Zero Temperature Dissipation & Holography

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PB & B. Sathiapalan (on going)

Outline

- AdS/CFT has been serving as a theoretician's best tool in studying strongly coupled systems analytically.
- Its predictions are mostly qualitative in nature, but they can be quantitative too (e.g, $\frac{\eta}{s} = \frac{1}{4\pi}$).
- The duality has glued many phenomena appearing in apparently different branches of physics together.
- Studying Brownian motion of a heavy particle using classical gravity technique is one such example.

Introduction

Langevin Dynamics

- The Langevin equation

$$M\frac{d^2x}{dt^2} + \gamma\,\frac{dx}{dt} = \xi(t) \tag{1}$$

$$\text{with} \qquad \langle \xi(t)\xi(t')\rangle = \Gamma\,\delta(t-t') \tag{2}$$

- The Langevin equation

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The generalized Langevin equation for a heavy particle under noise ξ

$$M_0\frac{d^2x(t)}{dt^2} + \int_{-\infty}^{t} dt'\;G_R(t,t')x(t') = \;\xi(t) \qquad \langle \xi(t)\xi(t')\rangle = \;iG_{\text{sym}}(t,t') \tag{3}$$

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The generalized Langevin equation for a heavy particle under noise ξ

$$\left[-M_0\omega^2 + G_R(\omega)\right] x(\omega) = \xi(\omega) \qquad \langle \xi(-\omega)\xi(\omega)\rangle = i\,G_{\mathrm{sym}}(\omega) \tag{3}$$

- The Langevin equation

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- Expanding $G_R(\omega)$ for small frequencies

$$G_R(\omega) = -\Delta M\omega^2 - i\gamma\omega + \ldots$$

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- Expanding $G_R(\omega)$ for small frequencies

$$G_R(\omega) = -\Delta M\omega^2 - i\gamma\omega + \ldots$$

- Fluctuation-Dissipation relation

$$\mathrm{i}\,G_{\text{sym}}(\omega) = -(1+2n_B)\ \text{Im}\ G_R(\omega) \tag{4}$$

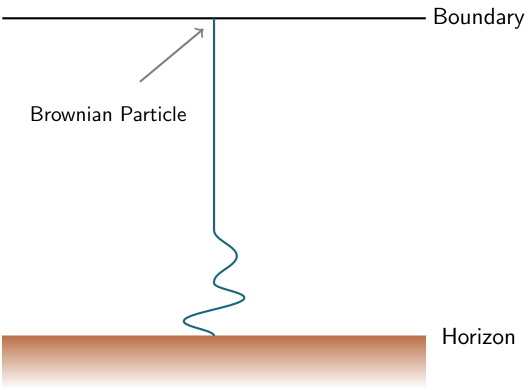
- For small frequency

$$G_R(\omega) = -i\gamma\,\omega - \Delta M\,\omega^2 - i\rho\,\omega^3 + \ldots$$

- Dimensional analysis

$$G_R(\omega) \sim [M]^3 \quad \Rightarrow \quad \begin{cases} \gamma \sim \mathrm{T}^2 \\ \Delta M \sim \mathrm{T} \\ \rho \sim \mathrm{T}^0 \end{cases}$$

- At zero temperature : $G_R(\omega) = -i\rho\,\omega^3 \quad \leftrightsquigarrow \quad$ Dissipative!



J. de Boer et al; Son & Teaney (2009)

- Strongly coupled field theory $\Leftrightarrow$ Weakly coupled gravity.

$$\left\langle \exp \left(\int_{S^d} \phi_0^i \mathcal{O}_i \right) \right\rangle_{\mathrm{CFT}} = Z_{\mathrm{QG}} \left(\phi_0^i \right) \tag{5}$$

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- Real time retarded Green's function for **scalar field** theory can be obtained by choosing ingoing boundary condition at the horizon.

$$G_R(k) = K\sqrt{-g}\, g^{rr} f_{-k}(r)\partial_r f_k(r)\Big|_{Boundary} \tag{6}$$

Son & Starinets (2002)

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- Real time retarded Green's function for **our case** can be obtained by choosing ingoing boundary condition at the horizon.

$$G_R(\omega) = T_0(r) f_{-\omega}(r) \partial_r f_\omega(r) \Big|_{Boundary} \tag{7}$$

1. Solve the EOM for the string in that non-trivial background

$$f_\omega(r) = C_1 f_\omega^{(1)}(r) + C_2 f_\omega^{(2)}(r) \tag{8}$$

2. Impose ingoing wave boundary condition at the horizon

$$f_\omega^R(r) := C_1 f_\omega^{(1)}(r) + \cancel{C_2 f_\omega^{(2)}(r)} \tag{9}$$

3. Properly normalize it at the boundary

$$F_\omega^R(r) := \frac{f_\omega^R(r)}{f_\omega^R(r_B)} \tag{10}$$

4. Compute the boundary action and take functional derivative w.r.t "source" to obtain the retarded Green's function $G_R^0(\omega)$.

String in BTZ

- String EOM

$$f''_\omega(r) + \frac{2(2r^2 - 4\pi^2 T^2 L^4)}{r(r^2 - 4\pi^2 T^2 L^4)} f'_\omega(r) + \frac{L^4\omega^2}{(r^2 - 4\pi^2 T^2 L^4)^2} f_\omega(r) = 0 \qquad (11)$$

- Two independent solutions

$$f_\omega(r) = C_1 \frac{P_1^{\frac{i\omega}{2\pi T}}\left(\frac{r}{2\pi T L^2}\right)}{r} + C_2 \frac{Q_1^{\frac{i\omega}{2\pi T}}\left(\frac{r}{2\pi T L^2}\right)}{r} \qquad (12)$$

- Ingoing & normalized solution

$$F^R_\omega(r) = \frac{\frac{P_1^{\frac{i\omega}{2\pi T}}\left(\frac{r}{2\pi T L^2}\right)}{r}}{\frac{P_1^{\frac{i\omega}{2\pi T}}\left(\frac{r_B}{2\pi T L^2}\right)}{r_B}} \qquad (13)$$

String in BTZ (cont.)

- Retarded propagator can be read off from the on-shell action

$$G_R(\omega) \equiv G_R^0(\omega) + \frac{\mu\ \omega^2}{2\pi}$$
$$= \frac{\mu\ \omega}{2\pi}\ \frac{(\omega^2 + 4\pi^2 T^2)}{(\omega + i\frac{\mu}{\sqrt{\lambda}})}$$

PB & B. Sathiapalan (2014)

- Viscous drag

$$\gamma = 2\sqrt{\lambda}\pi T^2$$

- Higher order "dissipation coefficient"

$$\rho = \frac{\sqrt{\lambda}}{2\pi} - \frac{2(\sqrt{\lambda})^3\pi T^2}{\mu^2} \xrightarrow{T\to 0} \frac{\sqrt{\lambda}}{2\pi}$$

Dissipation

Zero temperature dissipation is physical

- ✓ It is finite and therefore no need to renormalize by adding counterterms.
- ✓ It cannot be renormalized away in the boundary theory by any hermitian counter term.
- ✓ Quark moving at constant velocity doesn't feel any drag at T$=0$.

- Explanation : This zero temperature dissipation is due to radiation of accelerated charged particle. Remarkably the dissipation in this highly non-linear boundary theory is given by simple "Abraham-Lorentz"-like formula for radiation reaction in classical electrodynamics!

$$\vec{F}_{rad} = \frac{\sqrt{\lambda}}{2\pi}\, \dot{\vec{a}}$$

String in pure AdS_{d+1}

- String EOM

$$f''_\omega(r) + \frac{4}{r} f'_\omega(r) + \frac{L^4\omega^2}{r^4} f_\omega(r) = 0 \tag{14}$$

- Ingoing & normalized solution

$$F^R_\omega(r) = \frac{r_B}{r} \frac{e^{+i\frac{L^2\omega}{r}}(r - iL^2\omega)}{e^{+i\frac{L^2\omega}{r_B}}(r_B - iL^2\omega)} \tag{15}$$

- Retarded Green's function

$$G_R(\omega) := G^0_R(\omega) + \frac{\mu\omega^2}{2\pi} = \frac{\mu\omega^3}{2\pi} \frac{1}{(\omega + i\frac{\mu}{\sqrt{\lambda}})} \tag{16}$$

- Higher order "dissipation coefficient"

$$\rho = \frac{\sqrt{\lambda}}{2\pi} \quad (\mathit{Identical!}) \tag{17}$$

Dissipation

Is the zero temperature dissipation universal?

String in AdS_5-BH

- String EOM can *not* be solved exactly!

$$f''_\omega(r) + \frac{4r^3}{(r^4 - \pi^4 T^4 L^8)} f'_\omega(r) + \frac{\omega^2 L^4 r^4}{(r^4 - \pi^4 T^4 L^8)^2} f_\omega(r) = 0 \quad (18)$$

- Ingoing & normalized ansatz

$$F^R_\omega(r) = \left(1 - \frac{\pi^4 T^4 L^8}{r^4}\right)^{-i\frac{\Omega}{4}} \left(1 - i\Omega f_1(r) - \Omega^2 f_2(r) + i\Omega^3 f_3(r) + \dots\right) \quad (19)$$

- The limit we are interested in

$$\omega, T \to 0 \quad \text{and } \Omega := \frac{\omega}{\pi T} = \textit{ fixed} \quad (20)$$

- Solving $f_1(r), f_2(r)$ and $f_3(r)$ perturbatively and following the same procedure

$$G_R(\omega) \equiv G^0_R + \frac{\mu\omega^2}{2\pi} = -i\frac{\sqrt{\lambda}}{2\pi}\left(\frac{\pi - Log4}{4}\right)\omega^3 \quad (21)$$

Dissipation as T $\to 0$

- Calculating $G_R(\omega)$ in AdS_5-Schwarzschild black hole bulk geometry

$$\rho = \frac{(\pi - \log 4)}{4} \frac{\sqrt{\lambda}}{2\pi}$$

Dissipation at T$= 0$

- Calculating $G_R(\omega)$ in pure AdS_5 bulk geometry

$$\rho = \frac{\sqrt{\lambda}}{2\pi}$$

Dissipation as T $\to 0$

- Calculating $G_R(\omega)$ in AdS_5-Schwarzschild black hole bulk geometry

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Dissipation at T$= 0$

- Calculating $G_R(\omega)$ in pure AdS_5 bulk geometry

$$\rho = \frac{\sqrt{\lambda}}{2\pi}$$

- **Conclusion :** Possibly due to "deconfinement" transition at T$= 0$.

- Reissner-Nordström (RN) black hole in asymptotically AdS space time

$$ds^2 = \frac{L_{d+1}^2}{z^2}(-f(z)dt^2 + d\vec{x}^2) + \frac{L_{d+1}^2}{z^2}\frac{dz^2}{f(z)} \tag{22}$$

where,

$$f(z) = 1 + Q^2 z^{2d-2} - Mz^d$$

$$A_t(z) = \mu(1 - \frac{z^{d-2}}{z_0^{d-2}})$$

- We'll define a new length scale z_* where $Q := \sqrt{\frac{d}{d-2}\frac{1}{z_*^{d-1}}}$
- There are two possibilities :
  - Extremal BH ($T = 0$)
  - Non-extremal BH ($T \neq 0$)

Finite density and zero temperature

- The string EOM

$$x_\omega''(z) + \frac{\frac{d}{dz}\left(\frac{f(z)}{z^2}\right)}{\frac{f(z)}{z^2}}\, x_\omega'(z) + \frac{\omega^2}{[f(z)]^2}\, x_\omega(z) = 0 \qquad (23)$$

- Subtlety : At zero temperature the $f(z)$ has a double zero at the horizon. Thus this singular term dominates at the horizon irrespective of however small ω we choose.
- Matching technique : Isolate the 'singular' near horizon region and treat ω perturbatively "outside".
  - Inner/IR region : $AdS_2 \times \mathbb{R}^{d-1}$
  - Outer/UV region : Full RN-AdS background

Hong Liu et al. *(2009)*

Dissipation

Dissipation at finite density

- Matching the solutions in two regions near $z = z_*$ we obtain

$$G_R(\omega) := \frac{b_+ + \mathscr{G}_R(\omega) z_* b_-}{a_+ + \mathscr{G}_R(\omega) z_* a_-}$$

$$= \frac{(b_+^{(0)} + \omega^2 b_+^{(2)} + \ldots) - i\omega (b_-^{(0)} + \omega^2 b_-^{(2)} + \ldots) z_*}{(a_+^{(0)} + \omega^2 a_+^{(2)} + \ldots) - i\omega (a_-^{(0)} + \omega^2 a_-^{(2)} + \ldots) z_*} \qquad (24)$$

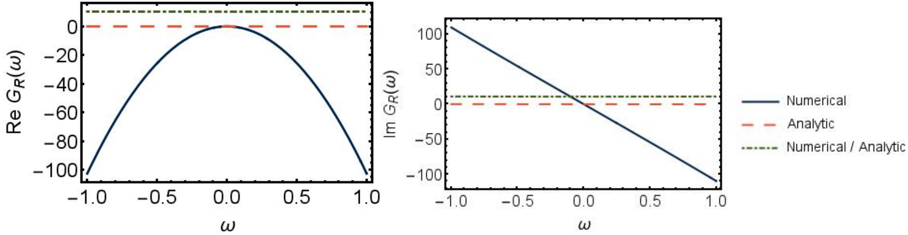
- The leading-order Green's function

$$G_R^{(0)}(\omega) = -\frac{i\ \omega z_*}{(1 + i\ \omega z_* a_-^{(0)})} \qquad (25)$$

- For small frequency

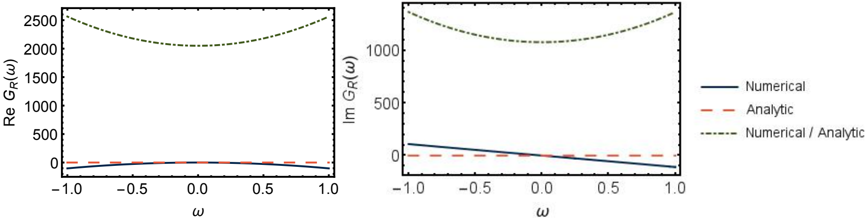
$$G_R^{(0)}(\omega) \approx -i\ \omega z_* (1 - i\ \omega z_* a_-^{(0)}) \qquad (26)$$

Choosing $a_{-}^{(0)} = 1$



$\frac{Numerical}{analytic}$ is nice a straight line in each case. Therefore the Green's functions match well up to some overall normalization.

Choosing $a_{-}^{(0)} = 5$



$\frac{\textit{Numerical}}{\textit{analytic}}$ is *not* a straight line!

Finite density and small temperature

- Inner Region changes to BH in $AdS_2 \times \mathbb{R}^{d-1}$
- The story is same with two possible modifications
  - The $\mathscr{G}_R(\omega)$ may change and can be T-dependent.
  - $a_\pm, b_\pm$ will be T-dependent.

- Actually the retarded Green's function at small T becomes

$$G_R^T(\omega) = \frac{b_+(\omega, T) + \mathscr{G}_R(\omega, T) z_* \ b_-(\omega, T)}{a_+(\omega, T) + \mathscr{G}_R(\omega, T) z_* \ a_-(\omega, T)} = \frac{b_+(\omega, T) - i\omega z_* \ b_-(\omega, T)}{a_+(\omega, T) - i\omega z_* \ a_-(\omega, T)} \tag{27}$$

- Leading order dissipation is same as zero temperature.

Conclusions

- The temperature independent dissipation is identical for all dimensions as long as the systems are in zero temperature (bulk is pure AdS).
- For higher dimensions $T \to 0$ and $T = 0$ (e.g, AdS_5-BH and pure AdS_5 bulk, say) the coefficients don't match!
- Retarded Green's function at $T = 0$ is computed at finite density. Zero temperature dissipation shows up as leading term.
- The form of the Retarded Green's function at finite density and small (but finite) temperature is also obtained. The leading dissipative part remains the same.
- The leading order Green's function is "matched"(or rather *compared*) with numerical results up to some overall normalization.


Thank you

Functional forms of $f_1(z), f_2(z), f_3(z)$

$$f_1(z) = \frac{1}{2} tan^{-1}(\pi T z) - \frac{1}{2} Log(1 + \pi T z) + \frac{1}{4} Log(1 + \pi^2 T^2 z^2) \quad (28)$$

$$\begin{aligned} f_2(z) = \frac{1}{32} [& 4\{-4 + tan^{-1}(\pi T z) - Log(1 + \pi T z)\}\{tan^{-1}(\pi T z) - Log(1 + \pi T z)\} \\ & - 4\{2 + tan^{-1}(\pi T z) - Log(1 + \pi T z)\} Log(1 + \pi^2 T^2 z^2) + Log(1 + \pi^2 T^2 z^2)^2] \end{aligned} \quad (29)$$

$$f_3(z) = \ldots \quad (30)$$

- The near horizon geometry for Near-extremal ($T \ll \mu$) RN Black hole

$$ds^2 = \frac{L_2^2}{\zeta^2}\left(-g(\zeta)dt^2 + \frac{d\zeta^2}{g(\zeta)}\right) + \mu_*^2 L_{d+1}^2 d\vec{x}^2 \tag{31}$$

$$A_t(\zeta) = \frac{1}{\sqrt{2d(d-1)}}\frac{1}{\zeta}(1 - \frac{\zeta}{\zeta_0}) \tag{32}$$

where $g(\zeta) := (1 - \frac{\zeta^2}{\zeta_0^2})$, $\zeta_0 := \frac{z_*^2}{d(d-1)(z_* - z_0)}$

- The corresponding temperature, $T = \frac{1}{2\pi\zeta_0}$
- This nice structure breaks down for large temperature ($T \approx \mu$).