Holographic Brownian Motion in 1+1 Dimensions

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Outline

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1. Introduction
   - Motivation

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   - Idea & Set-up

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   - Idea & Set-up
   - AdS/CFT : A theorist's tool

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- Lower dimensions are always interesting!

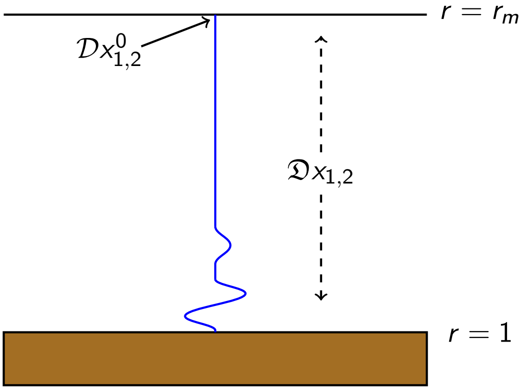

Figure: *The gravity set up for the boundary stochastic motion of the heavy particle .*

- Strongly coupled field theory $\Leftrightarrow$ Weakly coupled gravity.

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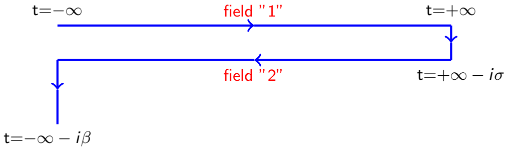
- Real time correlators for the field theory can be obtained by choosing appropriate boundary conditions.

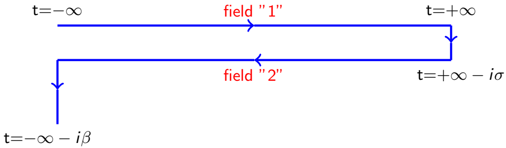
$$G_R(k) = -2\mathcal{F}(k,r)\Big|_{r_m} \tag{2}$$

where $\mathcal{F}(k,r) = K\sqrt{-g}g^{rr}f_{-k}(r)\partial_r f_k(r)$ *Son-Starinets (2002)*

Langevin Dynamics : A Review



Langevin Dynamics : A Review



- The partition function reads

$$Z = \left\langle \int [\mathcal{D}x_1][\mathcal{D}x_2] \; e^{i\int dt_1 M_Q^0 \dot{x}_1^2} \; e^{-i\int dt_2 M_Q^0 \dot{x}_2^2} \; e^{i\int dt_1 \phi_1(t_1) x_1(t_1)} \; e^{-i\int dt_2 \phi_2(t_2) x_2(t_2)} \right\rangle$$

$$= \int [\mathcal{D}x_1][\mathcal{D}x_2] \; e^{i\int dt_1 M_Q^0 \dot{x}_1^2} \; e^{-i\int dt_2 M_Q^0 \dot{x}_2^2} \; e^{-\frac{1}{2}\int dt dt' x_s(t)[\langle \phi(t)\phi(t')\rangle]_{ss'} x_{s'}(t')}$$

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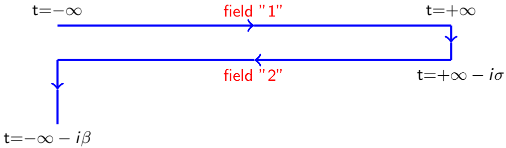

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- The Schwinger-Keldysh propagators

$$[\langle \phi(t)\phi(t')\rangle]_{ss'} \equiv i \begin{pmatrix} G_{11}(t,t') & -G_{12}(t,t') \\ -G_{21}(t,t') & G_{22}(t,t') \end{pmatrix} \quad (3)$$

"ra" formalism

$$x_r = \frac{x_1+x_2}{2} \qquad x_a = x_1 - x_2$$

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$$Z = \int [\mathcal{D}x_r]\,[\mathcal{D}x_a]\; e^{-i\int dt M_Q^0 x_a \ddot{x}_r}\; e^{-i\int dt dt' [x_a(t) i G_R(t,t') x_r(t') - \frac{1}{2} x_a(t) G_{\rm sym}(t,t') x_a(t')]}$$

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$$G_{\rm sym}(t,t') = \langle \phi_r(t)\phi_r(t')\rangle = \frac{1}{2}\left\langle \{\phi(t),\phi(t')\}\right\rangle \quad (4)$$

$$iG_R(t,t') = \langle \phi_r(t)\phi_a(t')\rangle = \theta(t-t')\left\langle [\phi(t),\phi(t')]\right\rangle \quad (5)$$

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$$iG_{sym}(\omega) = -(1+2n_B)\ \mathrm{Im}\ G_R(\omega) \tag{6}$$

- The path integral in fourier space

$$Z = \int [\mathcal{D}x_r]\,[\mathcal{D}x_a]\,\exp\left(-i\int \frac{d\omega}{2\pi} x_a(-\omega)[-M_Q^0\omega^2 + G_R(\omega)]x_r(\omega)\right) e^{-\frac{1}{2}\int \frac{d\omega}{2\pi} x_a(-\omega)[iG_{\rm sym}(\omega)]x_a(}$$

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- Introducing "noise"

$$e^{-\frac{1}{2}\int \frac{d\omega}{2\pi} x_a(-\omega)[iG_{\mathrm{sym}}(\omega)]x_a(\omega)} = \int [\mathcal{D}\xi]\ e^{i\int x_a(-\omega)\xi(\omega)} e^{-\frac{1}{2}\int \frac{d\omega}{2\pi} \frac{\xi(\omega)\xi(-\omega)}{iG_{\mathrm{sym}}(\omega)}} \qquad (7)$$

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and integrating out $x_a(-\omega)$

$$Z = \int [\mathcal{D}x]_r\ [\mathcal{D}\xi]\ e^{-\frac{1}{2}\int \frac{d\omega}{2\pi}\frac{\xi(\omega)(-\omega)}{iG_{\rm sym}(\omega)}}\ \delta_\omega\left[-M_Q^0\omega^2 x_r(\omega) + G_R(\omega)x_r(\omega) - \xi(\omega)\right]$$

Langevin Dynamics : A Review

- The partition function is an average over the classical trajectories of the heavy particle under noise ξ

$$\left[-M_Q^0\omega^2 + G_R(\omega)\right] x_r(\omega) = \xi(\omega) \qquad \langle \xi(-\omega)\xi(\omega)\rangle = iG_{\text{sym}}(\omega) \tag{8}$$

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- Expanding $G_R(\omega)$ for small frequencies

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- Expanding $G_R(\omega)$ for small frequencies

$$G_R(\omega) = -\Delta M\omega^2 - i\gamma\omega + \dots$$

- Then the Langevin equation reads

$$M_{\mathrm{kin}}\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} = \xi \tag{9}$$

$$\text{with} \quad \langle\xi(t)\xi(t')\rangle = \Gamma(t-t') \tag{10}$$

- The background metric AdS$_3$-BTZ is defined as

$$ds^2 = \frac{\bar{r}^2}{L^2}\left[-f(b\bar{r})\mathrm{d}t^2 + \mathrm{d}x^2\right] + \frac{L^2\mathrm{d}\bar{r}^2}{f(b\bar{r})\bar{r}^2} \tag{11}$$

Generalized Langevin Equation from Holography

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where, $b = \frac{1}{\pi T L^2}$, $f(r) = 1 - \frac{1}{r^2}$ and T is Hawking temperature.

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- The Nambu-Goto action is

$$S = -\frac{1}{2\pi {l_s}^2}\int \mathrm{d}\tau\mathrm{d}\sigma \sqrt{-\det h_{ab}} \tag{13}$$

Generalized Langevin Equation from Holography

- For small fluctuations

$$\sqrt{-h} = (\pi T)L^2\sqrt{1 + (\pi T)^2 r^4 f(r) x'^2 - \frac{\dot{x}^2}{f(r)}}$$
$$\approx (\pi T)L^2\left[1 + \frac{1}{2}(\pi T)^2 r^4 f(r) x'^2 - \frac{1}{2}\frac{\dot{x}^2}{f(r)}\right]$$

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- The string world sheet action becomes

$$S = -\int \mathrm{d}t\mathrm{d}r\left[m + \frac{1}{2}T_0(\partial_r x)^2 - \frac{m}{2f}(\partial_t x)^2\right] \tag{14}$$

where, $m \equiv \frac{(\pi T)L^2}{2\pi l_s^2} = \frac{1}{2}\sqrt{\lambda}T$ and $T_0(r) \equiv \frac{\sqrt{\lambda}\pi^2 T^3}{2} r^2(r^2 - 1)$

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- So the EOM is

$$0 = -\frac{m}{f}\partial_t^2 x + \partial_r(T_0(r)\partial_r x) \quad (15)$$

Generalized Langevin Equation from Holography

- In Fourier space

$$x(r,t) = \int \frac{\mathrm{d}\omega}{2\pi} e^{i\omega t} f_\omega(r) x_0(\omega)$$

$$x(r = r_m, t) = \int \frac{\mathrm{d}\omega}{2\pi} e^{i\omega t} x_0(\omega) \qquad \text{since,} \qquad f_\omega(r_m) = 1$$

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- The EOM reduces to

$$\partial_r^2 f_\omega + \frac{2(2r^2-1)}{r(r^2-1)} \partial_r f_\omega + \frac{\mathfrak{w}^2}{(r^2-1)^2} f_\omega = 0 \tag{16}$$

where we have defined $\mathfrak{w} \equiv \omega/(\pi T)$.

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- The solution to this EOM is given by

$$f_\omega(r) = C_1 \frac{P_1^{i\mathfrak{w}}}{r} + C_2 \frac{Q_1^{i\mathfrak{w}}}{r} \tag{17}$$

Generalized Langevin Equation from Holography

- Ingoing wave boundary condition at the horizon $\implies f_\omega^R(r) \sim \frac{P_1^{i\mathfrak{w}}}{r}$

Generalized Langevin Equation from Holography

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- At the boundary i.e, $r \to r_m$, $f^R_\omega(r) \to 1$.

$$f^R_\omega(r) = \frac{(1+r)^{i\mathfrak{w}/2}}{(1+r_m)^{i\mathfrak{w}/2}} \frac{(1-r)^{-i\mathfrak{w}/2}}{(1-r_m)^{-i\mathfrak{w}/2}} \frac{r_m}{r} \frac{{}_2F_1(-1,2;1-i\mathfrak{w};\frac{1-r}{2})}{{}_2F_1(-1,2;1-i\mathfrak{w};\frac{1-r_m}{2})}$$

$$= \frac{(1+r)^{i\mathfrak{w}/2}}{(1+r_m)^{i\mathfrak{w}/2}} \frac{(1-r)^{-i\mathfrak{w}/2}}{(1-r_m)^{-i\mathfrak{w}/2}} \frac{r_m}{r} \frac{\mathfrak{w}+ir}{\mathfrak{w}+ir_m} \qquad (18)$$

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&= \frac{(1+r)^{i\mathfrak{w}/2}}{(1+r_m)^{i\mathfrak{w}/2}} \frac{(1-r)^{-i\mathfrak{w}/2}}{(1-r_m)^{-i\mathfrak{w}/2}} \frac{r_m}{r} \frac{\mathfrak{w}+ir}{\mathfrak{w}+ir_m} \qquad (18)
\end{aligned}
$$

- The retarded correlator $G_R(\omega)$ is defined as

$$
\begin{aligned}
G^0_R &\equiv \lim_{r\to r_m} T_0(r) f^R_{-\omega}(r)\partial_r f^R_\omega(r) = -M^0_Q\omega^2 + G_R(\omega) \\
&= -\frac{\sqrt{\lambda}\pi^2 T^3}{2} \frac{r_m\mathfrak{w}(r_m\mathfrak{w}+i)}{(r_m-i\mathfrak{w})} \\
&= -\mu\omega \frac{(i\sqrt{\lambda}\pi^2 T^2+\mu\omega)}{2\pi(\mu-i\sqrt{\lambda}\omega)} \qquad (19)
\end{aligned}
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- Now expanding G_R in small frequencies

$$G_R(\omega) \approx \frac{\lambda\pi T^2}{2\mu}\omega^2 - i\left(\frac{\sqrt{\lambda}\pi T^2}{2}\,\omega + \left(\frac{\sqrt{\lambda}}{2\pi} - \frac{(\sqrt{\lambda})^3\pi T^2}{2\mu^2}\right)\omega^3\right) \tag{22}$$

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- Generically when $G_R(\omega)$ is expanded in small ω it takes the form

$$G_R(\omega) = -i\gamma\,\omega - \Delta M\omega^2 - i\rho\,\omega^3 + \dots \tag{23}$$

- Viscous drag

$$\gamma = \frac{1}{2}\sqrt{\lambda}\pi T^2 \tag{24}$$

Generalized Langevin Equation from Holography

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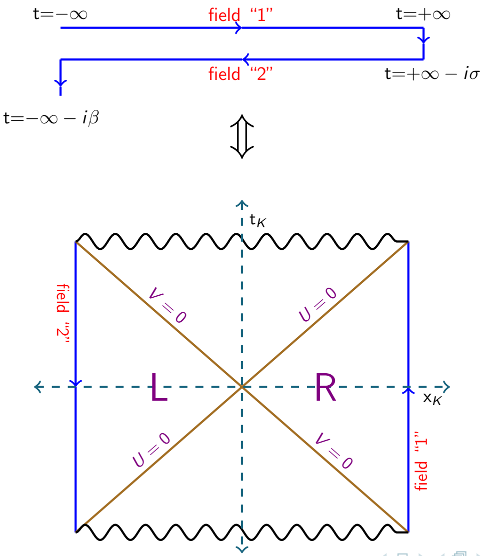
- Higher order "dissipation coefficient"

$$\rho = \frac{\sqrt{\lambda}}{2\pi} - \frac{(\sqrt{\lambda})^3\pi T^2}{2\mu^2} \tag{26}$$


t=−∞
field "1"
t=+∞
field "2"
t=+∞ − iσ
t=−∞ − iβ
⇕
t_K
field "2"
V = 0
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L
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- The EOM for the fluctuating string is solved subjected to the boundary conditions

$$\lim_{r \to r_m} x(\omega, r_1) = x_1^0(\omega) \tag{27}$$

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- The general solutions in L and R are

$$x(\omega, r_1) = a(\omega) f_\omega(r_1) + b(\omega) f_\omega^*(r_1) \tag{29}$$

$$x(\omega, r_2) = c(\omega) f_\omega(r_2) + d(\omega) f_\omega^*(r_2)$$

Exact Schwinger-Keldysh Propagators

Kruskal/Keldysh Correspondence : Review

- The EOM for the fluctuating string is solved subjected to the boundary conditions

$$\lim_{r \to r_m} x(\omega, r_1) = x_1^0(\omega) \tag{27}$$

$$\lim_{r \to r_m} x(\omega, r_2) = x_2^0(\omega) \tag{28}$$

- The general solutions in L and R are

$$x(\omega, r_1) = a(\omega) f_\omega(r_1) + b(\omega) f_\omega^*(r_1) \tag{29}$$

$$x(\omega, r_2) = c(\omega) f_\omega(r_2) + d(\omega) f_\omega^*(r_2)$$

- Analytically continue modes from R to L region to get

$$x(\omega, r_2) = a(\omega) f_\omega(r_2) + b(\omega) e^{+\omega/T} f_\omega^*(r_2) \tag{30}$$

- Solve for $a(\omega)$ & $b(\omega)$ in terms of $x_1^0(\omega)$ & $x_2^0(\omega)$.

- The next step is to plug this solution into the boundary action

$$S_{\text{b'dy}} = -\frac{T_0(r_m)}{2}\int_{r_1}\frac{d\omega}{2\pi}x_1(-\omega,r_1)\partial_r x_1(\omega,r_1) + \frac{T_0(r_m)}{2}\int_{r_2}\frac{d\omega}{2\pi}x_2(-\omega,r_2)\partial_r x_2(\omega,r_2) \tag{31}$$

to get

$$\begin{aligned} iS_{\text{b'dy}} = -\frac{1}{2}\int\frac{d\omega}{2\pi} & x_1^0(-\omega)\left[i\text{Re}G_R^0 - (1+2n_B)\text{Im}\,G_R^0\right]x_1^0(\omega) \\ & + x_2^0(-\omega)\left[-i\text{Re}G_R^0 - (1+2n_B)\text{Im}\,G_R^0\right]x_2^0(\omega) \\ & - x_1^0(-\omega)\left[-2n_B\text{Im}\,G_R^0\right]x_2^0(\omega) \\ & - x_2^0(-\omega)\left[-2(1+n_B)\text{Im}\,G_R^0\right]x_1^0(\omega) \end{aligned} \tag{32}$$

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to get

$$\begin{aligned} iS_{\text{b'dy}} = -\frac{1}{2}\int\frac{d\omega}{2\pi} x_1^0(-\omega)\left[i\mathrm{Re}\,G_R^0 - (1+2n_B)\mathrm{Im}\,G_R^0\right]x_1^0(\omega) \\ + x_2^0(-\omega)\left[-i\mathrm{Re}\,G_R^0 - (1+2n_B)\mathrm{Im}\,G_R^0\right]x_2^0(\omega) \\ - x_1^0(-\omega)\left[-2n_B\mathrm{Im}\,G_R^0\right]x_2^0(\omega) \\ - x_2^0(-\omega)\left[-2(1+n_B)\mathrm{Im}\,G_R^0\right]x_1^0(\omega) \end{aligned} \tag{32}$$

- Here retarded Green function is defined as

$$G_R^0(\omega) \equiv T_0(r)\frac{f_{-\omega}(r)\partial_r f_\omega}{|f_\omega(r)|^2}\bigg|_{r=r_m} \tag{33}$$

- Take functional derivative to get the Schwinger-Keldysh propagators

$$G_{\rm ab} = \begin{bmatrix} i\text{Re } G_R^0 - (1+2n_B) \text{ Im } G_R^0 & -2n_B \text{ Im } G_R^0 \\ -2(1+n_B) \text{ Im } G_R^0 & -i\text{Re } G_R^0 - (1+2n_B) \text{ Im } G_R^0 \end{bmatrix} \tag{34}$$

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<u>In "ra" basis</u> :

- And the boundary action in this set up becomes

$$S_{\text{b'dy}} = -\frac{T_0(r_m)}{2} \int_{r_m} \frac{d\omega}{2\pi} x_a(-\omega, r)\, \partial_r x_r(\omega, r) - \frac{T_0(r_m)}{2} \int_{r_m} \frac{d\omega}{2\pi} x_r(-\omega, r)\, \partial_r x_a(\omega, r)$$

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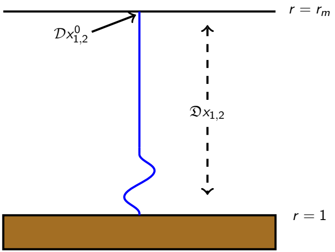
- Therefore the boundary action reduces to

$$iS_{\mathrm{b'dy}} = -i\int\frac{d\omega}{2\pi}x_a^0(-\omega)\ [G_R^0(\omega)]\ x_r^0(\omega) - \frac{1}{2}\int\frac{d\omega}{2\pi}x_a^0(-\omega)\ [iG_{\mathrm{sym}}(\omega)]\ x_a^0(\omega)$$

Exact Schwinger-Keldysh Propagators

Boundary stochastic motion

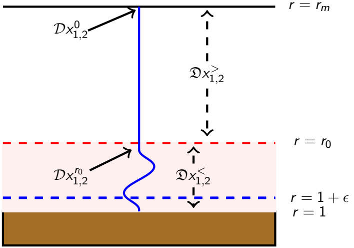


$$Z = \int [\mathcal{D}x_1^0][\mathcal{D}x_2^0] \underbrace{[\mathfrak{D}x_1][\mathfrak{D}x_2] e^{iS_1 - iS_2}}$$

$$\equiv \int [\mathcal{D}x_1^0][\mathcal{D}x_2^0] e^{iS^0_{\text{eff}}}$$

$$iS^0_{\text{eff}} = -i \int \frac{d\omega}{2\pi} x^0_a(-\omega)[G^0_R(\omega)] x^0_r(\omega) - \frac{1}{2} \int \frac{d\omega}{2\pi} x^0_a(-\omega)[iG_{\text{sym}}(\omega)] x^0_a(\omega)$$

Effective Action at General r



$$Z = \int [\mathcal{D}x_1^0 \mathfrak{D}x_1^> \mathcal{D}x_1^{r_0}]\,[\mathcal{D}x_2^0 \mathfrak{D}x_2^> \mathcal{D}x_2^{r_0}]\; e^{iS_1^> - iS_2^>}\; \underbrace{[\mathfrak{D}x_1^<]\,[\mathfrak{D}x_2^<]\; e^{iS_1^< - iS_2^<}}$$

$$= \int [\mathcal{D}x_1^0 \mathfrak{D}x_1^> \mathcal{D}x_1^{r_0}]\,[\mathcal{D}x_2^0 \mathfrak{D}x_2^> \mathcal{D}x_2^{r_0}]\; e^{iS_1^> - iS_2^>}\; e^{iS^{r_0}_{\text{eff}}}$$

$$iS_{\text{eff}} = -i \int \frac{d\omega}{2\pi} x_a^{r_0}(-\omega)[G_R^{r_0}(\omega)]x_r^{r_0}(\omega) - \frac{1}{2}\int \frac{d\omega}{2\pi} x_a^{r_0}(-\omega)[iG^{r_0}_{\text{sym}}(\omega)]x_a^{r_0}(\omega)$$

Effective Action at General r

- Retarded Green function at arbitrary $r = r_0$

$$
\begin{aligned}
G_R^{r_0}(\omega) &\equiv T_0(r)\frac{f_{-\omega}(r)\partial_r f_\omega}{|f_\omega(r)|^2}\Big|_{r=r_0} \\
&= -\frac{\sqrt{\lambda}\pi^2 T^3}{2}\,\frac{r_0\mathfrak{w}(r_0\mathfrak{w}+i)}{(r_0-i\mathfrak{w})} \\
&= -\mu_0\omega\,\frac{(i\sqrt{\lambda}\pi^2 T^2+\mu_0\omega)}{2\pi(\mu_0-i\sqrt{\lambda}\omega)}
\end{aligned} \tag{35}
$$

Effective Action at General r

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$$= -\mu_0\omega\;\frac{(i\sqrt{\lambda}\pi^2 T^2+\mu_0\omega)}{2\pi(\mu_0 - i\sqrt{\lambda}\omega)} \qquad (35)$$

- Softening of delta function

$$\lim_{t\to t_0}\;\int_{t_0}^{t} dt'\;\gamma(t') = \lim_{t\to t_0}\;\int_{t_0}^{t} dt'\;\int_{-\infty}^{\infty} d\omega\; e^{-i\omega t'}\gamma(\omega)$$
$$= -\lim_{t\to t_0}\;\int_{t_0}^{t} dt'\;\int_{-\infty}^{\infty} d\omega\; e^{-i\omega t'}\;\frac{\mu\omega}{2\pi}\;\frac{(\omega^2+\pi^2 T^2)}{(\omega + i\frac{\mu}{\sqrt{\lambda}})}$$
$$= -\lim_{t\to t_0}\;\int_{t_0}^{t} dt'\;2\pi i\; e^{-\frac{\mu}{\sqrt{\lambda}}t'}\;\frac{\mu(-i\frac{\mu}{\sqrt{\lambda}})}{2\pi}\left(\left(-i\frac{\mu}{\sqrt{\lambda}}\right)^2+\pi^2 T^2\right)\to 0$$

Effective Action at General r

- The action

$$
\begin{aligned}
iS_1^{>} - iS_2^{>} + iS_{\text{eff}}^{r_0} = &- i\int_{r_m} \frac{d\omega}{2\pi} x_a^0(-\omega, r)[T_0(r_m)\partial_r x_r^{>}(\omega, r)] \\
&- i\int_{r_0} \frac{d\omega}{2\pi} x_a^{r_0}(-\omega, r)[-T_0(r_0)\partial_r x_r^{>}(\omega, r) + G_R^{r_0}(\omega)x_r^{r_0}(\omega) - \xi^{r_0}(\omega)] \\
&- i\int \frac{d\omega}{2\pi}\ dr\ x_a^{>}(-\omega, r)\left[-\partial_r\left(T_0(r)\partial_r x_r^{>}(\omega, r)\right) - \frac{m\omega^2 x_r^{>}(\omega, r)}{f}\right]
\end{aligned}
$$

Effective Action at General r

- The action

$$iS_1^{>} - iS_2^{>} + iS_{\text{eff}}^{r_0} = -\, i\int_{r_m} \frac{d\omega}{2\pi} x_a^0(-\omega, r)[T_0(r_m)\partial_r x_r^{>}(\omega, r)]$$
$$- i\int_{r_0} \frac{d\omega}{2\pi} x_a^{r_0}(-\omega, r)[-T_0(r_0)\partial_r x_r^{>}(\omega, r) + G_R^{r_0}(\omega)x_r^{r_0}(\omega) - \xi^{r_0}(\omega)]$$
$$- i\int \frac{d\omega}{2\pi}\, dr\ x_a^{>}(-\omega, r)\left[-\partial_r\left(T_0(r)\partial_r x_r^{>}(\omega, r)\right) - \frac{m\omega^2 x_r^{>}(\omega, r)}{f}\right]$$

- The path integral reduces to

$$Z = \int [\mathcal{D}x_r^0 \mathfrak{D}x_r^{>} \mathcal{D}x_r^{r_0}][\mathcal{D}\xi^{r_0}] e^{-\frac{1}{2}\int \frac{\xi^{r_0}(\omega)\xi^{r_0}(-\omega)}{iG_{sym}^{r_0}(\omega)}} \underbrace{[\mathcal{D}x_a^0 \mathfrak{D}x_a^{>} \mathcal{D}x_a^{r_0}] e^{iS_1^{>} - iS_2^{>} + iS_{\text{eff}}^{r_0}}}$$
$$= \int [\mathcal{D}x_r^0 \mathfrak{D}x_r^{>} \mathcal{D}x_r^{r_0}][\mathcal{D}\xi^{r_0}] e^{-\frac{1}{2}\int \frac{\xi^{r_0}(\omega)\xi^{r_0}(-\omega)}{-(1+2n_B)\text{Im}\, G_R^{r_0}(\omega)}} \delta_\omega\left[-T_0(r_m)\partial_r x_r^{>}(\omega, r)\right]_{r=r_m}$$
$$\delta_\omega\left[-\partial_r(T_0(r)\partial_r x_r^{>}(\omega, r)) - \frac{m\omega^2 x_r^{>}(\omega, r)}{f}\right]$$
$$\delta_\omega[-T_0(r_0)\partial_r x_r^{>}(\omega, r) + G_R^{r_0}(\omega)x_r^{r_0}(\omega) - \xi^{r_0}(\omega)]_{r=r_0}$$

- Retarded correlator at stretched horizon

$$G_R^h(\omega) \sim -i\gamma\omega$$

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- The solution near the AdS boundary

$$x(\omega, r) = x_0(\omega) f_\omega(r) + \xi^0(\omega) \left[\frac{\mathrm{Im} f_\omega(r)}{-\mathrm{Im}\, G_R(\omega)} \right]$$

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- Dynamics of boundary end point

$$[-M_Q^0\omega^2 + G_R(\omega)]x_0(\omega) = \xi^0(\omega)$$

- Relation between boundary and stretched horizon fluctuation

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$$\xi^0(\omega) = \left(1 - i\frac{\omega}{\pi T}\right)\xi^h(\omega) \tag{37}$$

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- Use the horizon fluctuation-dissipation theorem to get

$$\langle \xi^0(-\omega)\xi^0(\omega)\rangle = -(1 + 2n_B)\mathrm{Im}\, G_R(\omega) \tag{38}$$

*** Boundary FDT*

Conclusions and frontiers

Have been done

- Natural softening of delta function in Langevin equation.
- Temperature dependent mass correction is zero (in the extreme UV limit).
- A temperature independent dissipation at all frequencies.
- The "stretched horizon" can be placed at an arbitrary radius and an effective action obtained.

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Can be done

- Study the holographic RG interpretation in this case.
- Same problem using a charged BTZ , thereby introducing a chemical potential.
- ...


Thank you
wiseGEEK


QUESTIONS