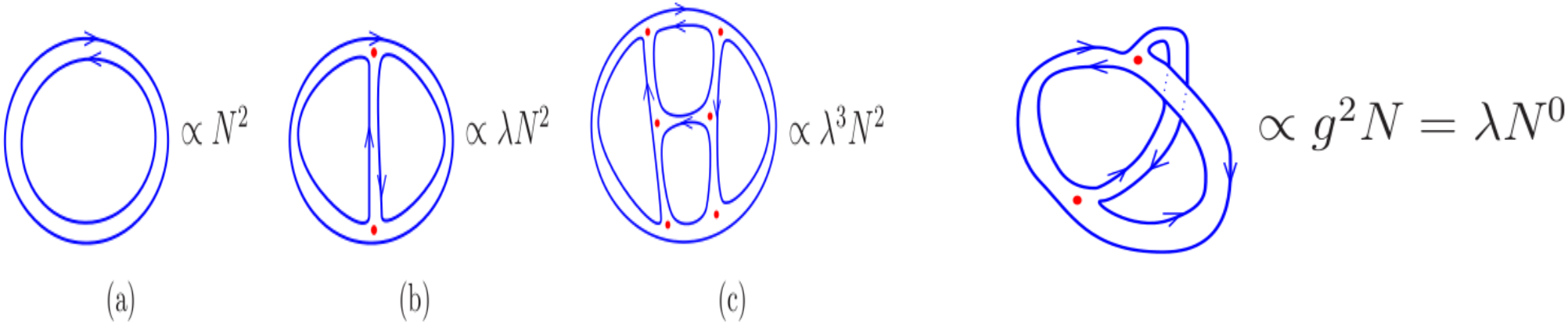


Motivating AdS/CFT

2. Large-N gauge theories

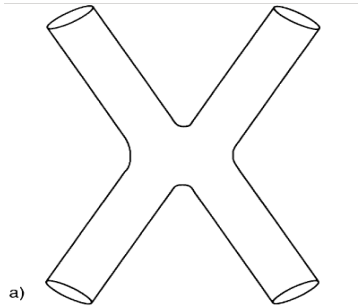


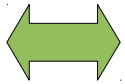


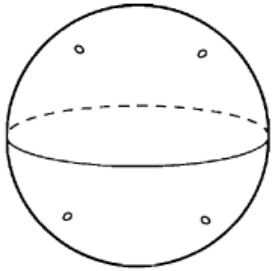


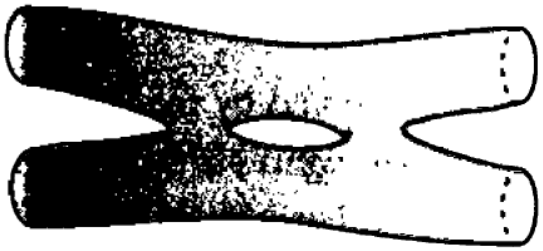
Motivating AdS/CFT

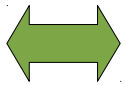
- String perturbation theory :

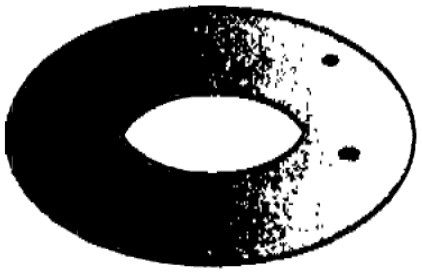












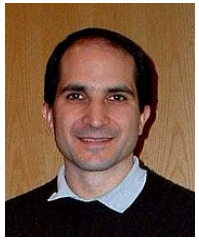
Motivating AdS/CFT

3. Idea of Holography

- Black hole Entropy

$$\approx \frac{Area}{4}$$

AdS/CFT Correspondence



Maldacena '97

- Type IIB string theory in $AdS_5 \times S^5$

$$\mathcal{N}=4 \;\; SU(N_c) \;\; \text{SYM in} \;\; M_4$$

Witten '98

- $$Z_{AdS} \equiv Z_{CFT}$$

$$Z_{QG}(\phi_0^i) = \left\langle \exp\left(\int_{S^d} \phi_0^i O\right) \right\rangle_{CFT}$$

GKPW formula

The AdS/CFT Dictionary

$AdS^5 \times S_5$	$\mathcal{N} = 4$ SYM
L_{AdS}	none
$G_{N(5)}$	$\frac{\pi}{2} L^3 / N_c^2$
(string length) ℓ_s	$\lambda^{-1/4} L$
$(L/\ell_s)^4$	λ
(string coupling) g_s	$g_{YM}^2 / 4\pi$

The AdS/CFT Dictionary

$AdS^5 \times S_5$	$\mathcal{N} = 4$ SYM
Fields ϕ	Operators $\mathcal{O}$
$\phi(r \to \infty)$	source, J coupled to $\mathcal{O}$
$g^{\mu\nu}$	$T^{\mu\nu}$
A^a_μ	J^a_μ

$\mathcal{N} = 4$ SYM	$AdS^5 \times S_5$
$T = 0$	AdS_5
$T > 0$ (eq. plasma)	AdS_5-Schwarzchild
entropy density $s = \frac{S}{V} = \frac{\pi^2}{2} N_c^2 T^3$	$\frac{A/4G}{V}$

Why so interesting?!!

- 1. $N_c \to \infty, \ \lambda$ is fixed

  ➡ Classical string theory

- 2. $L \gg l_s, \ \lambda \gg 1$

  ➡ Classical supergravity!

$$g_s = \frac{g_{YM}^2}{4\pi}$$

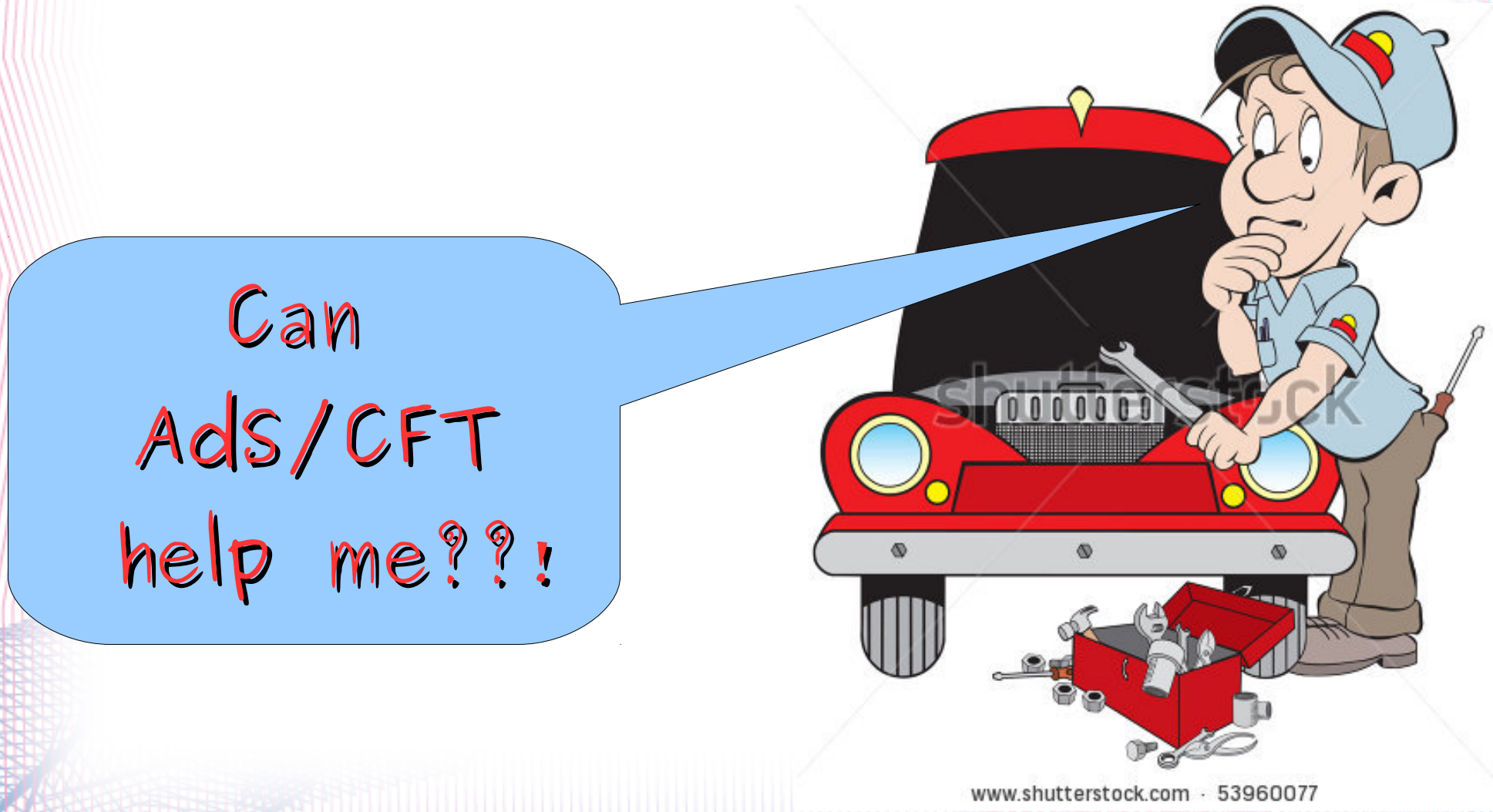
$$\frac{L^4}{l_s^4} = g_{YM}^2 N_c = \lambda$$

- Strongly coupled theory ⟺ Classical (super-)gravity

Some interesting applications

- Entropy to viscosity ratio of QGP : $\frac{\eta}{s} = \frac{1}{4\pi}$
- AdS/QCD
- Holographic superconductivity
- Quantum phase transition
- Quantum quench
- Physics out of equilibrium and ...

Disclaimer !!



QCD vs $\mathcal{N}$ = 4 SYM Plasma

QCD	$\mathcal{N} = 4$ SYM
adjoint gluons	adjoint gluons
fundamental quarks	adjoint fermions(4) + scalars(6)
asymptotic freedom	Conformal (scale invariant)

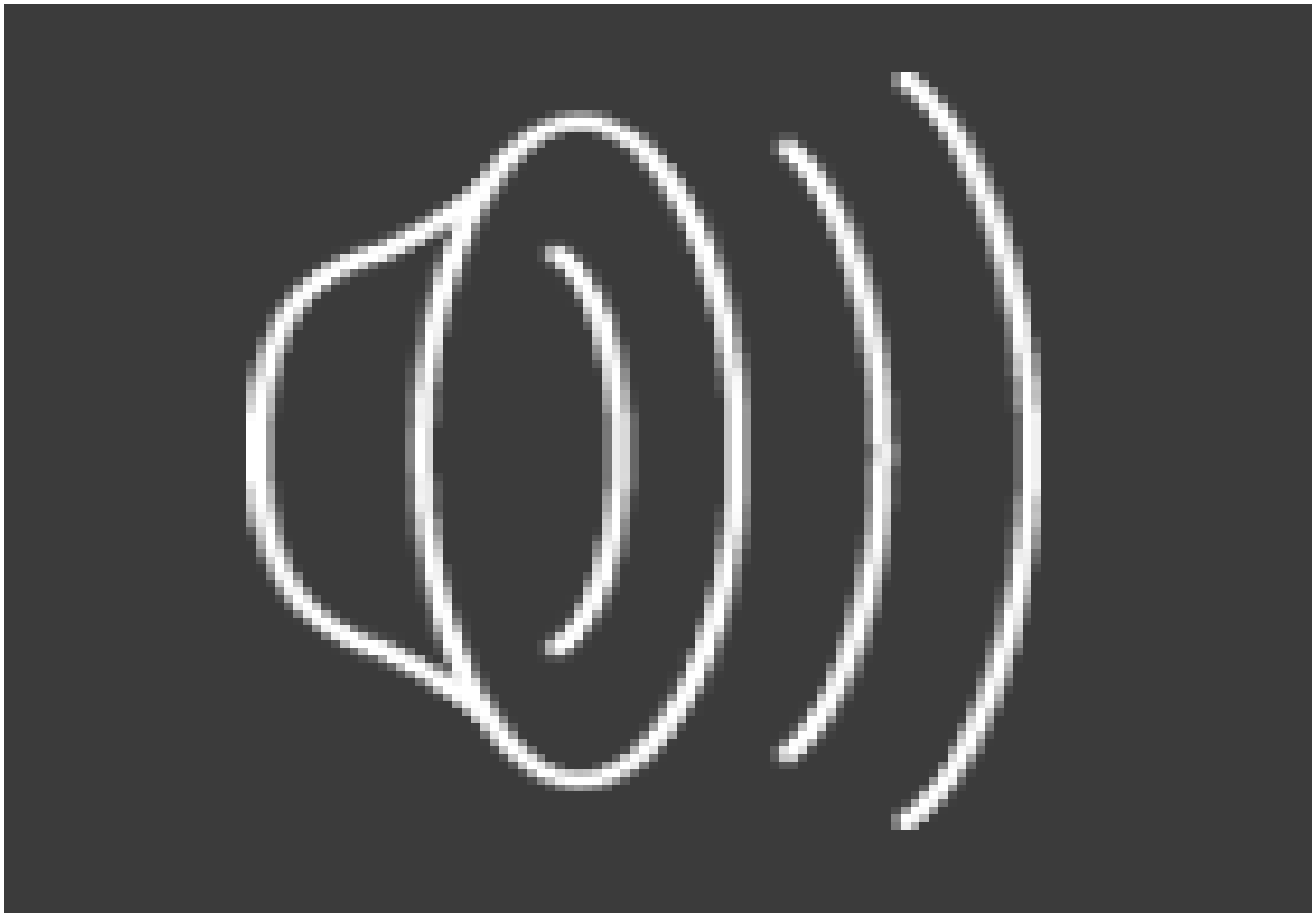
T = 0

confinement, hadrons, S-matrix	none

T> 0

non-abelian plasma	✓
Debye screening	✓
Corr length $\xi < \infty$	✓
Hydrodynamics	✓

Brownian Motion



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Langevin dynamics

- $$M\frac{d^2x}{dt^2}+\eta\frac{dx}{dt} = \xi(t)$$

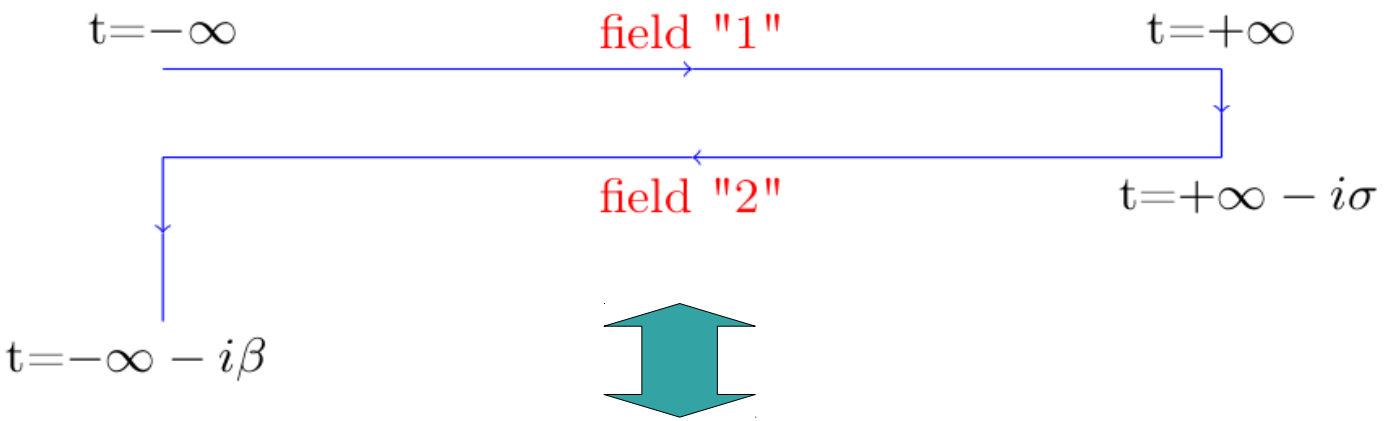
$$\langle \xi(t)\xi(\tilde{t})\rangle = 2\mathrm{T}\,\eta\,\delta(t-\tilde{t})$$

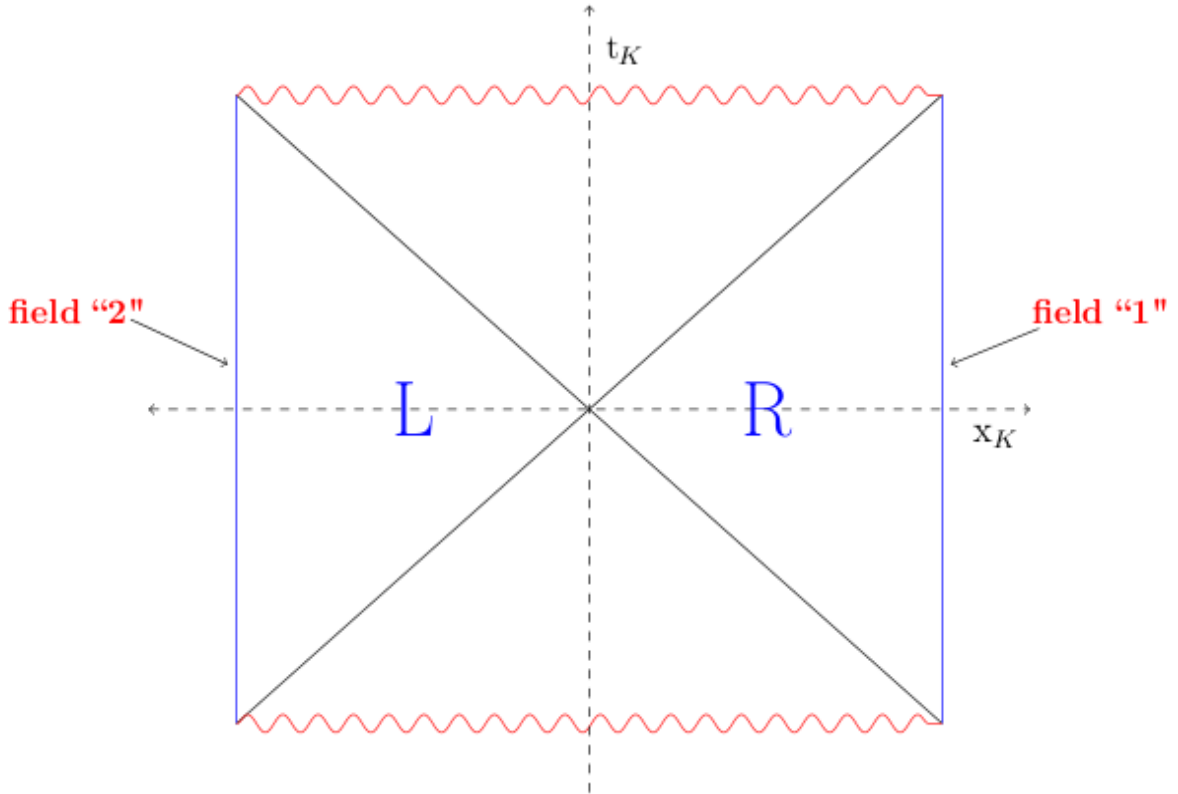
- This is a form of Fluctuation-dissipation theorem.
- Quantum version of FD theorem :

$$\langle \xi(t)\xi(\tilde{t})\rangle = -Im\,G_R(t,\tilde{t})$$

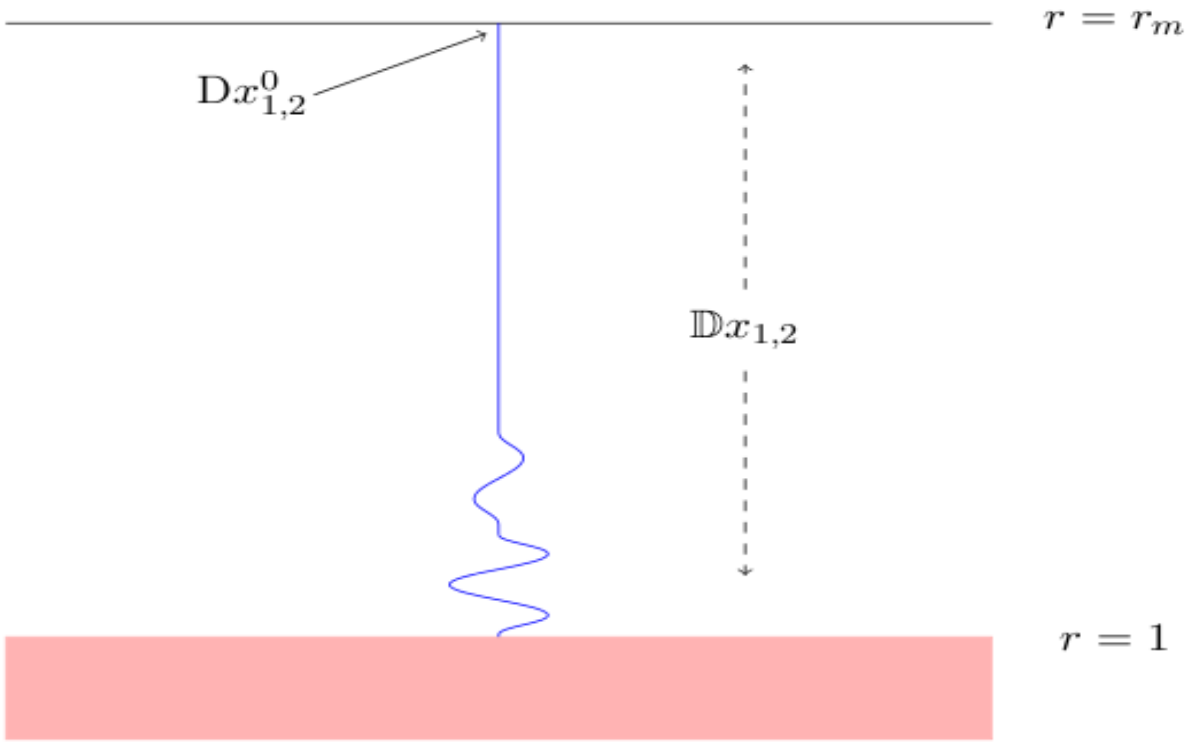
- For ultimate long time scale : $\eta\frac{dx}{dt} = \xi$

Contour & Gravity



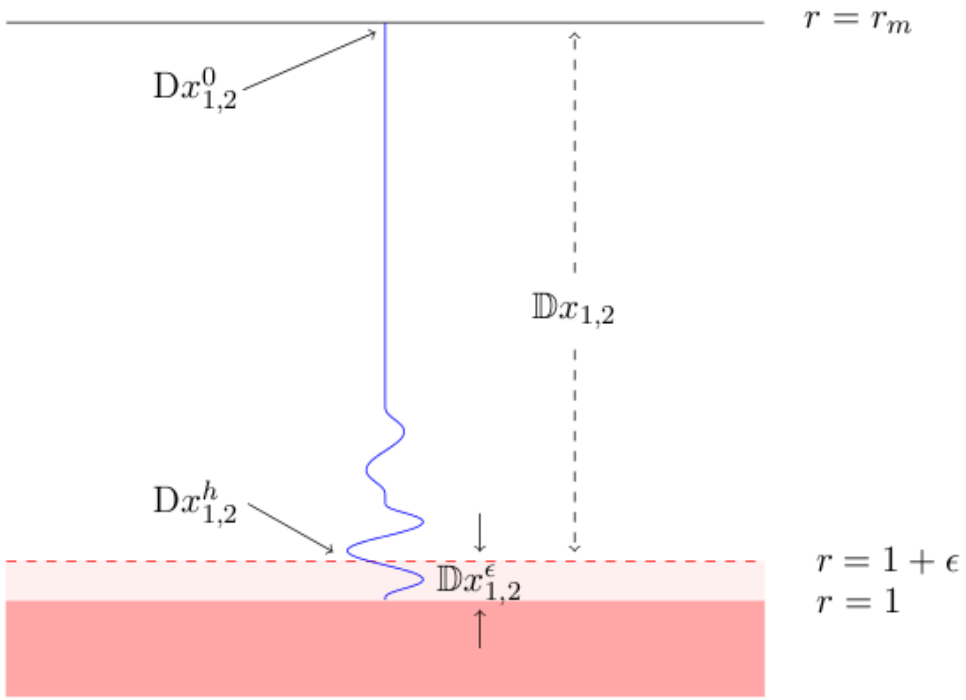


Brownian motion in AdS/CFT



$$Z = \int [\mathrm{D}x_1^0][\mathrm{D}x_2^0]\,\underbrace{[\mathbb{D}x_1][\mathbb{D}x_2]e^{iS_1 - iS_2}}$$
$$\equiv \int [\mathrm{D}x_1^0][\mathrm{D}x_2^0]e^{iS^0_{\mathrm{eff}}}$$

Brownian motion in AdS/CFT



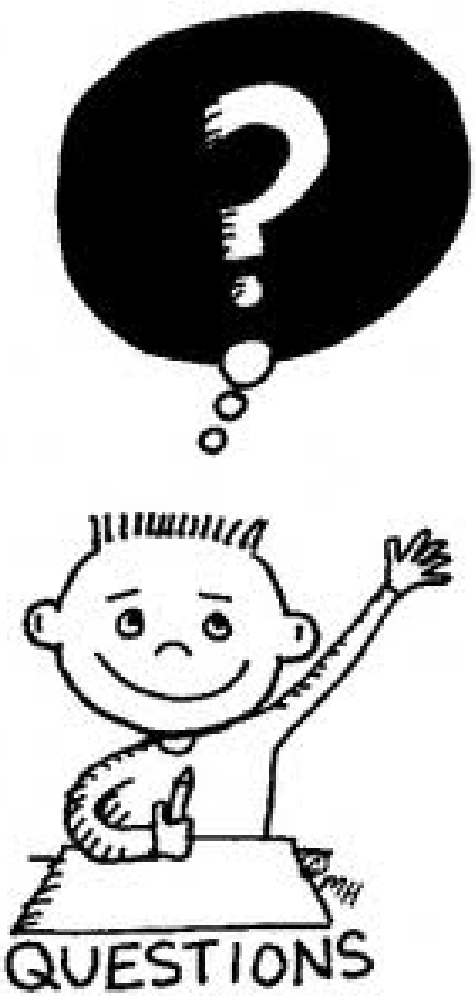
$$Z = \int [\mathrm{D}x_1^0 \mathbb{D}x_1 \mathrm{D}x_1^h][\mathrm{D}x_2^0 \mathbb{D}x_2 \mathrm{D}x_2^0] e^{iS_1 - iS_2} \underbrace{[\mathbb{D}x_1^\epsilon][\mathbb{D}x_2^\epsilon] e^{iS_1^\epsilon - iS_2^\epsilon}}$$

$$= \int [\mathrm{D}x_1^0 \mathbb{D}x_1 \mathrm{D}x_1^h][\mathrm{D}x_2^0 \mathbb{D}x_2 \mathrm{D}x_2^0] e^{iS_1 - iS_2} e^{iS^h_{\mathrm{eff}}}$$

Summary & Frontiers

- AdS/CFT is an extremely useful tool for theoretical physicists.
- It has versatile applications over different braches of physics viz., Quantum gravity, QCD, Cond. Mat, Stat. Mech ...
- Few thousands of work exist in literature and a lot more to be done.
- Still it is a "conjecture" since 1997. Yet to be proved!


Thank you
wiseGEEK


QUESTIONS