Chandrasekhar's contribution to Radiative Transfer: an appreciation

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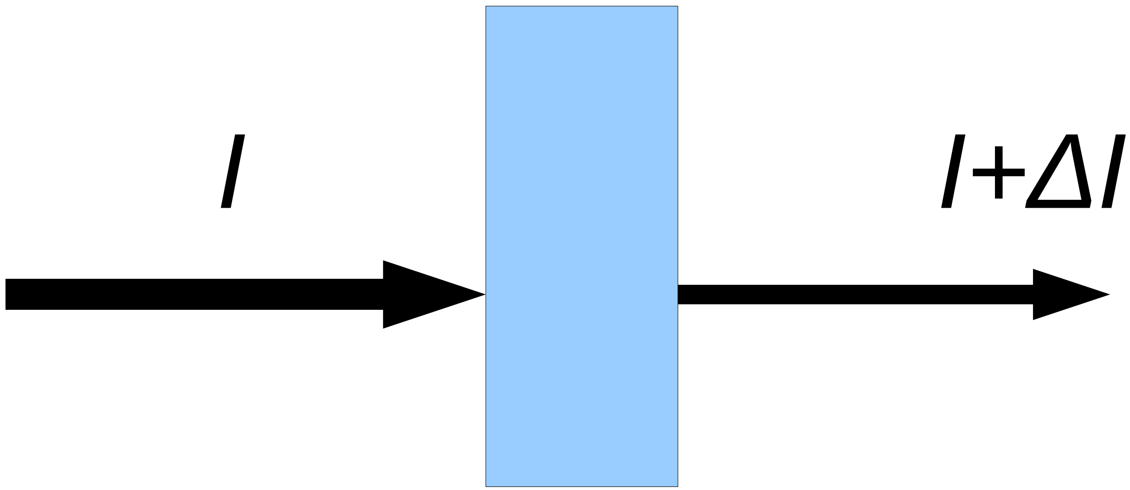
Plan

- Radiative transfer and how it stood ca. 1944
- A numerical scheme and some analytical fallout
- The principles of invariance
- Blue sky research - polarisation
- The negative ion of hydrogen
- Matters of taste, style, substance

Simple absorption



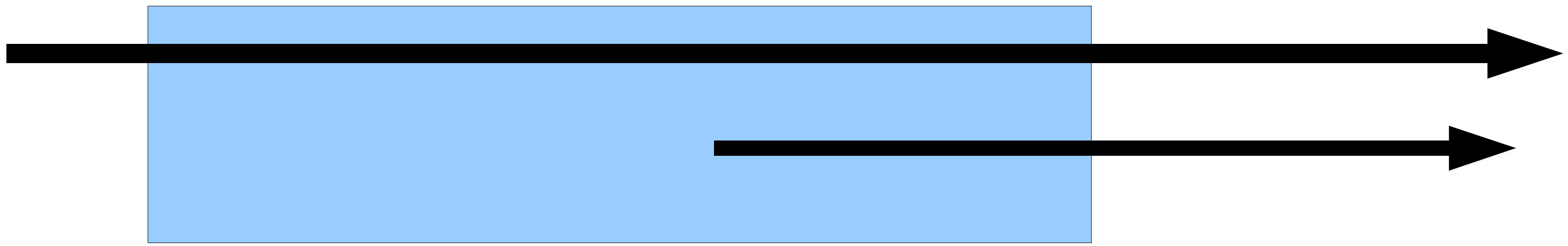
$$\Delta I / I = -\alpha\, dl = -d\tau$$

$$I(x) = I(0)\exp\left(-\int_0^x \alpha(y)\,dy\right) = I(0)\exp(-\tau)$$

Absorption plus emission

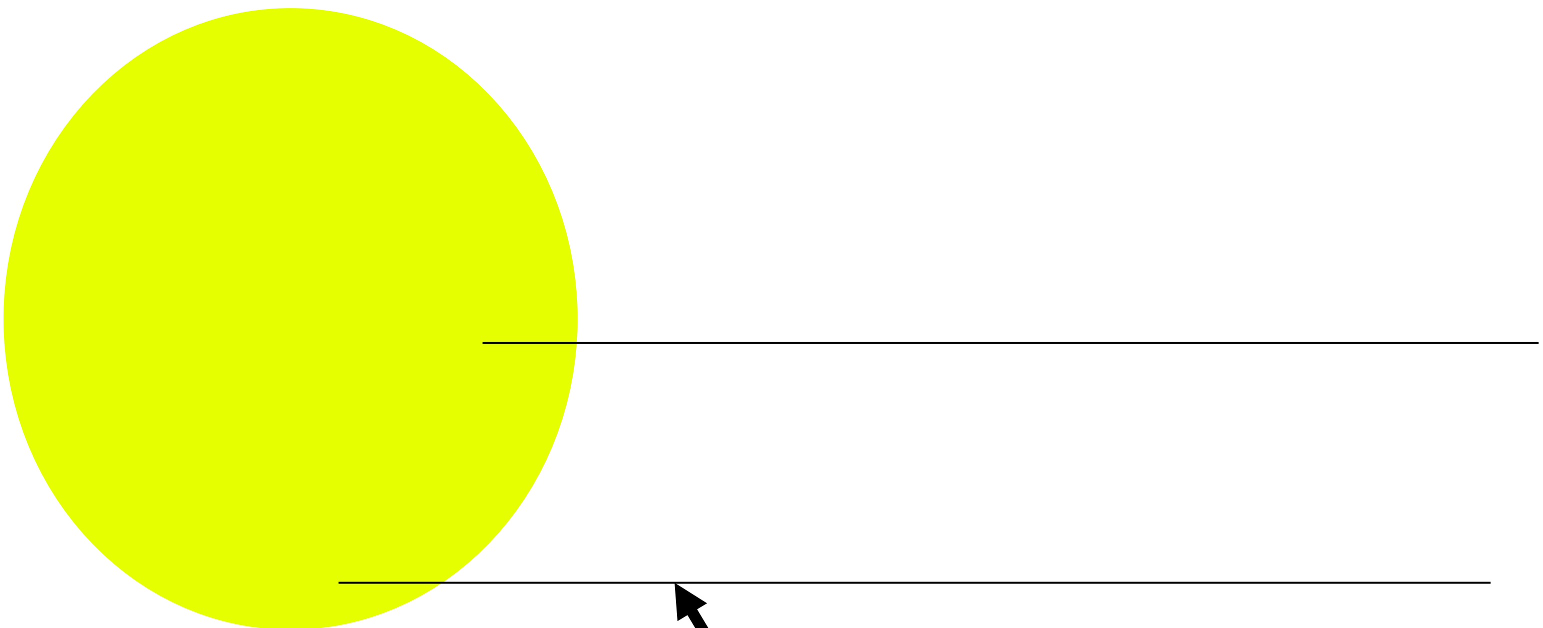
$$\Delta I = -\alpha\, dl\, I + \epsilon\, dl; \frac{dI}{dl} = -\alpha I + \epsilon$$

$$I = I(0)\exp(-\tau(0,x))$$
$$+\int_0^x \epsilon(z)\exp(-\tau(z,x))\,dz$$



- Note mean free path interpretation – we see down to the 'photosphere' "Optical depth" is the "tau of astrophysics:"distance in units of the mean free path"

Limb darkening



Looking at the edge of the Sun,
the surface of optical depth unity
is reached at a greater height
because of viewing at an angle.
This layer is darker

Stellar Atmospheres

- Given temperature as a function of height, the fraction of different ions (Saha) and the detailed behaviour of each for all wavelengths ((Los Alamos tables)
- The reward for modeling and studying stellar spectra is information on chemistry, rotation, magnetic field, turbulence, and even planets!

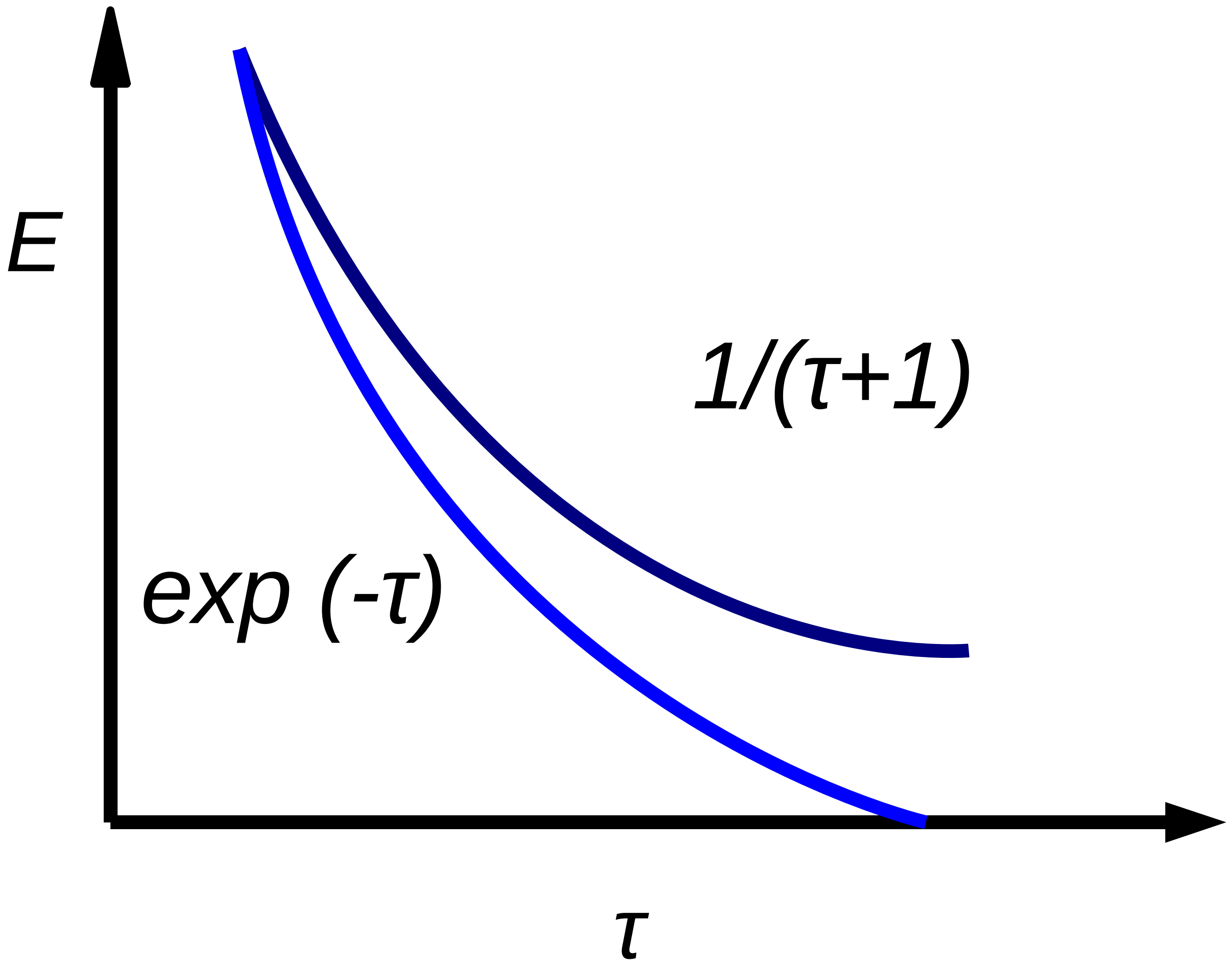
- In hot stars, radiation is a major player, for self consistency we need “non- LTE “
- Extreme – electron scattering atmosphere

Radiative transfer with scattering

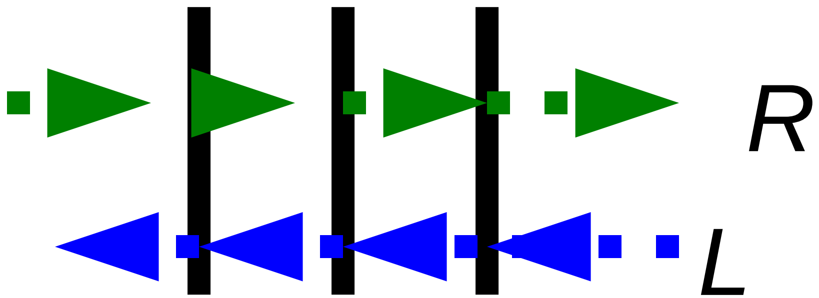
- Earth's atmosphere at visible wavelengths, and those of hot stars (free electrons cant absorb or emit single photons)
- Neutrons in reactors while they are slowing down
- Mathematically more complex and physically more subtle than the pure absorption-emission case

Scattering RT begins at home

Two kinds of attenuation



One-dimensional model with scattering



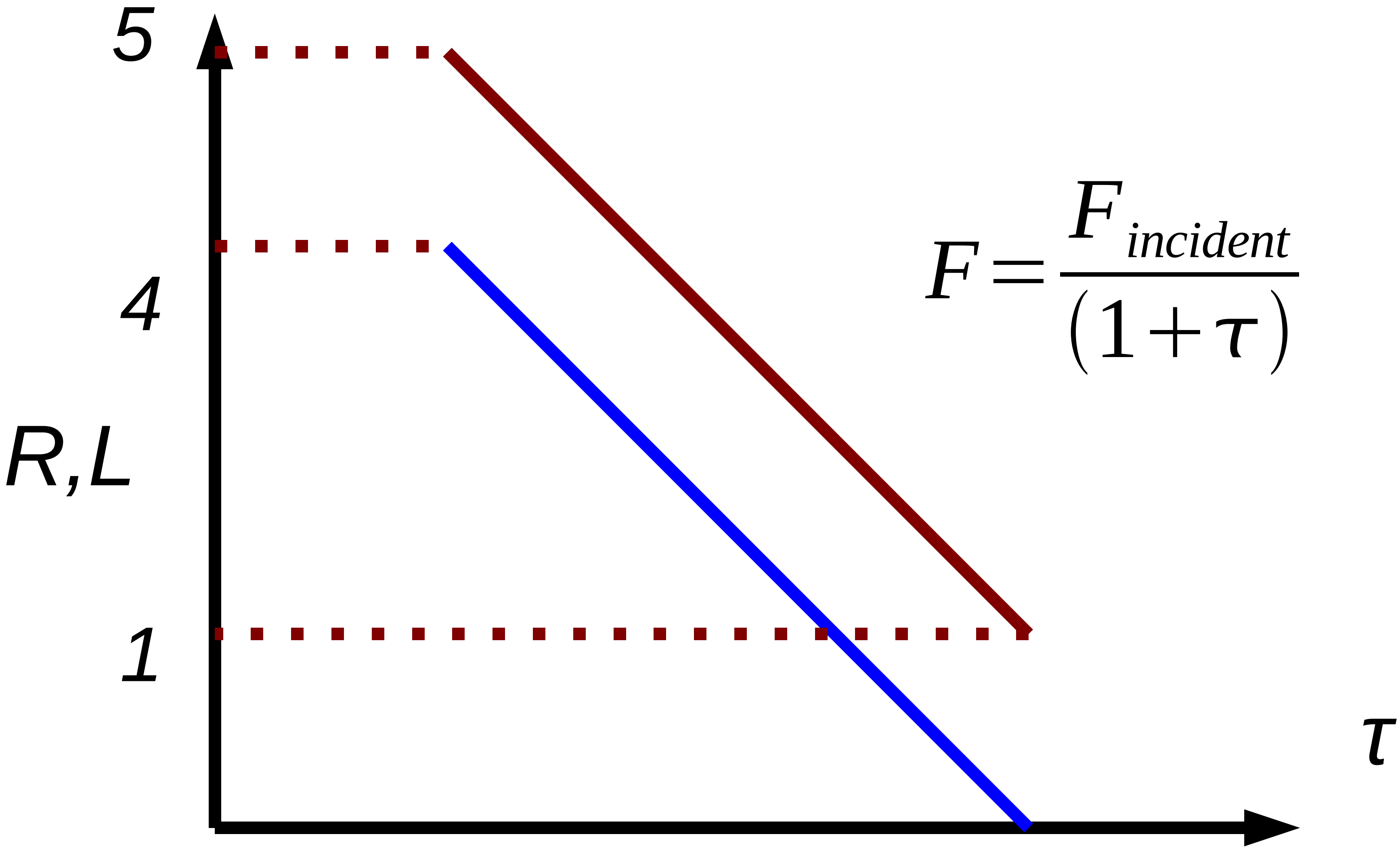
$$dR/dx = -\alpha_s(R) + \alpha_s(L+R)/2$$

$$-dL/dx = -\alpha_s(L) + \alpha_s(L+R)/2$$

$$R - L = F, R + L = E,$$

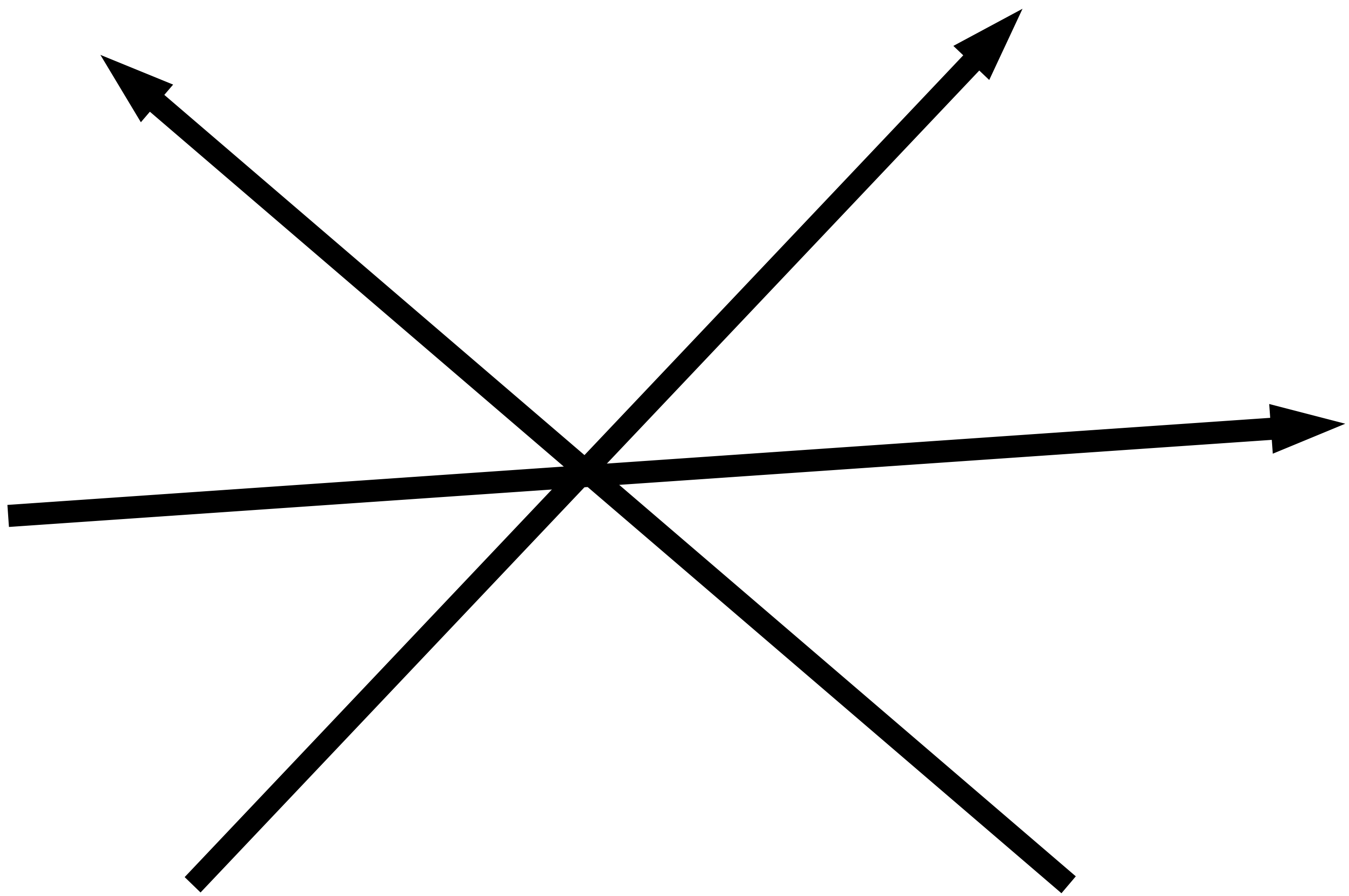
$$dF/dx = 0, dE/d\tau = -F$$

Forward and backward fluxes



Coupling of intensity along different rays by scattering

$$\epsilon(\mu) = \int d\mu' \sigma(\mu, \mu') I(\mu')$$



Coping with scattering

- Equations for moments of the radiation field in the axissymmetric case, e.g

$$\mu \frac{dI}{d\tau} = -I + \frac{1}{2}\int_{-1}^{+1} I \, d\mu = -I + J$$

$$\int I \, d\mu = J ; \int I \mu \, d\mu = F : \int I \mu^2 = K , etc.$$

- But the equations do not close because of the extra μ on left side

- Eddington cut the Gordian knot by setting *K=J/3* which is exact for a sphere and hemisphere! Good enough for many people but not for Chandrasekhar

Wiener-Hopf integral equation for the energy density, isotropic scattering case

$$\mu \frac{dI}{d\tau} = -I + \frac{1}{2}\int_{-1}^{+1} I\, d\mu = -I + J$$

$$I = \int_0^{\infty} J(\tau')\exp\left(-\left|\frac{\tau' - \tau}{\mu}\right|\right) d\left(\left|\frac{\tau' - \tau}{\mu}\right|\right)$$

$$J(\tau) = \frac{1}{2}\int_0^{\infty} d\tau'\, J(\tau')\, E_1(|\tau - \tau'|)$$

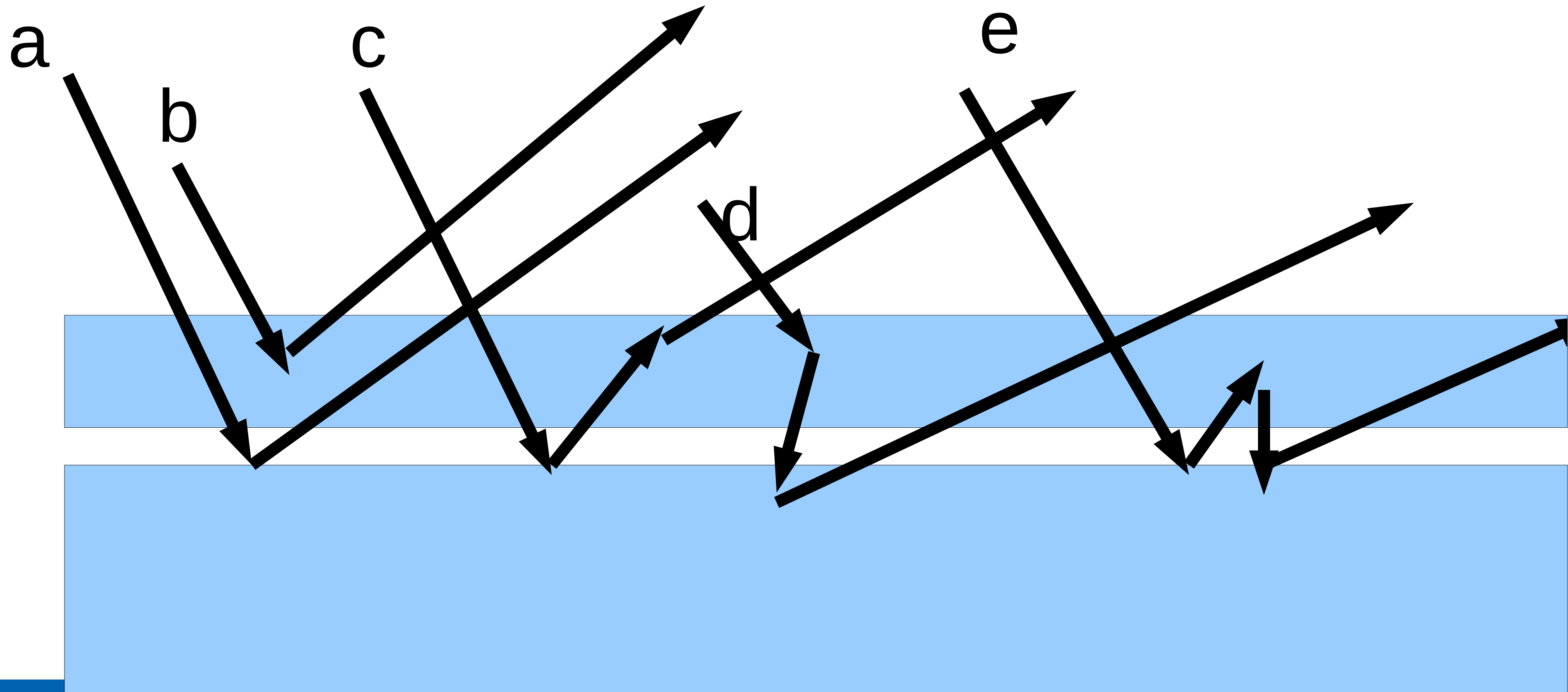
Becomes messier for anisotropic scattering!

The book ...

- ***Radiative transfer*** starts with 'discrete ordinates' i.e Gaussian quadrature (Wick) – seems a pedestrian exercise in coupled ODE's with constant coefficients until ....
- Exact and mysterious analytic results emerge from identities between zeros of Legendre polynomials and of the characteristic polynomial e.g $F = J/\sqrt{3}$; factorisation of the 'reflection probability ' $S(\mu, \varphi, \mu_{0,} \varphi_0)$
- They fall into place with the help of a physical principle of invariance (Ambartsumian)

Ambartsumian's principle of invariance, in pictures ....

All corrections due to adding a thin layer dτ must cancel !



....and in (not quite) full analytical glory

$$S(\mu,\mu_0)\left(\frac{1}{\mu}+\frac{1}{\mu_0}\right)=1+\frac{1}{2}\int S(\mu',\mu_0)\,d\mu'/\mu'$$
$$+\frac{1}{2}\int S(\mu,\mu'')\,d\mu''/\mu''$$
$$+\frac{1}{4}\int\int S(\mu,\mu'')S(\mu',\mu_0)\frac{d\mu'}{\mu'}\frac{d\mu''}{\mu''}$$

Enter the H-function

$$H(\mu) = 1 + \frac{1}{2}\int S(\mu, \mu')\,d\mu'/\mu'$$

$$S(\mu, \mu_0)(1/\mu + 1/\mu_0) = H(\mu)\,H(\mu_0)$$

$$H(\mu) = 1 + \frac{\mu H(\mu)}{2}\int \frac{H(\mu')}{(\mu + \mu')}\,d\mu'$$

More complex problems like finite slabs, anisotropic scattering, polarisation all have their own H functions
Guiding principles enable C to navigate through these

The polarisation of skylight

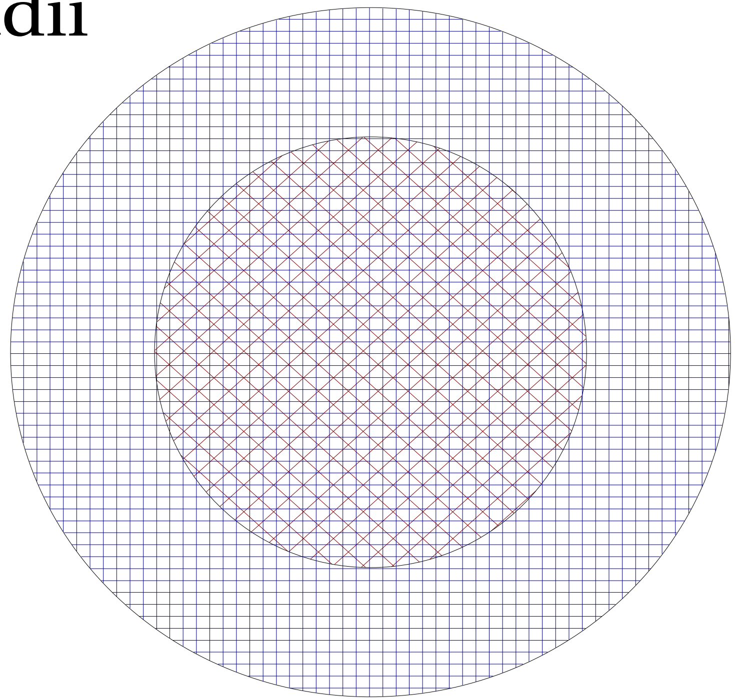
- Rayleigh's 1871 paper is single scattering, so maximum (100 per cent) polarisation at 90 degrees to the solar direction, zero at 0 degrees
- In reality, zero polarisation is displaced from the solar and antisolar directions, and maximum is about 90 per cent. Qualitatively understood in early 20 century
- Chandrasekhar formulated a matrix transfer equation for the Stokes parameters, and found the 'eleven per cent solution' for stars and a quantitative fit to the sky

Chandrasekhar as quantum chemist

- Bethe, Hylleraas used a very complicated wave function to prove that the Hydride ion is bound
- Wildt realised the role of H- in fitting the solar spectrum in near IR
- Chandrasekhar produced a much better energy from a much simpler wave function
- And followed it up with hero(in)ic numerical work on energies and oscillator strengths

The snowflake in hell: hydride ion in the Sun

Chandrasekhar's wave function for H- reduces Coulomb repulsion by putting the two electrons in orbitals of different radii



Used different methods for computing transition proba-bilities from bound to continuum, and more elaborate wave functions

Some questions arising while reading RT

- Why replace one approximation by another?
- Why replace a linear integral equation by nonlinear ones?
- Why so much numerical work and tables?
- Why spend so much time and effort on a problem which was essentially solved?
- Why was it one of the 'happiest periods' of his scientific life?

(My) answers

- The loose ends left behind by Rayleigh scattering drew him to the field. Earlier methods and even formulations were inadequate (e.g Stokes parameters)
- The numerical work started out as such. His happy choice of Gaussian division and weights plus his rapport with equations drew him further and further into the problem
- The invariance principle deeply attracted him and he generalised it as far as he could
- His aim: not just answers, but a coherent structure,: RT sytlistically closest to his ideal