The Many Faces of Black Holes

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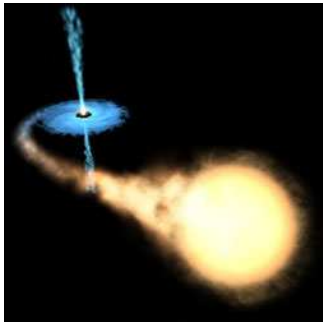
(a) BH Merger



(b) BH Accretion

(c) BH x-ray image

Public Lecture during CHANDRAYANA;   Chennai, January 4th, 2011

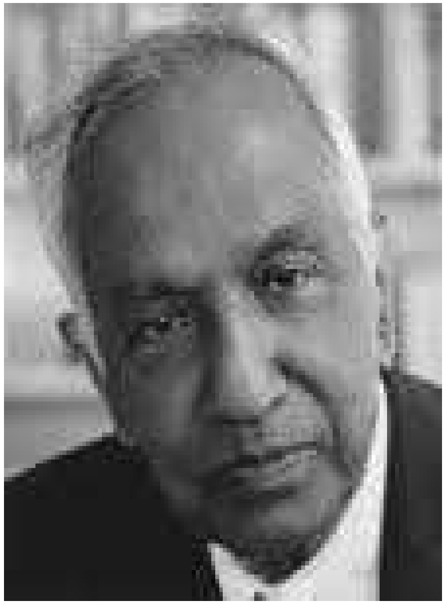
Dedication

Respectfully dedicated to the memory of

Professor S. Chandrasekhar

During the Centennial Celebration,

CHANDRAYANA.



Pre-eminent Astrophysicist of the 20th Century,
Nobel Laureate,
Recipient of Highest Honors
from Heads of three Countries.

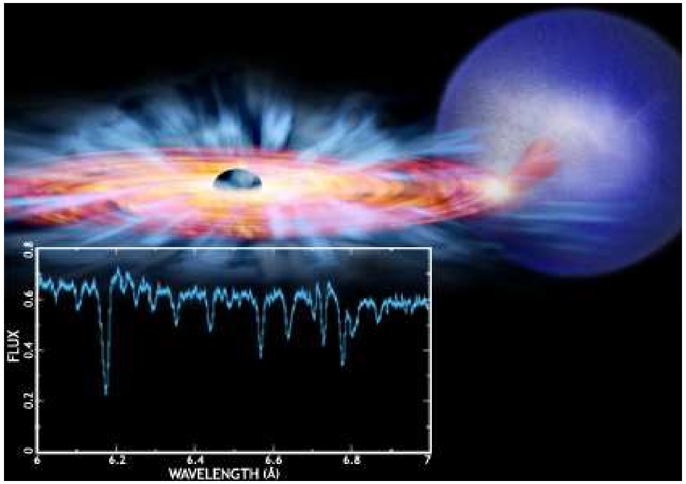
Teacher, Mentor and Sage
Who led by Example

• Black holes are widely recognized as the engines that drive the most energetic phenomena in astrophysics (Narayan's public talk on Thursday). But they have also been the engines behind some of the most unexpected and fascinating advances in mathematical and fundamental physics for over three decades. Professor Chandrasekhar made seminal contributions to the astrophysically important perturbation theory.

• Goal of this Talk: An overview of the profound impact black holes have had on the conceptual fabric of general relativity, why they continue to be fascinating, intriguing and vexing.

Organization:

1. Historical & Conceptual Setting
2. Some Strangeness in Proportion
3. New Challenges, Unexpected Vistas
4. Summary



1. Historical and Conceptual Setting

• Escape Velocity in Newtonian Gravity: $V_e = \sqrt{\frac{2GM}{R}}$

If $V_e \geq c \;\Leftrightarrow\; \frac{2GM}{Rc^2} \geq 1$: light will not escape from the surface of the body: Black Hole!

• Suppose for simplicity the body has uniform density. Then,
$M = \frac{4\pi}{3} R^3 \rho \;\Rightarrow$ Black hole iff $\frac{8\pi}{3}\frac{G}{c^2}\rho R^2 \geq 1$.

Two ways of achieving this:

$\star$ ρ large; Say $\rho \sim 6 \times 10^{16} \mathrm{gm/cm^3}$; then can form a BH of radius $\sim 1\mathrm{km}$. [Recall, Nuclear density $\sim 10^{14}\mathrm{gm/cm^3}$.]

$\star$ ρ small, say density of water. Then if $R \sim 2.5 \times 10^8 \mathrm{km}$, again we have a black hole!

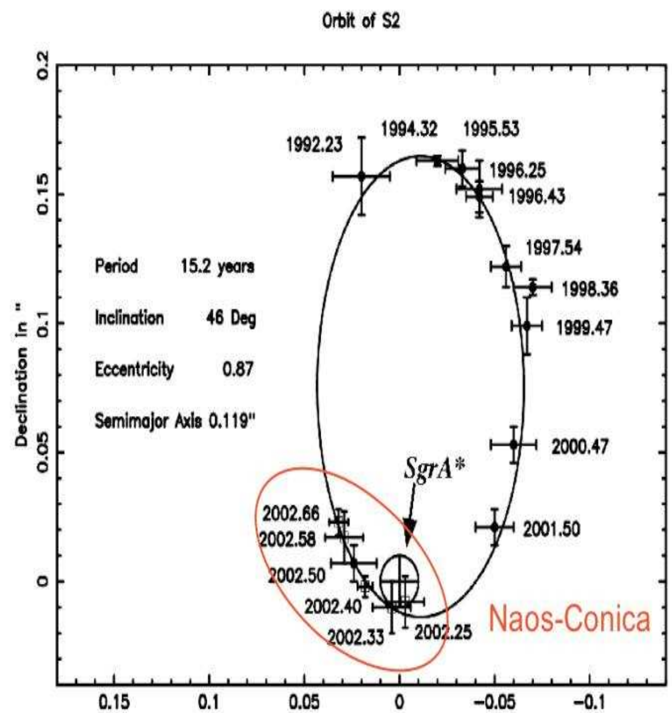
• Interestingly, Nature uses both these avenues. First type of black holes result from stellar collapse; few km in size; a few times $M_\odot$. The second exist in galactic centers; 10^6 to $10^9\; M_\odot$.

These ideas are very old. Surge of interest in the late 1700's:

If there should exist in nature any [such] bodies .... we could have no information from sight; yet if any other luminous bodies should happen to revolve around them we might still perhaps from the motions of these revolving bodies infer the existence of the central ones with some degree of probability, as this might afford a clue to some of the apparent irregularities of the revolving bodies, which would not be easily explicable on any other hypothesis.

John Mitchell

Phil. Trans. R. Soc. (Lon) (1784)



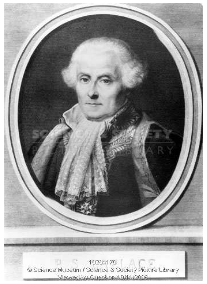
These ideas are very old. Surge of interest in the late 1700's:

A luminous body of the same density as earth, whose diameter is 250 times larger than that of the sun, can by its attractive power prevent its light rays from reaching us, and consequently, largest bodies in the universe could remain invisible to us.

... there exist, in the immensity of space, opaque bodies as considerable in magnitude, and perhaps equally as numerous as stars.

M Le Marquis de Laplace/ Peter Simon Laplace

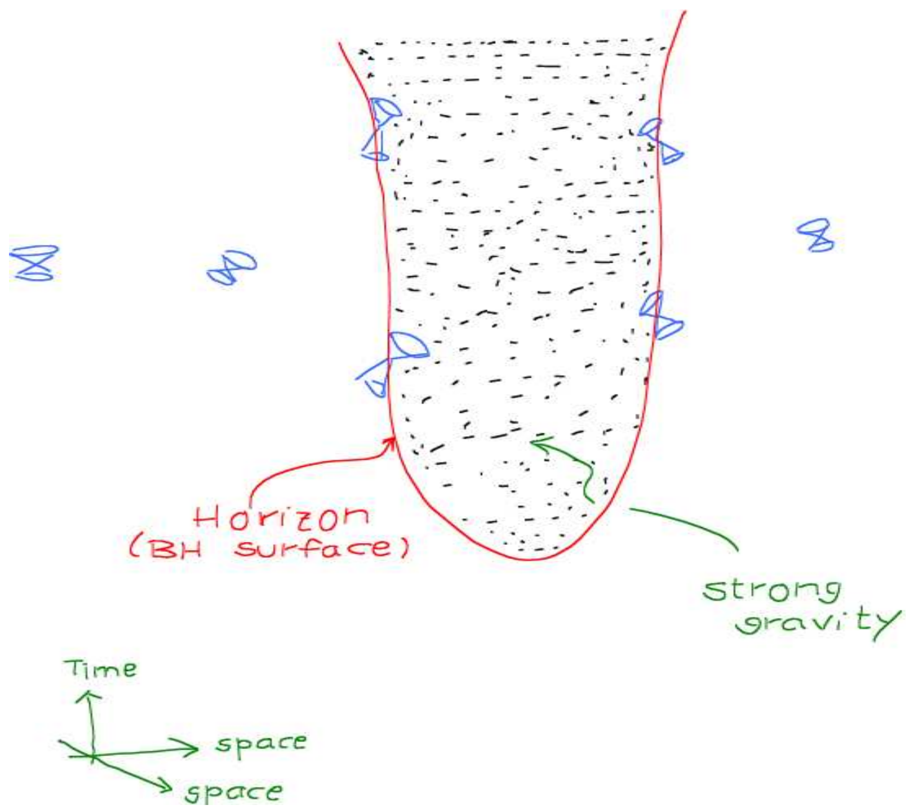
Exposition du système du Monde, Part II (1798/1799)



GENERAL RELATIVITY IS ESSENTIAL!

• Attractive as they seem, the Mitchell/Laplace arguments are conceptually flawed because speed of light is observer dependent in Newtonian physics. Strictly, No black holes in newtonian gravity.

• The notion of black holes requires an observer independent speed of light and gravity $\Rightarrow$ General Relativity is essential.



Black Holes and General relativity

"Black holes are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time."

— S. Chandrasekhar, *Mathematical Theory of Black Holes*

"General theory of relativity is a theory of gravitation; and like the Newtonian theory of gravitation which it refines and broadens, its natural home is astronomy."

— S. Chandrasekhar, *Address to the International Astronomical Union*

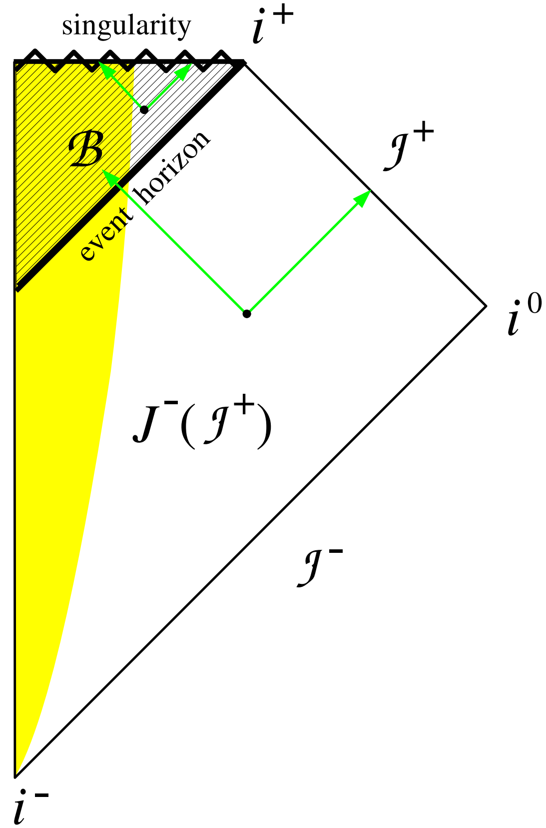
Horizons

• What exactly is a BH in GR? Precise definition?
Event horizon! (Hawking, early 1970s)

• Idea: Black hole $\mathcal{B}$ is a region of space-time from which light cannot escape to infinity. More precisely: Penrose diagram.

$\mathcal{I}^+$ denotes future (null) infinity.

$J^-(\mathcal{I}^+)$ is the 'exterior space-time region' from which light can escape to $\mathcal{I}^+$. $\mathcal{B}$ is the black-hole region from where it cannot. Event horizon E is the outer boundary of $\mathcal{B}$. Once you cross E, cannot escape out! According to GR: Will crush into the singularity.



• In the Schwarzschild space-time, E is the $r = 2M$ surface. Curvature $\sim M/r^3 \sim 1/M^2$.
Can be quite weak at E of $10^9\, M_\odot$ black holes
(~ 100 times weaker than that on earth's surface!)

Simplest Black Holes in General Relativity

• The Schwarzschild Solution: Discovered two months after publication of Einstein's paper on General Relativity, while serving the German army in the First World War (1915).

• Difficulty with the $r = 2m$ surface: Black hole interpretation was not established firmly till Kruskal's 1960 paper! Example of the reluctance to accept that BHs could exist in Nature because of gravitational collapse:

⋆ The Chandrasekhar-Eddington Episode

Various accidents may intervene to save the star. But I want more protection than that. I think there should be a law of Nature to prevent the star from behaving in this absurd way!
— A. Eddington, 1931

⋆ The Einstein (P.G. Bergmann & H.P. Robertson) Episode

Einstein paper: Ann. Math. XI, 922–936 (1939):
Impossibility of formation of a black hole through gravitational collapse!
Oppenheimer-Volkoff paper just a few months later. No mention of black holes in Bergmann's 1942 book, the first monograph on General Relativity.

• Kerr Solution: Rotating, stationary black hole (1963) two parameters: Mass and Angular Momentum.

2. Some Strangeness in Proportion

There is no excellent beauty that hath not some strangeness in proportion

— Sir Francis Bacon

• Kerr Black holes: Solutions to Einstein's vacuum solutions. Manifestations of pure geometry.

• Uniqueness theorems for stationary, Electro-vac black hole solutions (Israel, Hawking, Robinson, Mazur, Bunting) $\Rightarrow$ Kerr-Newman family suffices!
Striking contrast with stationary stars.
Uniqueness holds *only* in the Einstein Maxwell theory (long range fields) and only in 4-dimensional space-times: Simplicity *precisely* in the physically relevant situations!

• Uniqueness theorems: Black holes very very rare? No! Higher multipoles get radiated away (Penrose (60's), Price (70's), Dafermos & Rodnianski (2008-09)). Black holes very common!

• The astronomy community was reluctant to accept existence of black-holes till the early eighties! Now, the tide has reversed: Essentially every galaxy is thought to have a super-massive black hole at its center.

Unforeseen properties of event horizons E

General Relativity & Thermodynamics are related! Black holes of GR are subject to three laws: (Bardeen, Carter, Hawking)

i) Surface gravity κ is constant on E, if the BH is in equilibrium (stationary), even when E is non-spherical! ($\kappa \sim g$ on earth's surface)

ii) If a BH makes a transition from an equilibrium state to a nearby equilibrium state, the mass M of the BH, the area a of E, and κ, are related by

$$\delta M = \frac{\kappa}{8\pi G}\,\delta a + \delta[\text{Work done on the BH}]$$

iii) If matter satisfies 'energy conditions', the area a of E cannot decrease.

• Striking similarity with the laws of thermodynamics: (a multiple of) κ plays the role of temperature, and (a multiple of) a of entropy! (Bekenstein)

• Hawking's discovery: BHs radiate quantum mechanically as though they are black bodies at temperature $T = \hbar\kappa/2\pi$. From first law, one is led to assign entropy $S = a/4G\hbar = a/4\ell_{\rm Pl}^2$ to E.

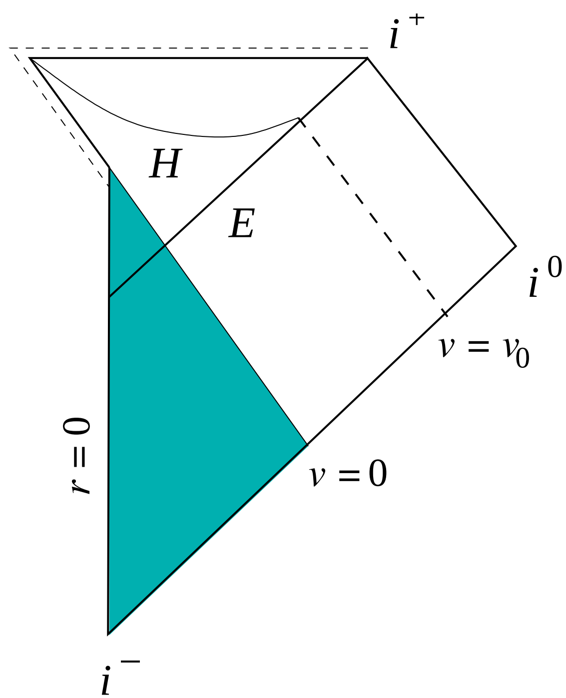
• The three pillars of fundamental physics, Quantum Mechanics, General Relativity and Statistical mechanics, unexpectedly brought together!

3. New Challenges: ‘Spookiness’ of event horizons

• Event horizons are too global. Refer to the entire past of $\mathcal{I}^+$. A smooth change in space-time geometry in a small neighborhood of singularity can shift them drastically and even make them disappear! (Hajicek).

• Event horizons are teleological: One may be developing right now, in this room in anticipation of a gravitational collapse in our region of the Milky Way a million years from now!

Explicit solution to Einstein’s equation (Vaidya metric) in which E develops in a flat space-time region and grows in anticipation of a future gravitational collapse!



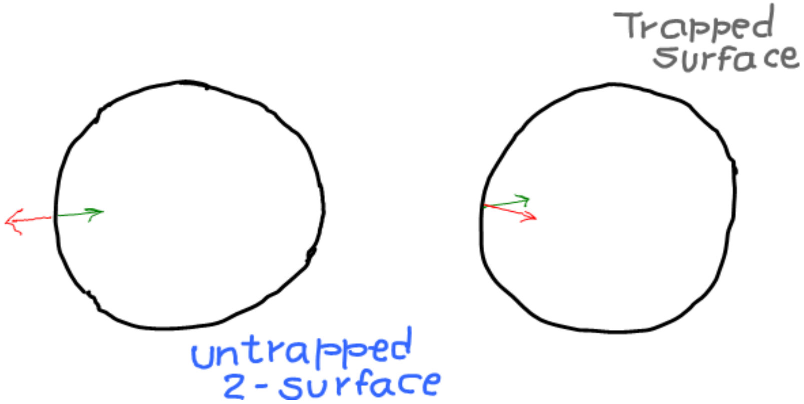
• Can not hope to generalize the first law to fully dynamical situations using E: Area can grow even when nothing is falling across E!

• To construct E, one needs to know full space-time. Not very useful to characterize a BH during numerical simulations.

Strategy: Quasi-local horizons

• To overcome these limitations of E in dynamical situations, quasi-local horizons were introduced more recently (Hayward, AA,).

• Idea: Trapped and Marginally Trapped Surfaces (MTSs) (Penrose).

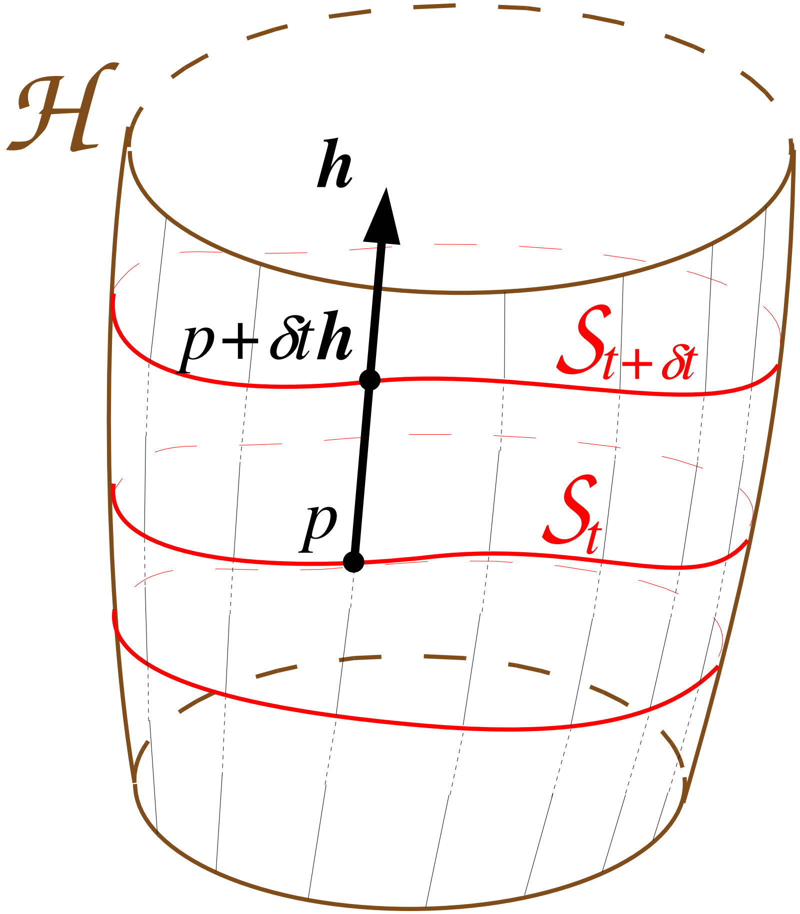


Event horizons replaced by world tubes $\mathcal{H}$ of MTSs obtained by stacking MTSs.(Unrelated to apparent horizons.)

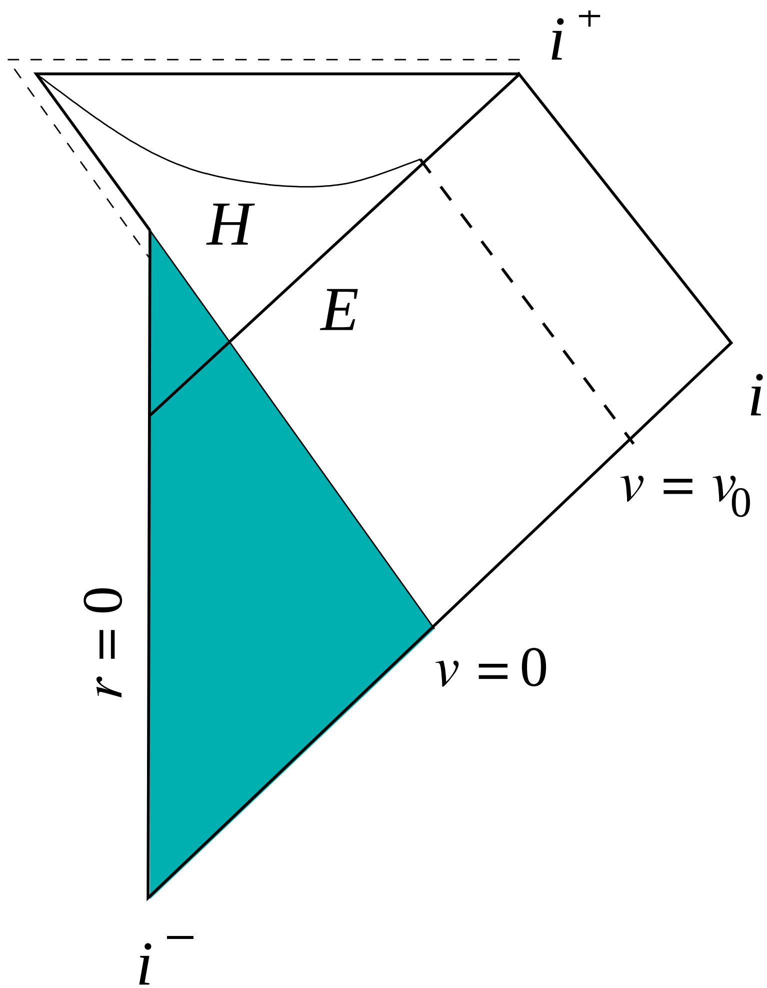
• Quasi-local notions $\Rightarrow$ nothing teleological. HUGE generalization from stationary event horizons. No quasi-local horizon in this room!

• But what about BH Thermodynamics??

Quasi-local horizons



(d) $\mathcal{H}$ obtained by stacking MTSs S_t



(e) Vaidya space-time

Unexpected Vistas

• But what about BH Thermodynamics?? Extends to Quasi-local horizons!! (AA, Krishnan, Beetle, Booth, Fairhurst, Hayward, Liko, Lewandowski, Pawlowski....) Black hole and cosmological horizons covered in one stroke!

• Even when there is radiation arbitrarily close to a quasi-local horizon, the zeroth law holds as long as none falls in: surface gravity κ is constant. Non-trivial example: A family of Robinson-Trautmann solutions. (Chrusciel) Huge generalization from stationarity.

• Furthermore, generalization of the first law to fully dynamical situations. A new differential geometric monotonicity formula directly tells us how BHs grow, i.e., how the area of H increases (AA, Krishnan) :

$$R(t_2) - R(t_1) = 2G\,\text{[Flux of matter energy]} + 2G\,\underbrace{\text{[positive definite geometric quantity]}}_{\text{flux of gravitational wave energy}}$$

• Bonus: Formula for gravitational energy flux. (positive) precisely if the 3-manifold is a dynamical horizon. Beautiful interplay between Geometry and Physics.

• In presence of angular momentum: Integral form of the first law for finite processes within a given space-time. (AA, Krishnan)

New Challenge: Black Hole Entropy

• First law of BH Mechanics + Hawking's discovery that $T_{\rm BH} = \kappa\hbar/2\pi \;\; \Rightarrow$ for isolated horizons, $S_{\rm BH} = a_{\rm hor}/4\ell_{\rm Pl}^2$

• Entropy: Why is the entropy proportional to area? For a $M_\odot$ black hole, we must have $\exp 10^{76}$ micro-states, a HUGE number even by standards of statistical mechanics. Where do these micro-states come from?

For gas in box, the microstates come from molecules; for a ferromagnet, from Heisenberg spins; Black hole ?
Cannot be gravitons: gravitational fields stationary.

• To answer these questions, must go beyond the classical space-time approximation used in the Hawking effect. Must take into account the quantum nature of gravity.

• Distinct approaches. Attractive features but none completely satisfactory. In Loop Quantum Gravity, this entropy arises from the huge number of microstates of the quantum horizon geometry. 'Atoms' of geometry itself!

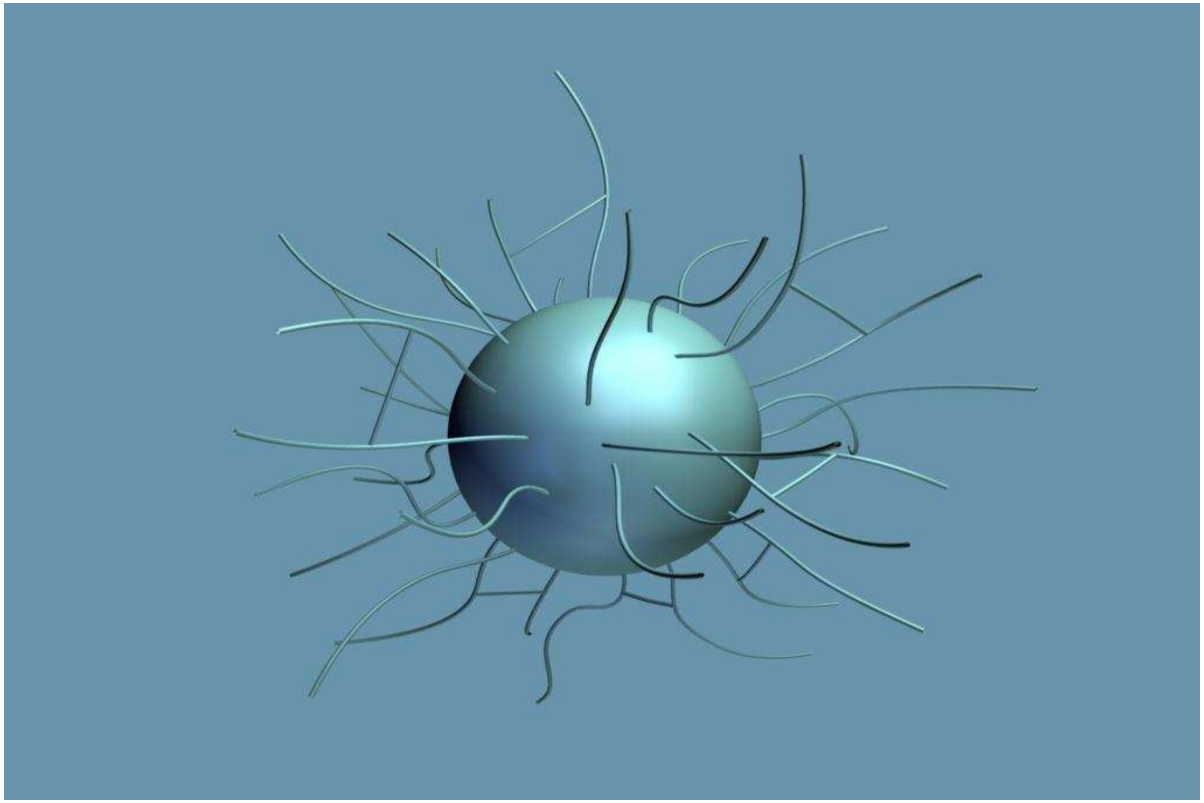
Quantum Horizon Geometry & Entropy

• Heuristics: Wheeler's It from Bit
Divide the horizon into elementary cells, each carrying area $\ell_{\rm Pl}^2$.
Assign to each cell a 'Bit' i.e. 2 states.
Then, # of cells $n \sim a_o/\ell_{\rm Pl}^2$; No of states $\mathcal{N} \sim 2^n$;
$S_{\rm hor} \sim \ln \mathcal{N} \sim n \ln 2 \sim a_o/\ell_{\rm Pl}^2$. Thus, $S_{\rm hor} \propto a_o/\ell_{\rm Pl}^2$.

• Argument made rigorous in quantum geometry. Many inaccuracies of the heuristic argument have to be overcome: Calculation has to know that the surface is black hole horizon; What is a quantum horizon? In Loop quantum gravity, horizon is quasi-local and isolated. This horizon boundary condition is made into an operator equation whose solutions represent quantum horizons of interest. Quanta of area not $\ell_{\rm Pl}^2$ but $4\pi\gamma \sqrt{j(j+)}\, \ell_{\rm Pl}^2$.

• Interesting mathematical structures $\mathrm{U}(1)$ Chern-Simons theory; non-commutative torus, quantum $\mathrm{U}(1)$, mapping class group, ...(AA, Baez, Corichi, Krasnov; Kaul and Majumdar; Gosh & Mitra; Domagala, Lewandowski; Meissner; AA, Engle, Van Den Broeck, ...)

Quantum Horizon

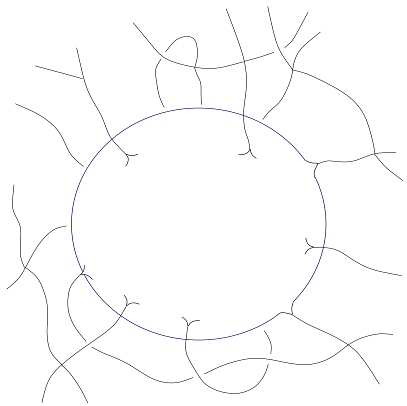


Continuum only an approximation. At Planck scale, fundamental excitations of bulk geometry 1-dimensional, polymer-like. Each quantum thread pierces the horizon $\mathcal{H}$ and deposits a quantum of area on $\mathcal{H}$. Quantum geometry of $\mathcal{H}$ described by the Chern-Simons theory.

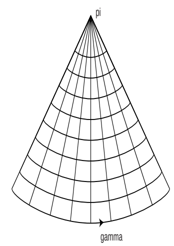
Quantum Horizon Geometry and Entropy

• Horizon geometry flat everywhere except at punctures. At punctures the bulk polymer excitations cause a ‘tug’ giving rise to quantized deficit angles. They add up to 4π providing a 2-sphere quantum geometry. ‘Quantum Gauss-Bonnet Theorem’.

• As in statistical mechanics,
count the number of horizon quantum states compatible with pre-specified macroscopic parameters (multipoles) characterizing the intrinsic geometry of the quasi-local, isolated horizon. $S_{\rm hor} = \ln \mathcal{N}$.



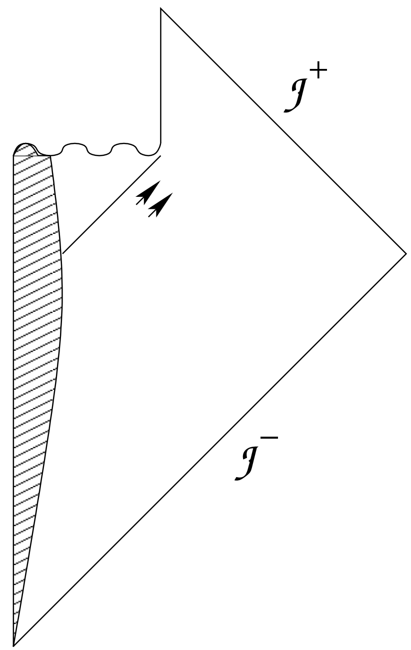
• Result:
$S_{\rm hor} = a_{\rm hor}/4\ell_{\rm Pl}^2 - (1/2)\ln(a_{\rm hor}/\ell_{\rm Pl}^2) + o(a_{\rm hor}/\ell_{\rm Pl}^2)$
for a specific value of the parameter γ.
Procedure incorporates all physically interesting BHs and Cosmological horizons in one swoop.



Another Fundamental Challenge: Information loss!

• Hawking radiation: Quantum Field Theory in a fixed BH space-time.
Energy conservation $\Rightarrow$ BH must loose mass and evaporate.

• Suppose BH was formed by collapsing matter in a pure state. Hawking radiation is thermal. So when the BH is 'gone', we are left with a mixed state of maximum entropy (for the given energy). So, in this approximation, pure states seem to evolve to mixed states in BH formation and evaporation. Process seems non-unitary; information is lost! If so: Basic structure of quantum mechanics has to be modified!!



• Where did the information go??
Although BH has evaporated, in Hawking's original scenario, a singularity still remains. (The Cheshire cat disappears but the smile remains!) Acts as a sink of information.

Information Loss: Status

• Does this situation persist in full quantum gravity which appropriately incorporates the quantum nature of gravity/geometry, not just of matter?

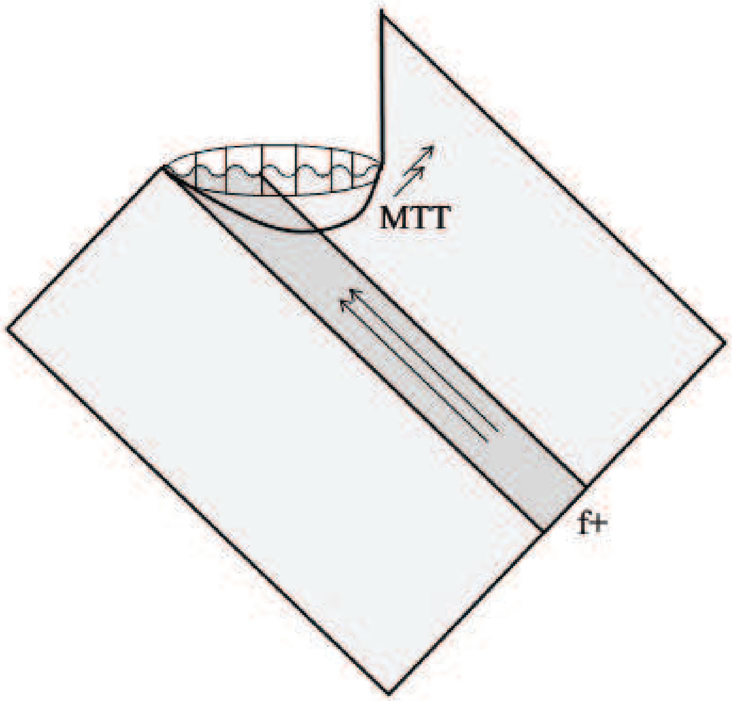
• Problem still open in 4-d! String theory suggests that pure states must evolve into pure states (AdS/CFT); information cannot be lost. But the reasoning requires $\Lambda < 0$ More importantly, 'space-time understanding' still lacking.

• In Loop Quantum Gravity, we know in detail that quantum geometry creates a brand new repulsive force in the Planck regime and resolves cosmological singularities. Strong indications that this is also true for black holes. No singularity $\Rightarrow$ No sink of information. So the S-matrix should be unitary. No need to modify this premise of quantum mechanics.

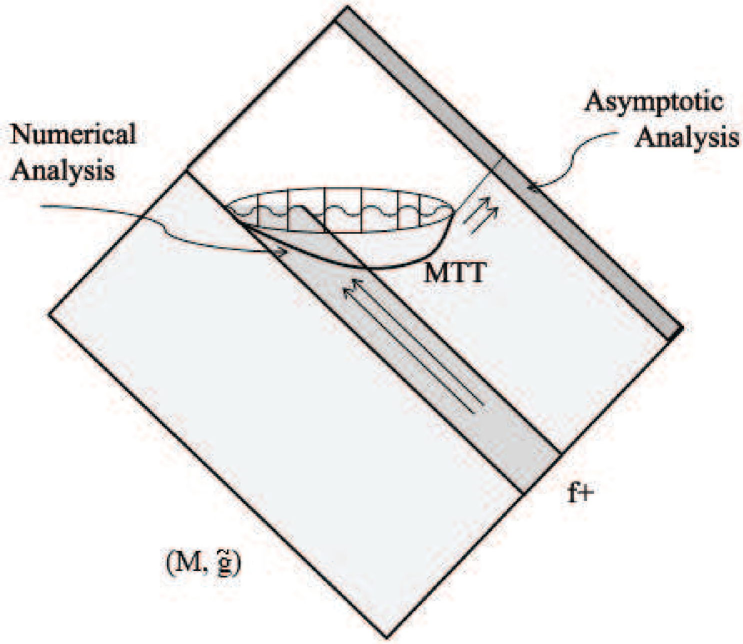
• Singularity resolution $\Rightarrow$ Quantum space-time larger than what Einstein told us. Information can be recovered on the larger null infinity of this extended space-time. (Varadarajan's talk on Friday).

• Scenario realized in 2-dimensional CGHS BHs. (AA, Pretorius, Ramazanoğlu; Varadarajan, AA, Taveras).

Quantum process: Older and the New descriptions



(f) Remnant singularity: sink of info



(g) No singularity & no info loss

There is no information loss because the quantum space-time is sufficiently larger than the classical one.

4. Summary: BHs and Fundamental Physics

• Stationary BHs have been well understood for some time. Hawking et al showed an astonishing connection between GR and Thermodynamics. To make it precise, need quantum physics! Beautiful convergence of ideas from the three pillars of fundamental physics.

• But based on an excessively global and teleological event horizons. Euphoria followed by frustration.

• Quasi-local horizons. Free of these difficulties and provide even stronger analogs of the laws of thermodynamics. Have had applications to computational relativity, mathematical physics and quantum gravity.

• New questions: Why is the entropy proportional to the Isolated horizon area? Is there information lost during BH formation and evaporation? Lot of progress in recent years. But still not completely settled.

• Black holes seem to have a vast potential to tease us, vex us and then lead us to deeper insights. Cycles: euphoria, followed by vexing puzzles and challenges, followed by new paradigms, ... !!

BHS: Our Triumph, Our Consolation

"There is a square; there is an oblong. The players take the square and place it upon the oblong. They place it very accurately; they make a perfect dwelling-place. Very little is left outside. The structure is now visible; what is inchoate is here stated; we are not so various or so mean; we have made squares and stood them upon oblongs. This is our triumph; this is our consolation."

— Virginia Wolf, *The Waves*