Black Hole Evaporation from the perspective of Quantum Gravity

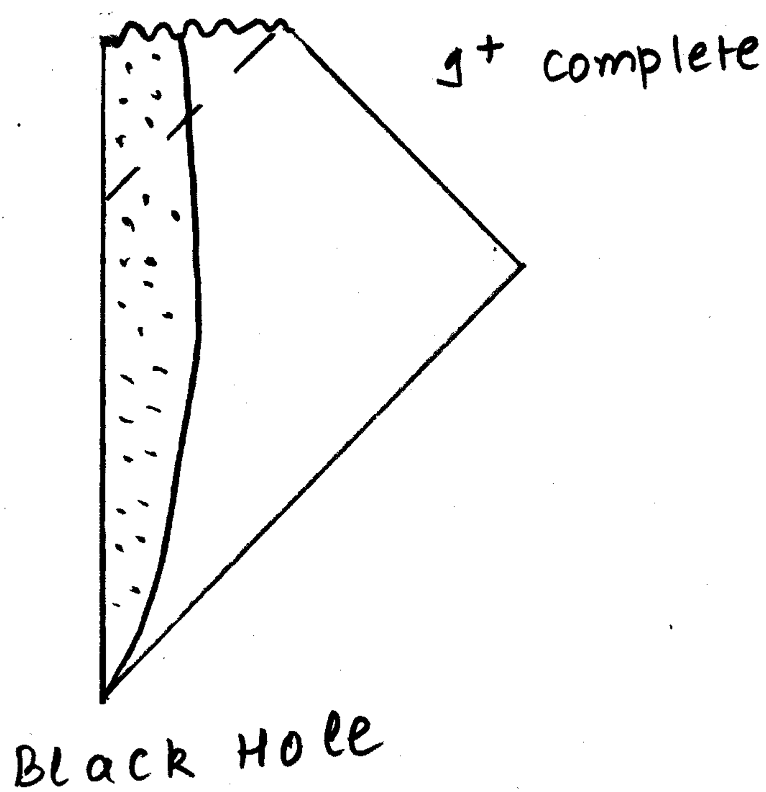
Madhavan Varadarajan

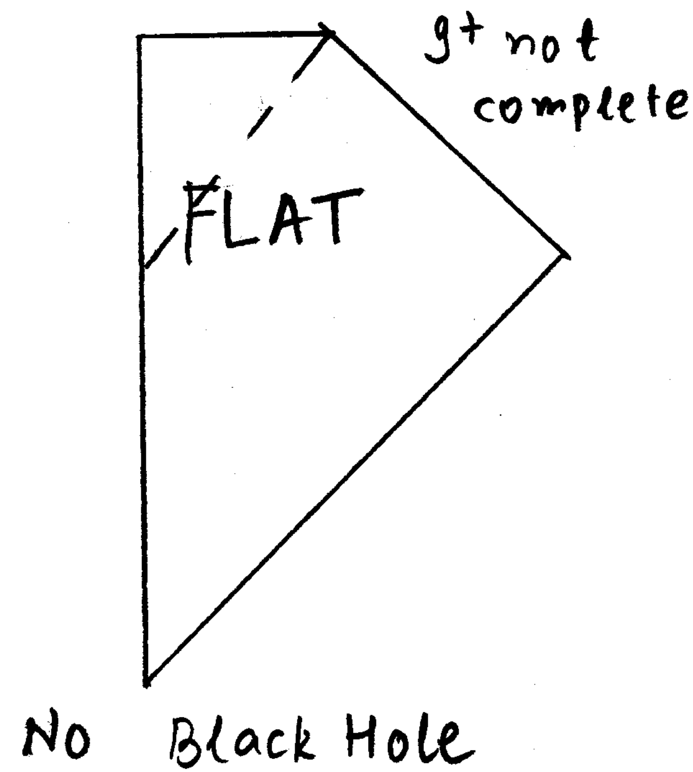
Raman Research Institute, Bangalore, India

Plan:

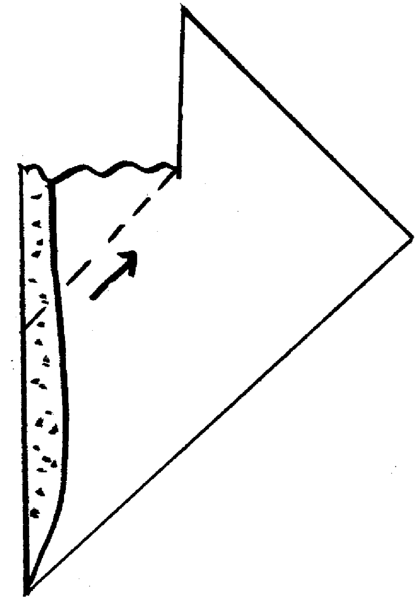
- Standard picture of BH Evaporation and ensuing Information Loss Problem
- The Ashekar- Bojowald paradigm as a possible solution to Info Loss Problem
- Detailed analysis of 2 dimensional BHs

Classical Black Hole:



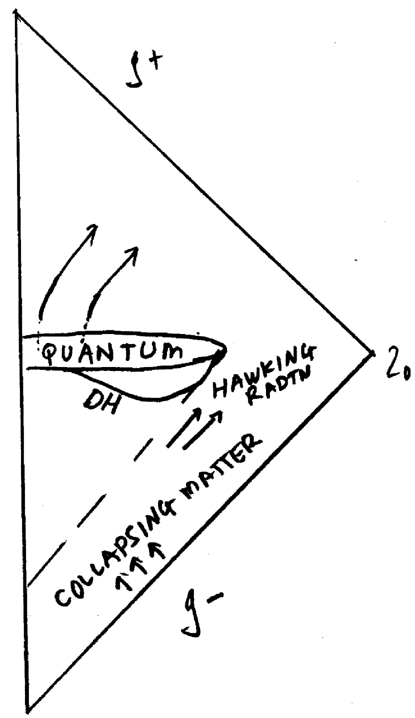


Hawking's Picture of BH Evaporation:



- BH radiates at $\mathbf{kT_H = \frac{m_P}{M}\, m_P c^2}$.
- More it radiates, the hotter it gets. But temp small for large black holes. $\mathbf{M}$ =Solar mass, $\mathbf{T_H \sim 10^{-8\circ}K}$
- So evaporation very slow till $\mathbf{M \sim m_P}$. Quasistatic process.
- Endpt $= \mathbf{m_P}$ + Hawking Radiation. Initial matter = pure quantum state $\Rightarrow$ **INFO LOSS**.

AB Paradigm:



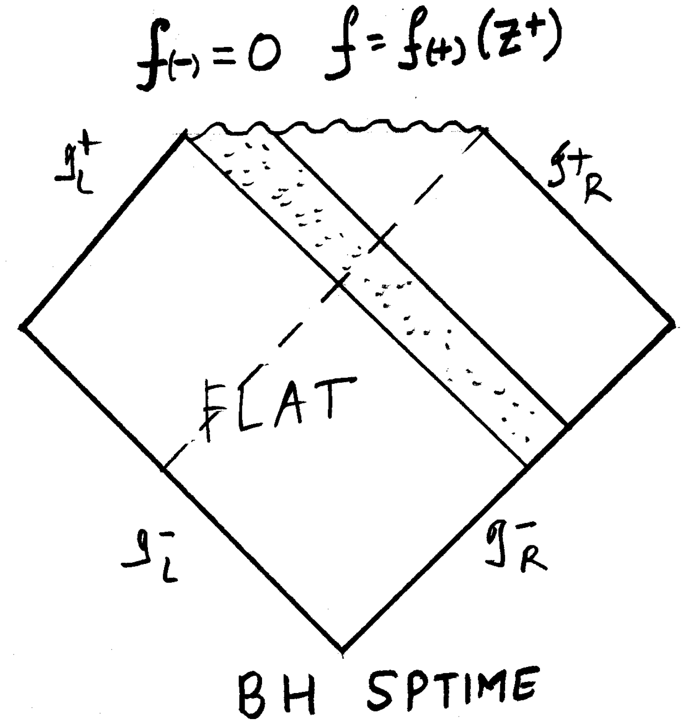
- A quantum extension of classical sptime opens up beyond singularity.
- Info recovered thru correlations of Hawking Radiation with matter on “other side of singularity”

CGHS Model:

$\mathbf{S} = \mathbf{S}(\mathbf{g_{ab}}, \phi) - \frac{1}{2}\int \mathbf{d^2x}\sqrt{\mathbf{g}}\mathbf{g^{ab}}\nabla_\mathbf{a}\mathbf{f}\nabla_\mathbf{b}\mathbf{f}$

- Coupling constants: $[\mathbf{G}] = \mathbf{M^{-1}L^{-1}}$ $\;[\kappa] = \mathbf{L^{-1}}$
- 2d: $\mathbf{g^{ab}} = \mathbf{\Omega}\eta^{\mathbf{ab}}$, $\eta \to -(\mathbf{dt})^2 + (\mathbf{dz})^2$, null coordinates: $\mathbf{z}^\pm = \mathbf{t} \pm \mathbf{z}$
- Equations of Motion: $\partial_+\partial_-\mathbf{f} = \mathbf{0} \Rightarrow \mathbf{f} = \mathbf{f}_+(\mathbf{z}^+) + \mathbf{f}_-(\mathbf{z}_-)$
- Remaining eqns can be solved for the metric and dilaton in terms of stress energy of $\mathbf{f}$. Thus, true degrees of freedom $= \mathbf{f}_+(\mathbf{z}^+), \mathbf{f}_-(\mathbf{z}^-)$

BH solution:



- QFT on CS calculation a′la Hawking (Giddings, Nelson) yields Hawking radiation at $\mathcal{I}_{\mathbf{R}}^{+}$ with $\mathbf{kT_H} = \kappa\hbar$ indep of mass.
- Remark: BH Sptime occupies only part of $(\mathbf{z}^+, \mathbf{z}^-)$ plane.

FULL QUANTUM THEORY:

- $\partial_+\partial_-\hat{\mathbf{f}} = \mathbf{0} : \hat{\mathbf{f}} = \hat{\mathbf{f}}_+(\mathbf{z}^+) + \hat{\mathbf{f}}_-(\mathbf{z}_-)$
  $\hat{\mathbf{f}} =$ free scalar field on $\eta_{\mathbf{ab}}$.
  Fock repn:$\mathcal{F}^+ \times \mathcal{F}^-$.
  Arena for Quantum Theory is entire Minkowskian Plane
- Note: $\mathcal{F}^+ \times \mathcal{F}^-$ is Hilbert space for gravity-dilaton-matter system, not only for matter.
- Dilaton, Metric are operators on this Hilbert space and satisfy (at the moment, formal) operator eqns relating them to $\hat{\mathbf{T}}_{\mathbf{ab}}$.
- Open Issue: **QFT on Quantum sptime**, $\hat{\mathbf{T}}_{\mathbf{ab}} = \hat{\mathbf{T}}_{\mathbf{ab}}(\hat{\mathbf{\Omega}})$
- Despite this, framework itself allows an analysis of Info Loss Problem.

- Info Loss Issue Phrased in Full Quantum Theory Terms:

- Choose "quantum black hole" state $|\mathbf{f_+}\rangle \times |\mathbf{0_-}\rangle$ - analog of classical data $\mathbf{f} = \mathbf{f_+}(\mathbf{z^+}), \mathbf{f_-} = \mathbf{0}$

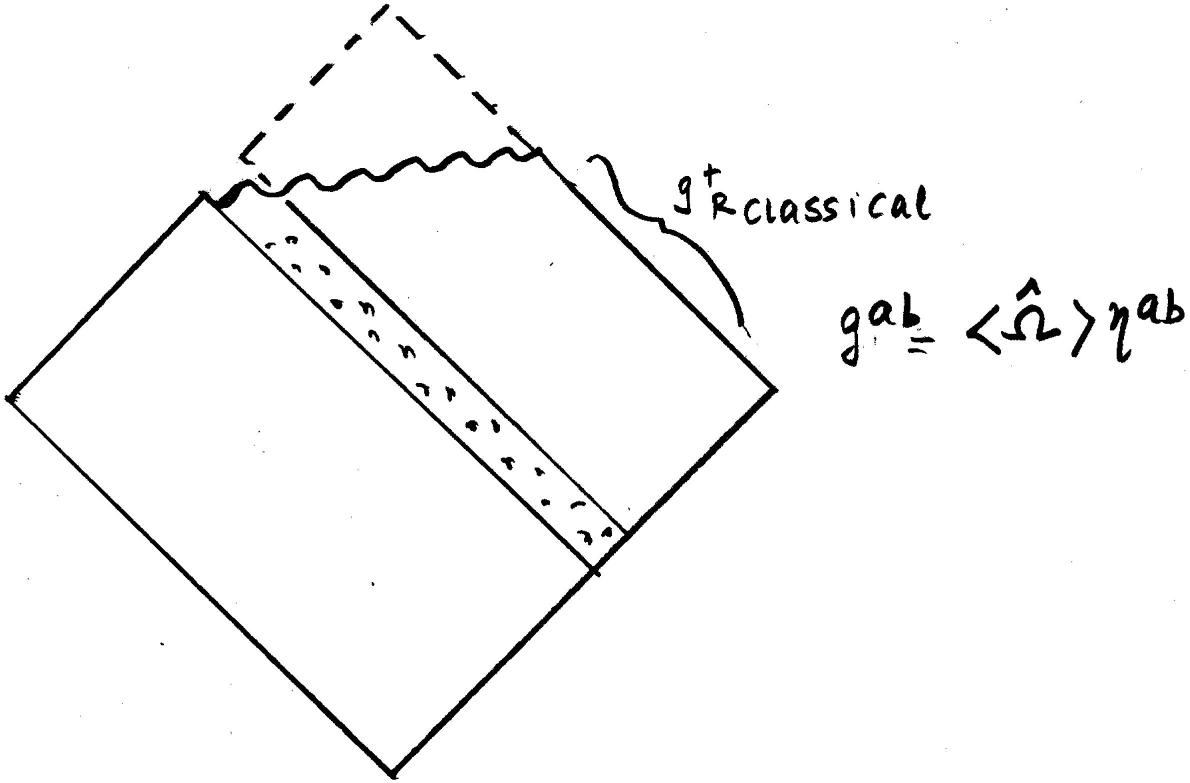
- Info loss issue takes the form: **What happens to $|\mathbf{0_-}\rangle$ part of the state during BH evaporation?**

We shall extract physics from the operator equations using different approximations/Ansatz:

- Trial Solution to the Operator eqns using $\eta_{\mathbf{ab}}$ to define stress energy operator
- Mean Field approximation (analog of semiclassical gravity)

Trial Solution to Oprtr Eqns:

- Use $\eta_{\mathbf{ab}}$ to define $\hat{\mathbf{T}}_{\mathbf{ab}}$.Then $\hat{\mathbf{T}}_{+-} = \mathbf{0}$, can solve oprtr equations explicitly
- Exp value $\langle\hat{\mathbf{\Omega}}\rangle = \mathbf{\Omega}_{\mathbf{classical}}$!
- On singularity $\langle\hat{\mathbf{\Omega}}\rangle = \mathbf{0}$ but $\hat{\mathbf{\Omega}}$ still well defined as operator.
- Near classical singularity: Certain physically relevant operators (related to the quasilocal energy) have large fluctuations
  $\Rightarrow$ Exp values not to be trusted.
- $\hat{\mathbf{\Omega}}$ well defined on whole
  Minkowskian plane, even “above” singularity: **Quantum Extension of Classical Spacetime**.



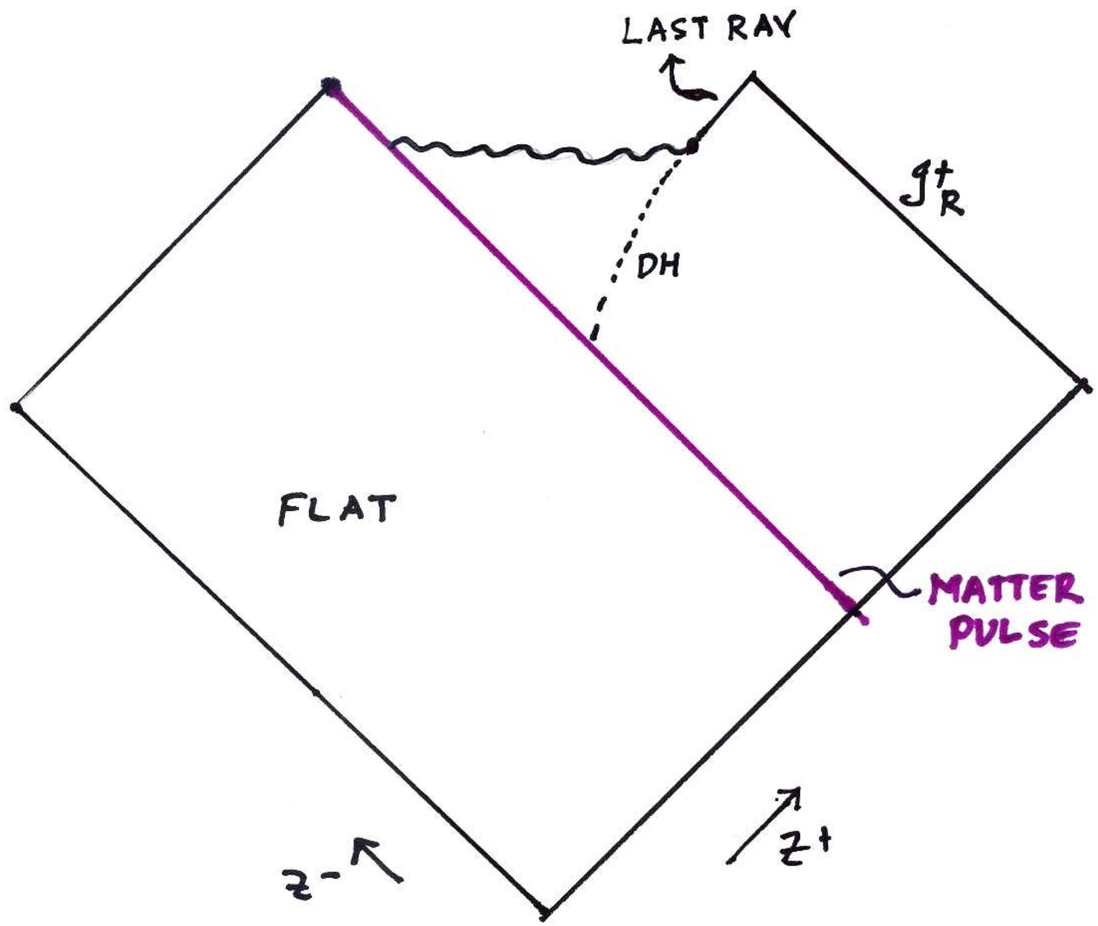
- **Hawking Effect:** Quantum State of gravity-dilaton-matter system $|\mathbf{f}_+\rangle \times |\mathbf{0}_-\rangle$. $|\mathbf{0}_-\rangle$ interpreted by asymptotic inertial observers in expectation-value- geometry at $\mathcal{I}^+_{\mathbf{Rclassical}}$ as Hawking radiation!
- *But*: No backreaction of this radtn

Mean Field Approximation:

- Take exp value of oprtr equations w.r.to $|\mathbf{f_+}\rangle \times |\mathbf{0_-}\rangle$.
- Neglect fluctuations of gravity-dilaton but not of matter
- Get exact analog of "semiclassical gravity" 4d eqns, "$\mathbf{G_{ab} = 8\pi G\langle \hat{T}_{ab}\rangle}$".
- Here $\langle \mathbf{\hat{T}_{ab}}\rangle \sim$ classical $+\mathbf{0(\hbar)}$ (geometry)

Mean Field Numerical Soln:

MF eqns for CGHS studied numerically by Piran-Strominger-Lowe, analytically by Susskind-Thorlacius:

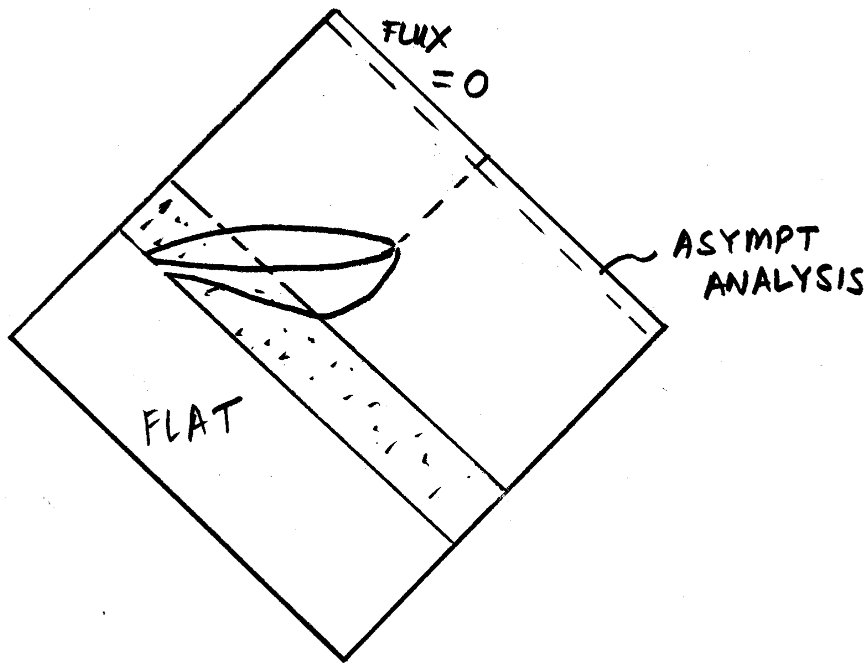


Asymptotic Analysis near $\mathcal{I}_R^+$:

- Knowledge of underlying quantum state of CGHS system + MFA eqns near $\mathcal{I}_{\mathbf{R}}^+$ dictate the response of asympt geometry to energy flux at $\mathcal{I}_{\mathbf{R}}^+$.
- Analysis of eqns implies:

- If Bondi flux smoothly vanishes along $\mathcal{I}_{\mathbf{R}}^+$ then $\mathcal{I}_{\mathbf{R}}^+|_{\mathrm{MFA}}$ is exactly as long as $\mathcal{I}_{\mathbf{R}}^+|_{\eta_{\mathbf{ab}}}$
- $|\mathbf{0}-\rangle$ is a normalized pure state in Hilbert space of freely falling observers (for $\mathbf{g_{ab}}$) at $\mathcal{I}_{\mathbf{R}}^+$
$\Rightarrow$ **NO INFO LOSS**.

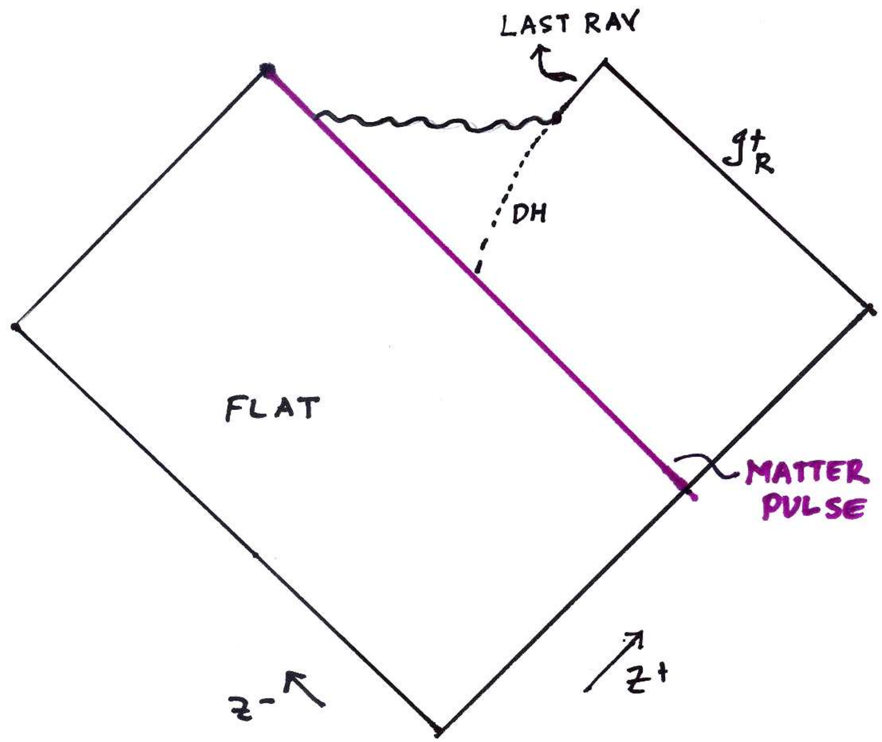
FINAL PICTURE:



- Interior to past of MFA singularity: MFA numerics.
- Near $\mathcal{I}^+_{\mathbf{R}}$: Asymptotic Analysis
- Conceptual underpinnings provided by oprtr equations suggest:
  - **singularity resolution**
  - **extension of classical sptime**

- $|\mathbf{0}_-\rangle$ is pure state populated with particles in Hilbert space of asymp observers
- Information emerges in correlations between ptcles emitted at early and late times
- Open issue: How fast does the information come out? What exactly is information in QFT?

BACK TO NUMERICS



- Interesting questions: Is $\mathcal{I}^+_R$ of Mean Field Sptime complete? Is curvature finite near last ray?
- No! and Yes! due to state of the art numerical simulations (see Ashtekar- Pretorius- Ramazanoglu)

SUMMARY:

Non-pert quantization + MFA numerics + asympt analysis point to unitary pic of BH evaporation with key features:

- **Singularity Resolution.**
- **Extension of Classical Sptime.**
- **No such thing as classically empty sptime.**

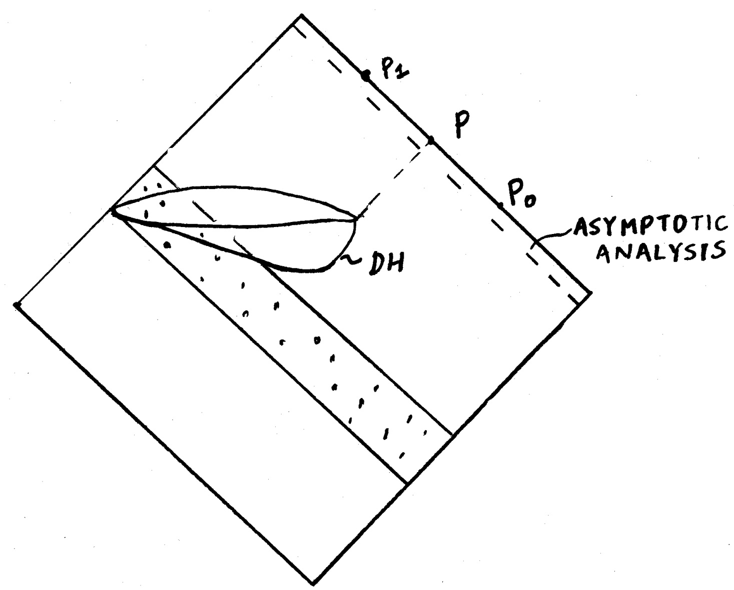
NOTE: MFA requires large N, can be taken care of.

CGHS wrk in collaboration with Abhay Ashtekar and Victor Taveras.

IMPORTANT CAVEAT:

- "Asymptotic analysis $\Rightarrow \mathcal{I}^+_R|_{MFA} = \mathcal{I}^+_R|_{\eta_{ab}}$" rests on key assumption of **SMOOTH** vanishing of Bondi flux.
- With a $(\frac{1}{\text{distance}})^2$ fall off, Asympt Analysis admits the pbility that $\mathcal{I}^+_R$ of MFA sptime ends **BEFORE** $\mathcal{I}^+_R|_{\eta_{ab}}$ ends. This pbility implies Info Loss with respect to observers at physical $\mathcal{I}^+_R$ and suggests a "Thunderbolt" null singularity.

INFO LOSS PROBLEM:



- $|\mathbf{0}_-\rangle$ is pure state in Hilbert space of asymp observers
- Intuitively "nothing emmitted after $\mathbf{P}$", "All info emerges before $\mathbf{P}$ in Hawking radtn".
  $\Rightarrow |\mathbf{\Psi}\rangle = |\mathbf{vac}\rangle_{>\mathbf{P}} \otimes |\mathbf{purestate}\rangle_{<\mathbf{P}}$.
  NOT TRUE! $\langle \hat{\mathbf{f}}(\mathbf{P_1})\hat{\mathbf{f}}(\mathbf{P_0})\rangle \neq \mathbf{0}$ - **Correlations!**

- Where does intuition go wrong?
  Impossible (?) to localise states (Reeh-Schlieder?)to before/after $\mathbf{P} \Rightarrow$ no split $\mathcal{H} = \mathcal{H}_{>\mathbf{P}} \otimes \mathcal{H}_{<\mathbf{P}}$
- Can we do this split "approximately" and say that Hawking radtn is "approximately" pure?
- Use ptcle basis. Ptcle concept *nonlocal*. Can't localise ptcles only to future/past of $\mathbf{P}$. Use orthornormal set of peaked modes. Localiztn approximate because modes always have tails.
- Find $\hat{\rho}_{\mathbf{P}} = \mathbf{Tr}_{>\mathbf{P}}|\mathbf{0}_{-}\rangle\langle\mathbf{0}_{-}|$. Calculate $\mathbf{S}_{\mathbf{P}} = -\mathbf{Tr}\hat{\rho}_{\mathbf{P}} \ln \hat{\rho}_{\mathbf{P}}$.
- Is $\mathbf{S}_{\mathbf{P}}$ "approx" zero? How fast does $\mathbf{S}_{\mathbf{Q}}|_{\mathbf{Q}\to\mathbf{P}}$ decrease? (Depends on how peaked the modes are.Ones in use have very long tails. Can we do better?) Imp to know vis a vis remnants.

Asymptotic Analysis near $\mathcal{I}_R^+$:

- Knowledge of underlying quantum state of CGHS system + MFA eqns near $\mathcal{I}_{\mathbf{R}}^+$ dictate the response of asympt geometry to energy flux at $\mathcal{I}_{\mathbf{R}}^+$.
- But where is $\mathcal{I}_{\mathbf{R}}^+$ located ?

- want MFA soln = classical soln at early times i.e., at $\mathcal{I}_{\mathbf{L}}^-, \mathcal{I}_{\mathbf{R}}^-$ and early on $\mathcal{I}_{\mathbf{R}}^+$
- $\mathcal{I}_{\mathbf{L},\mathbf{R}}^-|_{\mathrm{class}} = \mathcal{I}_{\mathbf{L},\mathbf{R}}^-|_{\eta_{\mathbf{ab}}}$, $\mathcal{I}_{\mathbf{R}}^+|_{\mathrm{class}}^{\mathrm{early}} = \mathcal{I}_{\mathbf{R}}^+|_{\eta_{\mathbf{ab}}}^{\mathrm{early}}$,
  $\mathcal{I}_{\mathbf{R}}^+|_{\mathrm{MFA}} =$ null line
  $\Rightarrow \mathcal{I}_{\mathbf{R}}^+|_{\mathrm{MFA}}^{\mathrm{early}} = \mathcal{I}_{\mathbf{R}}^+|_{\eta_{\mathbf{ab}}}^{\mathrm{early}}$ (along $\mathbf{z}^+ = +\infty$)

- Analysis of eqns implies (almost) uniquely:
  - If Hawking flux smoothly vanishes along $\mathcal{I}^+_{\mathbf{R}}$ then $\mathcal{I}^+_{\mathbf{R}}|_{\mathrm{MFA}}$ is exactly as long as $\mathcal{I}^+_{\mathbf{R}}|_{\eta_{\mathbf{ab}}}$
  - $|\mathbf{0}-\rangle$ is a normalized pure state in Hilbert space of freely falling observers (for g_{ab}) at $\mathcal{I}^+_{\mathbf{R}}$ $\Rightarrow$ **NO INFO LOSS**.

In Detail

- Ansatz for $\mathbf{\Phi}, \mathbf{\Theta}$ consistent with asymp flatness near $\mathcal{I}^+_{\mathbf{R}}$
- Eqns constrain fnal dependence of Ansatz. Left with 1 eqn relating 2 functions $\mathbf{y}^-(\mathbf{z}^-), \beta(\mathbf{z}^-)$
- $(\mathbf{y}^-, \mathbf{z}^+)$ are asymp inertial null coordinates, $\mathbf{ds^2}|_{\mathbf{MF}} \rightarrow -\mathbf{dy^-dz^+} \Rightarrow \mathbf{y}^- \rightarrow \infty \equiv$ complete $\mathcal{I}^+_{\mathbf{R}}$
- $\mathbf{ds^2}|_{\mathbf{MF}} = -\frac{\mathbf{dy^-dz^+}}{\mathbf{1}+\beta\mathbf{e}^{-\kappa\mathbf{y}^-}\mathbf{e}^{-\kappa\mathbf{z}^+}}$ near $\mathcal{I}^+_{\mathbf{R}}$
- Eqn relates β to $\langle\mathbf{T}_{\mathbf{y}^-\mathbf{y}^-}\rangle$. Since state is vacuum wrto $\mathbf{z}^-$, can show that $\langle\mathbf{T}_{\mathbf{y}^-\mathbf{y}^-}\rangle = \frac{\hbar\mathbf{G}}{\mathbf{48}}((\frac{\mathbf{y}^{-\prime\prime}}{\mathbf{y}^{-\prime 2}})^{\mathbf{2}} + \mathbf{2}(\frac{\mathbf{y}^{-\prime\prime}}{\mathbf{y}^{-\prime 2}})^\prime)$.
- Reinterpret eqn as balance eqn for Bondi mass $\frac{\mathbf{dBondi}}{\mathbf{dy}^-} = -\frac{\hbar\mathbf{G}}{\mathbf{48}}(\frac{\mathbf{y}^{-\prime\prime}}{\mathbf{y}^{-\prime 2}})^{\mathbf{2}}$, Bondi determined by $\beta, \mathbf{y}^-$.
- Bondi stops decreasing $\Rightarrow \mathbf{y}^- = \mathbf{C}\mathbf{z}^-$ so $\mathcal{I}^+_{\mathbf{R}}$ coincides with $\mathcal{I}^+_{\mathbf{R}}|_{\eta_{\mathbf{ab}}}$, $\langle\mathbf{T}_{\mathbf{y}^-\mathbf{y}^-}\rangle$ vanishes, $|\mathbf{0}_-\rangle$ is pure state in $\mathbf{y}^-$ Hilbert space.

Oprtr Evoltn Eqns for $\hat{\mathbf{\Phi}}, \hat{\mathbf{\Theta}}$:

$$\partial_+\partial_-\hat{\mathbf{\Phi}} + \kappa^{\mathbf{2}}\hat{\mathbf{\Theta}} - \hat{\mathbf{\Phi}}\partial_+\partial_- \ln\hat{\mathbf{\Theta}} = \mathbf{0}$$

$$\partial_+\partial_-\hat{\mathbf{\Phi}} + \kappa^{\mathbf{2}}\hat{\mathbf{\Theta}} = \mathbf{2G}\hat{\mathbf{T}}_{+-}$$

Oprtr Valued Bdry Condtns.

$$-\partial_+^{\mathbf{2}}\hat{\mathbf{\Theta}} + \partial_+\hat{\mathbf{\Theta}}\partial_+ \ln\hat{\mathbf{\Theta}} = \mathbf{G}\hat{\mathbf{T}}_{++} \text{ on } \mathcal{I}^-_{\mathbf{R}}$$

$$-\partial_-^{\mathbf{2}}\hat{\mathbf{\Theta}} + \partial_-\hat{\mathbf{\Theta}}\partial_- \ln\hat{\mathbf{\Theta}} = \mathbf{G}\hat{\mathbf{T}}_{--} \text{ on } \mathcal{I}^-_{\mathbf{L}}$$