Topological Parameters in Theory of Gravity

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This analysis provides a topological interpretation for the BI parameter.

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To repeat, classical dynamics (the classical equations of motion) do not see tot. div. terms of $\mathcal{L}$; that is, classical dynamics does not depend on the parameter θ.

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Such topological density terms are *universal*.

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This just measures how many times do we wrap or wind $x(t)$ over the interval 0 to 2π as we go around t once over its full range.

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Its integral is characterized by the homotopy group $\Pi_1(S^1)$, which is the set of integers, $\mathbb{Z}$.

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If we were to disregard tunnellings, the quantum energy eigenstates would be an infinitely many degenerate states, each concentrated at the bottom of one of the wells.

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This suggests that θ like the electromagnetic coupling is *not* determined by the theory but is prescribed as a given parameter and *is to be fixed by the experiment.*

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This is the *winding number* of the homotopy maps $S^3 \to S^3$ characterised by the homotopy group $\Pi_3(SU(N)) = \mathbb{Z}$.

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True (nonperturbative) quantum vac state is the linear superposition of perturbative vac states associated with these classical ground states: $|vac\rangle = \sum_{n\in\mathbb{Z}} \exp(in\theta)\,|n\rangle$.

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The constraints from the electric dipole moment of the neutron suggest $\theta\ <\ 10^{-10}\ rad$.

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$$\mathcal{L}_{HP} \;=\; \tfrac{1}{2}\, e\, \Sigma_{IJ}^{\mu\nu}\, R_{\mu\nu}{}^{IJ}(\omega)$$

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We set up a theory of pure (*i.e.*, no matter couplings) gravity in terms of the $24\ \ SO(1,3)$ gauge connections ω_μ^{IJ} and 16 tetrad fields e_μ^I as the *independent* fields, described by Hilbert-Palatini Lagrangian density:

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There are three possible top. densities that can be added:

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So in such a quantum theory, besides the Newton's coupling constant, we can have additional two CP violating (η, θ) and one CP preserving (ϕ) couplings.

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From these, we define a convenient set of tetrad variable:

$$\left[E_i^a \; = 2e\Sigma_{0i}^{ta} \; \equiv e\left(e_0^t e_i^a - e_i^t e_0^a\right),\; \chi_i \; = -M_i/M^0\,,\; N\,,\; N^a\right]$$

Standard parametrization for the tetrad fields:

$$e_t^I = NM^I + N^a V_a^I, \quad e_a^I = V_a^I \; ; \; M_I V_a^I = 0, \quad M_I M^I = -1$$

with N and N^a as the lapse and shift fields.

The inverse tetrads are:

$$e_I^t = -\frac{M_I}{N} \; , \qquad e_I^a = V_I^a + \frac{N^a M_I}{N}; \qquad M^I V_I^a = 0 \; ,$$
$$V_a^I V_I^b = \delta_a^b \; , \qquad V_a^I V_J^a = \delta_J^I + M^I M_J$$

In this parametrization, we have, instead of the 16 tetrad component fields e_μ^I, the following 16 fields:

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where in the last expression $E_i^a \equiv \hat{E}_i^a + \frac{1}{1+\eta^2}\hat{F}_i^a - 2e_{0i}^a(A,K)$

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Constraints $\chi_i^a \approx 0$ are of particular interest.

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Similar discussion is valid for the diffeomorphism generators H_a; Dirac brackets of H_a with various fields yield the Lie derivatives of these fields respectively, modulo $SU(2)$ gauge transformations.

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Hamiltonian analysis, in the time-gauge, can again be set up in terms of a real $SU(2)$ gauge theory with η^{-1} as its coupling constant.