The Big Bang and the Quantum

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Loop Quantum Cosmology $\gtrsim\ 500$ articles in PRL, PRD, CQG and JCAP

Short Review; Proceedings of the Cosmology Conference, Paris, arXiv:1005.5491

Pedagogical articles: http://www.gravity.psu.edu/outreach/

Understanding emerged from the work of many researchers, especially:

Bianchi, Bojowald, Campiglia, Corichi, Chiou, Date, Henderson, Kaminski, Lewandowski, Pawlowski, Rovelli, Singh, Sloan, Taveras, Vandersloot, Vidotto, Wilson-Ewing ....

Chandrayana, IMS Chennai; Jan 3-7, 2011

1. Big-Bang: An Historical Introduction

- General Relativity: Gravity encoded in Geometry. Space-time geometry became a physical and dynamical entity. Spectacular consequences: Black holes, Gravitational Waves.

- But this fusion comes with a price: Now space-time itself ends at singularities. Big Bang: Absolute Beginning.

Friedmann (1921-1924) Lemaître (1926-1965)

The assumption of spatial homogeneity & isotropy implies that the metric has the FLRW form: $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\ \mathrm{d}\vec{x}^2$

$a(t)$: Scale Factor; Volume $v(t) \sim [a(t)]^3$ Curvature $\sim [a(t)]^{-n}$

Einstein Equations $\Rightarrow$ volume $\to 0$ and Curvature $\to \infty$:

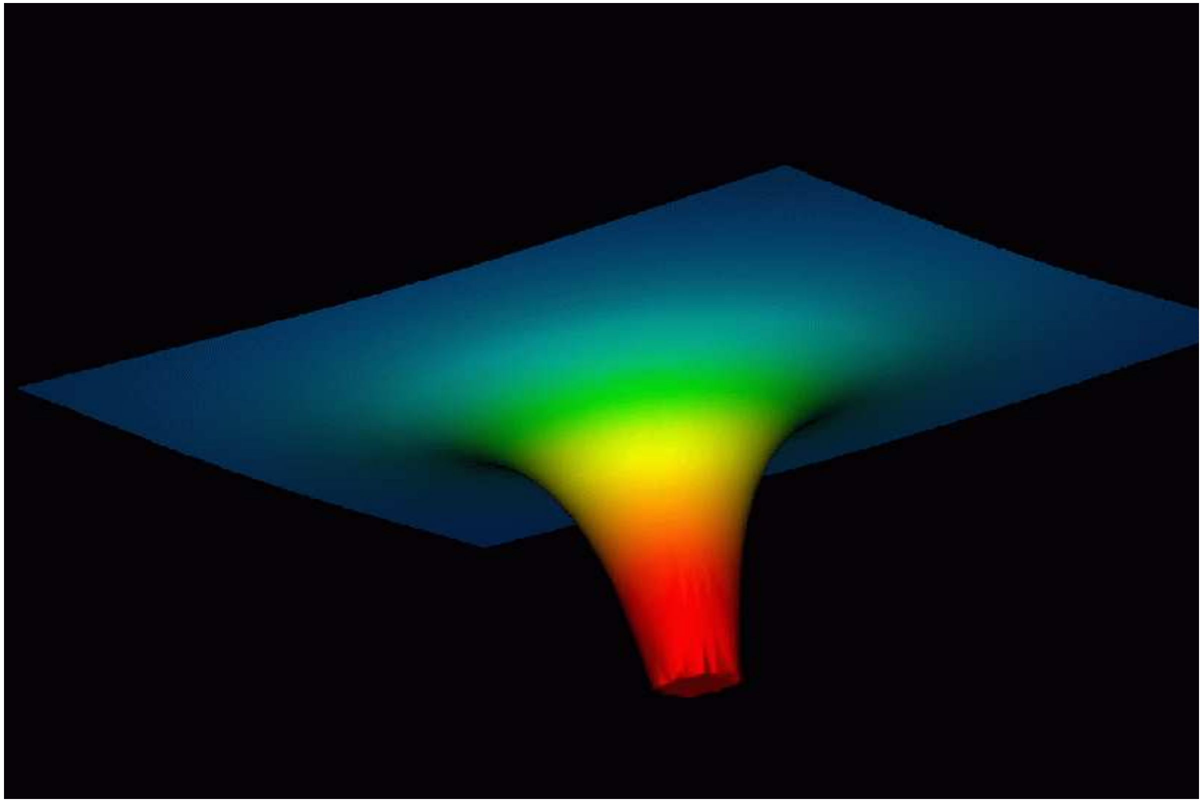
BIG BANG!! CLASSICAL PHYSICS STOPS!!

- Gamow, Alpher, Herman (1948-1967) (Detailed Nucleosynthesis).

  Gamow strongly disliked the emphasis on Big-Bang/Beginning.
- Dicke, Peebles, Roll, Wilkinson (1965 $\to$) (CMB Background)

Dicke also disliked the Absolute Beginning; Preferred an "oscillating" universe.

The Big Bang in classical GR



Artist's conception of the Big-Bang. Credits: Pablo Laguna.

In classical general relativity the fabric of space-time is violently torn apart at the Big Bang singularity.

Big Bang Singularity: Twists and Turns

- Friedmann was delighted to prove Einstein wrong but not very interested in the physics of the solutions. It is Lemaître who understood the implications and took their physical significance seriously.

- Even afterwards, Einstein did not take the Big-Bang/Beginning seriously. Suggested inhomogeneities may wash it away. This view persisted.

- The Khalatnikov-Lifshitz program: “General Solution” to Einstein’s equation will be singularity free (late 50’s - early 60’s). Gamow disliked the term *‘big-bang’* and preferred to emphasize *‘dynamical universe.’* Preferred to think the universe had a pre-big-bang branch.

- Paradigm Shift:

Penrose-Hawking Singularity Theorems (mid 60s): If matter satisfies ‘energy conditions’ then according to general relativity, cosmological space-times will necessarily have a singularity!

(Lemaître’s views realized.)

Beyond General Relativity

- But general expectation: theory is pushed beyond its domain of applicability. Must incorporate Quantum Physics. (Example: Instability of the Hydrogen atom in classical electrodynamics and $E_o = -me^4/2\hbar^2$ in quantum theory.)

- Big-bang is the prediction of General Relativity precisely in a domain in which it is inapplicable! Classical singularities are gates to Physics Beyond Einstein.

- Any viable quantum gravity theory should answer the questions: What *really* happened in the Planck regime? In the standard model, CMB occurs 380,000 years after the Big Bang. At the onset of inflation, matter density is less than $10^{-11}\,\rho_{\rm Pl}$. Far from 'proofs' that Big Bang occurred! Does quantum physics really stop if we went further back? Is there a finite Beginning? If not, what was really there before the GR era?

Beyond General Relativity

Goal of this talk: Show that in cosmological models *Quantum Physics* does not stop at singularities. Quantum geometry extends its life. Models simple but, in contrast to string theory, singularities treated are of direct physical interest. Will focus on conceptual and mathematical physics issues.

Idea in Loop Quantum Gravity: Retain the gravity $\leftrightarrow$ geometry duality by encoding new physics in Quantum Riemannian Geometry which was developed rigorously in the mid 1990's. (AA, Baez, Lewandowski, Rovelli, Smolin,...)

Organization:

$\surd$ 1. Big-Bang: An Historical Introduction $\surd$

2. Conceptual Setting & Older Quantum Cosmology

3. Loop Quantum Cosmology: Paradigm Shift

4. Illustrative Application: Conceptual and Phenomenological

5. Summary.

2. Big Bang: Conceptual Setting

Some Long-Standing Questions expected to be answered by Quantum Gravity Theories from first principles:

$\star$ How close to the big-bang does a smooth space-time of GR and a QFT on it make sense? (e.g. At the onset of the GUT era?)

$\star$ Is the Big-Bang singularity naturally resolved by quantum gravity?
(answer is ‘No’ in older quantum cosmology —the Wheeler-DeWitt theory)

$\star$ Is a new principle/ boundary condition at the Big Bang essential?
(e.g. The Hartle-Hawking ‘no-boundary proposal’.)

$\star$ Is the quantum evolution across the ‘singularity’ deterministic?
(answer ‘No’ e.g. in the Pre-Big-Bang and Ekpyrotic/cyclic scenarios)

$\star$ What is on the ‘other side’? A quantum foam? Another large, classical universe?
(Fascinating history within classical GR: Lemaître, Tolman, Gamow, Zanstra, Dicke, Sakharov, Weinberg ...)

Some Long Standing Questions (contd)

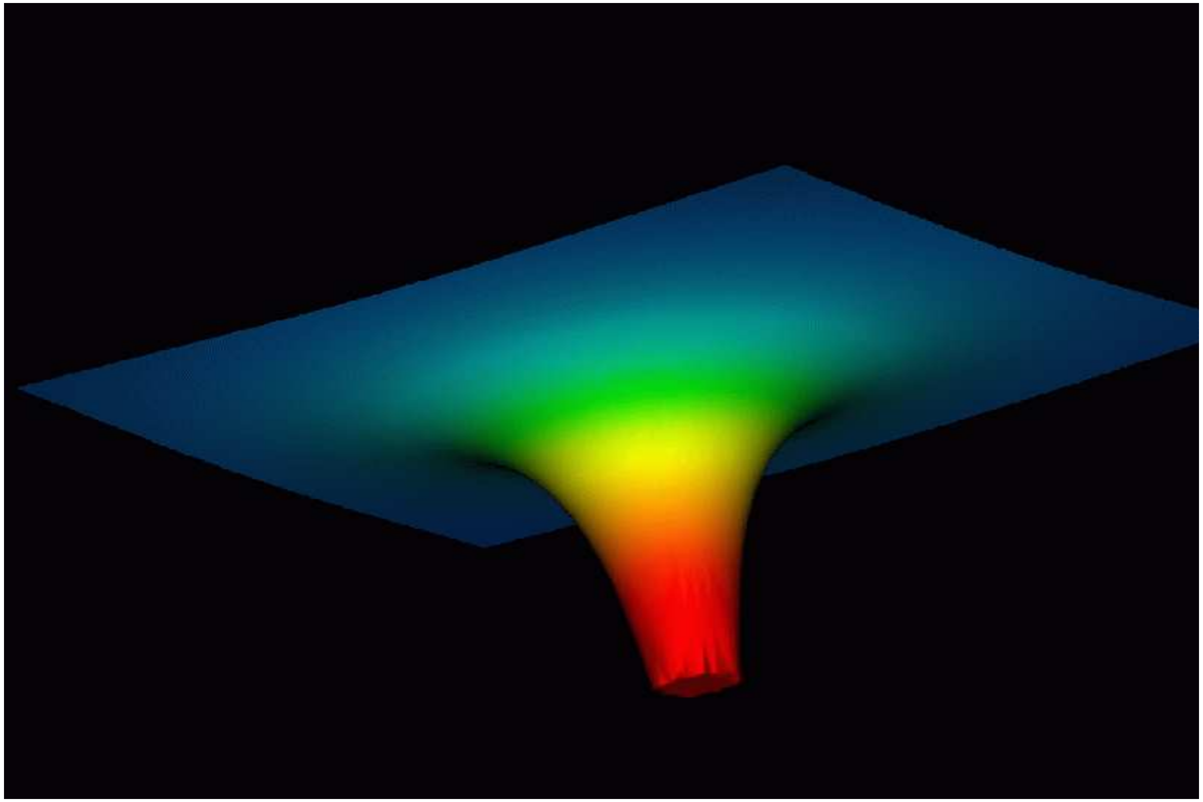
- UV - IR Tension: Can one have a deterministic evolution across the singularity and agreement with GR at low curvatures? (Background dependent perturbative approaches have difficulty with the first while background independent approaches, with second.) (Green and Unruh)

- These questions have been with us for 30-40 years since the pioneering work of DeWitt, Misner and Wheeler.

- In LQC, these issues have been resolved for general homogeneous cosmological models (Bianchi space-times). Physical observables which are classically singular (eg matter density) at the big bang have a dynamically induced upper bound on the physical Hilbert space. Mathematically rigorous and conceptually complete framework. (AA, Bojowald, Corichi, Pawlowski, Singh, Vandersloot, Wilson-Ewing, ...)

- Emerging Scenario:
In simplest models: Vast classical regions bridged deterministically by quantum geometry. No new principle needed to join the pre-big bang and post-big-bang branches.

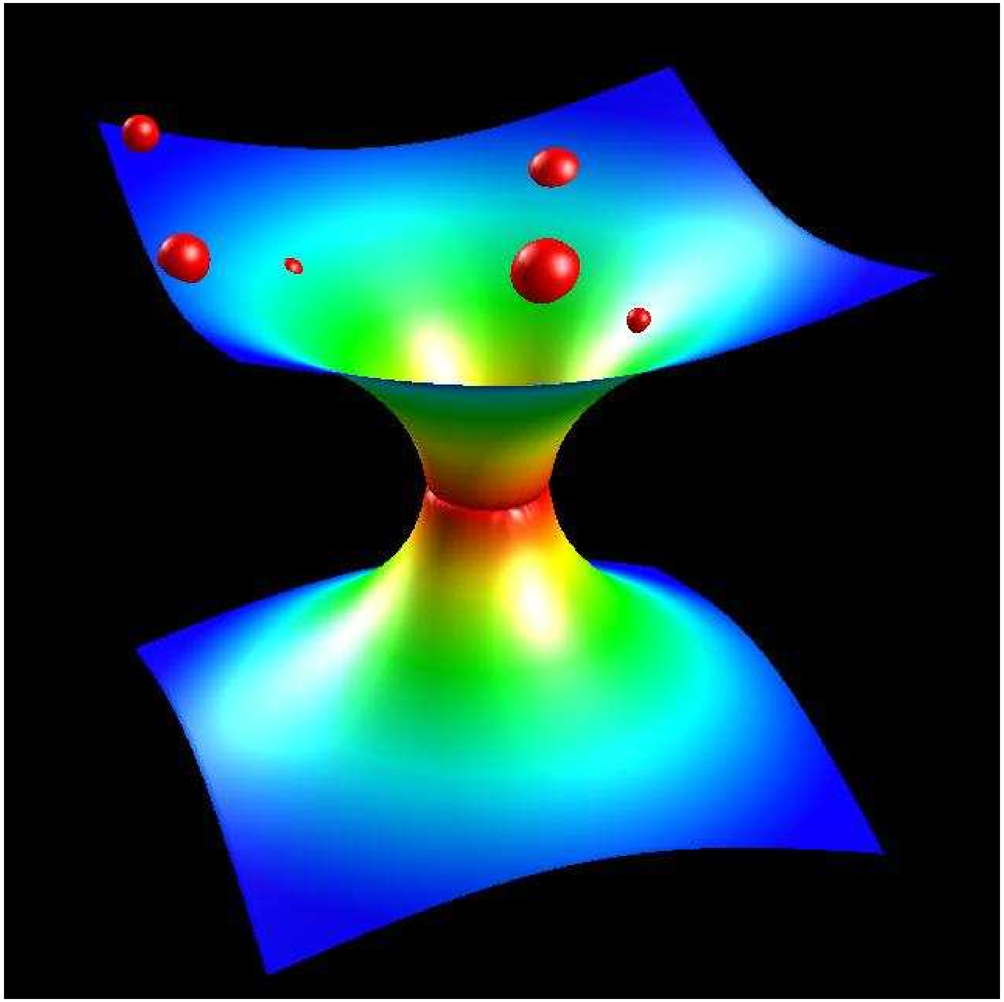
The Big Bang in classical GR



Artist's conception of the Big-Bang. Credits: Pablo Laguna.

In classical general relativity the fabric of space-time is violently torn apart at the Big Bang singularity.

The Big Bang in LQC: k= 0, Λ=0 Model

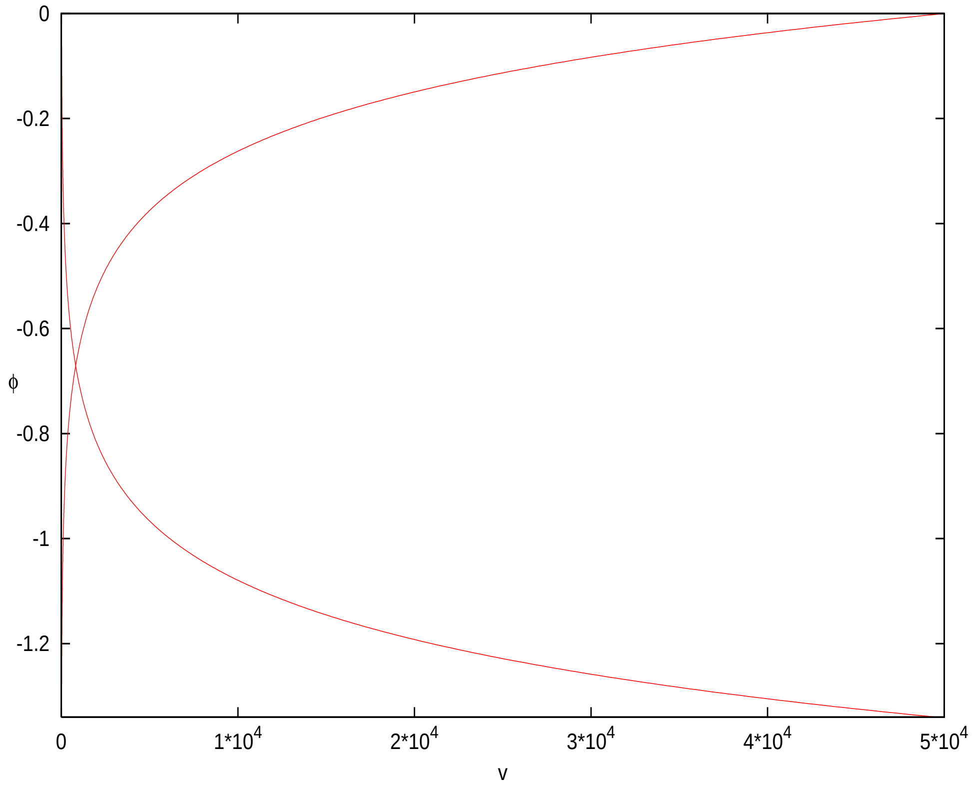


Artist's depiction of the Quantum Bounce Credits: Dr. Cliff Pickover.

In loop quantum cosmology, our post-big-bang branch of the universe is joined to a pre-big-bang branch by a quantum bridge: Gamow's bounce

The k=0 FLRW Model

FLRW, k=0 Model coupled to a massless scalar field ϕ. Instructive because every classical solution is singular. Provides a foundation for more complicated models.



Classical Solutions

Older Quantum Cosmology (DeWitt, Misner, Wheeler …70's)

- Since only finite number of DOF $v(t) \sim a^3(t), \phi(t)$, field theoretical difficulties bypassed; analysis reduced to standard quantum mechanics.

- Quantum States: $\Psi(v, \phi)$; $\hat{v}\Psi(v, \phi) = v\Psi(v, \phi)$ etc.
Quantum evolution governed by the Wheeler-DeWitt differential equation

$$\ell_{\rm Pl}^4 \, \frac{\partial^2}{\partial v^2}(f(v)\Psi(v, \phi)) = \text{const}\, G \; \hat{H}_\phi \, \Psi(v, \phi)$$

Without additional assumptions, singularity is not resolved.

- General belief since the 70's: Impasse because of the von Neumann's uniqueness theorem of QM.

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• In Loop Quantum Cosmology, situation is very different because of underlying Quantum Riemannian Geometry.

How is this possible? What about von Neumann's theorem?

• In the WDW quantum cosmology, one did not have guidance from a full quantum gravity theory. Therefore, in quantum cosmology, one just followed standard QM and constructed the Schrödinger representation of the fundamental Weyl algebra.

• By contrast, quantum kinematics of LQG has been rigorously developed. Background independence $\Rightarrow$ unique representation of the kinematic algebra (Lewandowski, Okolow, Sahlmann, Thiemann; Fleishhack) Provides the arena to formulate quantum Einstein equations.

• If one follows the procedure used in LQG, one of the assumptions of the von Neumann theorem violated $\Rightarrow$ uniqueness result bypassed.

[von Neumann's uniqueness theorem: There is a unique IRR of the Weyl operators $\hat{U}(\lambda), \hat{V}(\mu)$ by 1-parameter unitary groups on a Hilbert space satisfying: i) $\hat{U}(\lambda)\,\hat{V}(\mu) = e^{i\lambda\mu}\,\hat{V}(\mu)\,\hat{U}(\lambda)$; and ii) Weak continuity in λ, μ. This is the standard Schrödinger representation. ($U(\lambda) = e^{i\lambda x}$ and $V(\mu) = e^{i\mu p}$)]

Inequivalent representations even for cosmological models. New quantum mechanics! (AA, Bojowald, Lewandowski) Novel features precisely in the deep Planck regime.

3. Loop Quantum Cosmology

• Because of these key differences, the LQC kinematics does not support the WDW dynamics. *Quantum dynamics has to be built from scratch by incorporating quantum geometry of LQG.* The WDW differential equation is replaced by a difference equation. (AA, Bojowald, Lewandowski, Pawlowski, Singh)

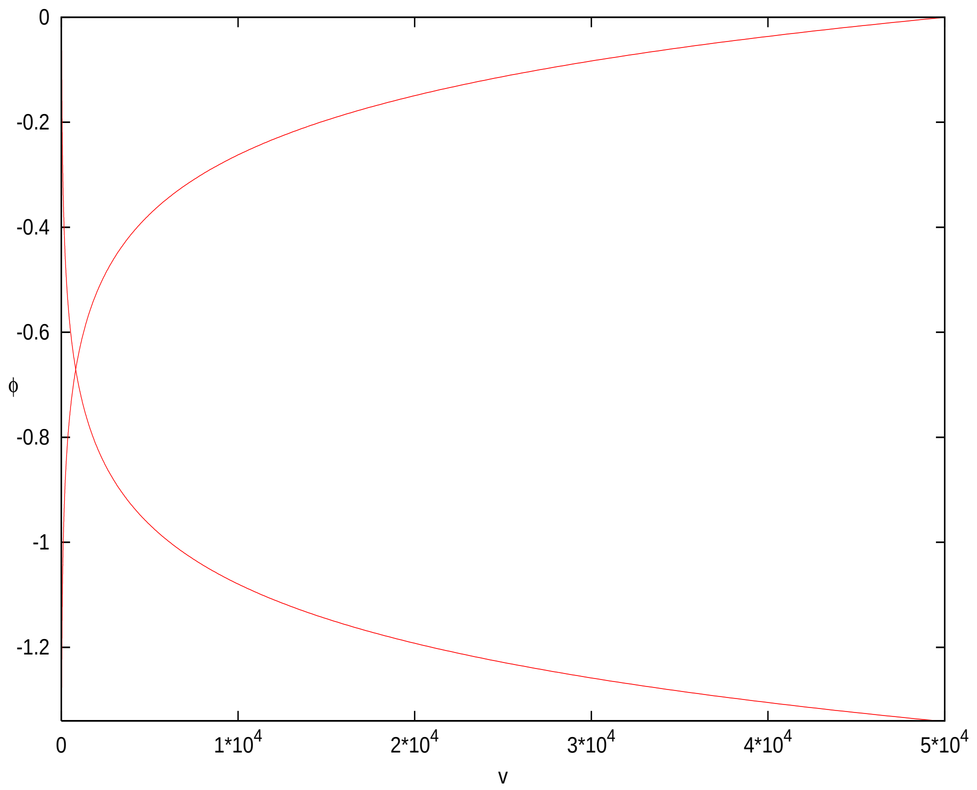
$$C^{+}(v)\,\Psi(v{+}4,\,\phi)+C^{o}(v)\,\Psi(v,\phi)+C^{-}(v)\Psi(v{-}4,\,\phi)\;=\;\gamma\ell_P^2\;\hat{H}_\phi\Psi(v,\phi)\quad(\star)$$

• In LQG, basic geometrical observables such as areas and volumes are quantized. The area operator has a smallest eigenvalue, the area gap Δ.

It turns out that the step size in $(\star)$ is governed by the area gap Δ. Good agreement with the WDW equation at low curvatures but drastic departures in the Planck regime precisely because the WDW theory ignores quantum geometry.

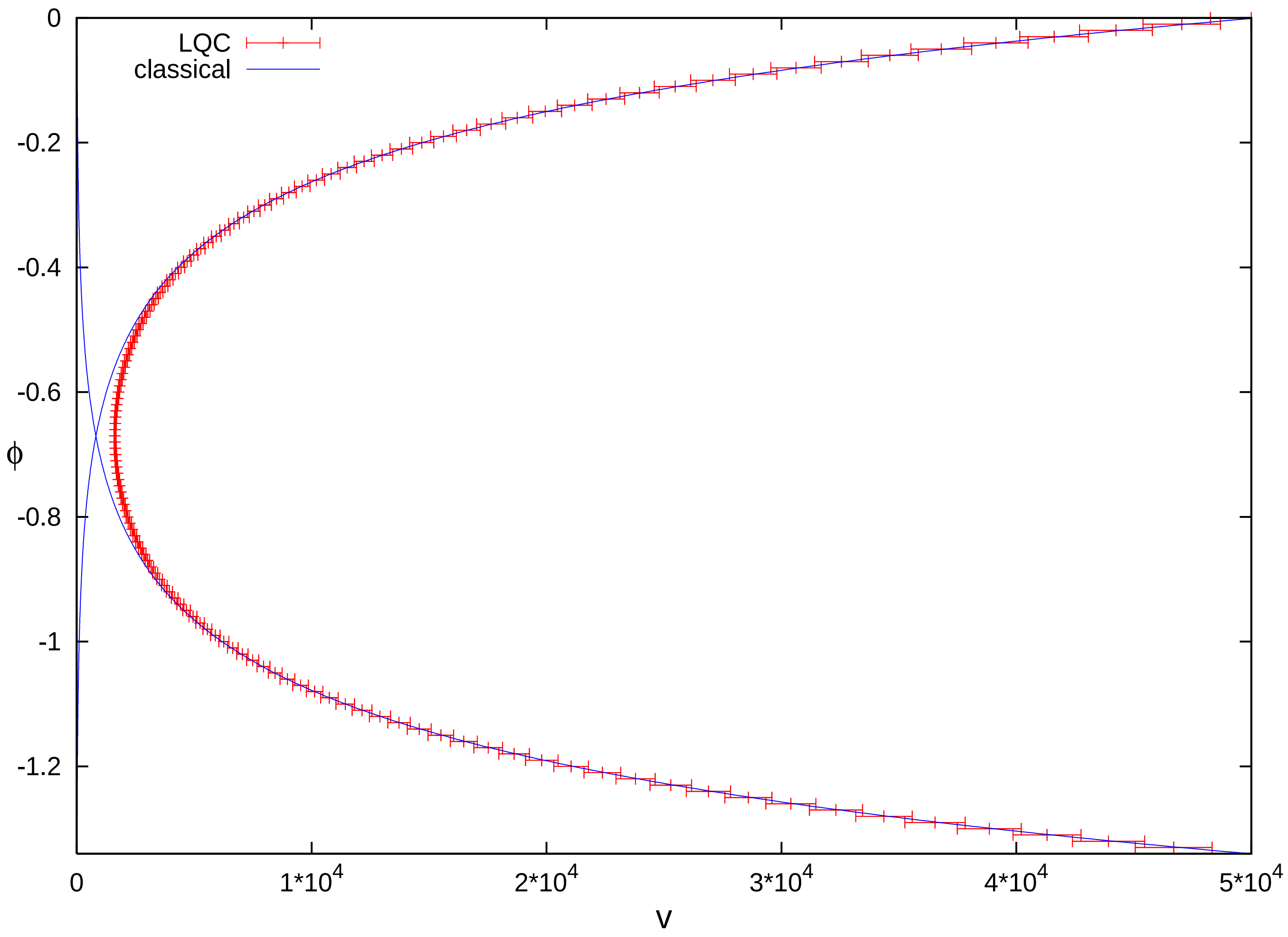
Return to the k=0 FLRW Model

FLRW, k=0 Model coupled to a massless scalar field ϕ. Instructive because every classical solution is singular. Provides a foundation for more complicated models.



Classical Solutions

k=0 LQC

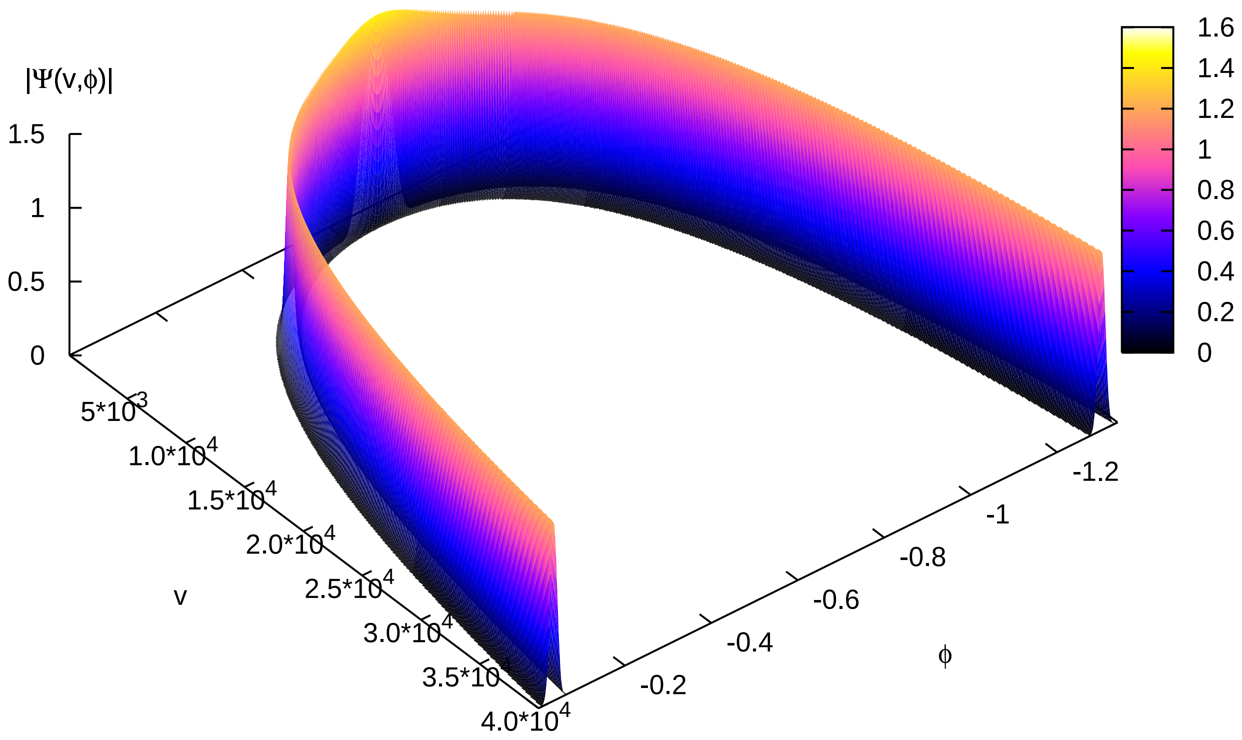


(AA, Pawlowski, Singh)

Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

Gamow's favorite paradigm realized.

k=0 LQC



Absolute value of the physical state $\Psi(v, \phi)$

(AA, Pawlowski, Singh)

k=0 Results

Assume that the quantum state is semi-classical at a late time and evolve backwards and forward. Then: (AA, Pawlowski, Singh)

• The state remains semi-classical till *very* early and *very* late times, i.e., till $R \approx 1/lp^2$ or $\rho \approx 10^{-3} \rho_{\rm Pl}$. $\Rightarrow$ We know 'from first principles' that space-time can be taken to be classical at the GUT scale.
(since $\rho < 10^{-11} \rho_{\rm Pl}$ at the onset of the GUT era).

• In the deep Planck regime, semi-classicality fails. But quantum evolution is well-defined through the Planck regime, and remains deterministic unlike in other approaches. No new principle needed.

• No unphysical matter. All energy conditions satisfied. But the left side of Einstein's equations modified because of quantum geometry effects (discreteness of eigenvalues of geometric operators.): Main difference from WDW theory.

k=0 Results

• To compare with the standard Friedmann equation, convenient to do an algebraic manipulation and move the quantum geometry effect to the right side. Then:

$$(\dot{a}/a)^2 = (8\pi G\rho/3)[1 - \rho/\rho_{\rm crit}] \quad \text{where } \rho_{\rm crit} \sim 0.41\rho_{\rm Pl}.$$

Big Bang replaced by a quantum bounce.

• The matter density operator $\hat{\rho}$ has an absolute upper bound on the physical Hilbert space (AA, Corichi, Singh):

$$\rho_{\rm sup} = \sqrt{3}/16\pi^2\gamma^3G^2\hbar \approx 0.41\rho_{\rm Pl}!$$

Provides a precise sense in which the singularity is resolved.

• Quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Physics does not end at singularities. A robust super-inflation phase immediately after the bounce.

Generalizations

• More general singularities: At finite proper time, scale factor may blow up, along with similar behavior of density or pressure (Big rip) or curvature or their derivatives diverge at finite values of scale factor (sudden death). Quantum geometry resolves all strong singularities in homogeneous isotropic models with $p = p(\rho)$ matter (Singh).

• Beyond Isotropy and Homogeneity: Inclusion of a cosmological constant and the standard $m^2\phi^2$ inflationary potential. Inclusion of anisotropies. $k = 1$ closed cosmologies. The Gowdy model with inhomogeneities and gravitational waves. Singularities are resolved and Planck scale physics explored in all these cases.

(AA, Bentevigna, Date, Pawlowski, Singh, Vandersloot, Wilson-Ewing, ...)

4. Illustrative Application: Inflation

• Removing conceptual incompleteness: the new paradigm well suited because all physical quantities remain finite. Example:

• Inflationary models have had tremendous success particularly with structure formation. These implications require a 'slow roll inflation' with 60-70 e-foldings (i.e. $a_{\rm end}/a_{\rm start} \approx e^{60} - e^{70}$).

• How natural is inflation? How did the inflaton get sufficiently high up the potential to yield 60-70 e-foldings as it rolls down slowly? Difficult to set initial conditions in GR because of the initial singularity

• So: Even if a theory allows for inflation, can we say that a sufficiently long slow roll will actually occur with high probability? Controversy in the literature. Recently Gibbons and Turok argued that in general relativity, the a priori probability of obtaining N e-foldings goes as e^{-3N}! This would put a tremendous burden on the fundamental theory as to why a sufficiently long, slow roll inflation actually occurred.

• In LQC the situation is reversed; probability of getting at least 68 e-foldings is bigger than 0.999! (AA, Sloan; Barrau, Grain, Mielczarek; Corichi, Karami;..)

• Conceptual issues: Paradigm shift in all Bouncing Models. No horizon problem.

• Phenomenology: Spectral tilts due to quantum geometry effects. For power law inflation, LQC corrections important at the onset on inflation and become negligible at the end. Very small red or blue tilts in the power spectrum. (Barrau, Grain, Gorecki; Calcagni, Hossain;...). Several such examples of conceptually interesting but observationally very small quantum corrections. EX: Nucleosynthesis. (Bojowald, Das, Scherrer)

• Observational Possibilities: Structure formation. Normally, initial conditions (Adiabatic or Bunch Davis vacuum for the inflation) specified at the onset of inflation. Great phenomenological success but conceptually strange time to set initial conditions. Big bang would be a more natural place but there we have a singularity!

• Current work (Agullo, AA, Nelson): Use QFT on Cosmological, Quantum Space-times (AA, Lewandowski, Kaminski) to specify the natural initial state for perturbations at the big bounce and then evolve. Viable idea? Big burden: Should evolve to a state near the Bunch-Davis vacuum at the onset of inflation and yet not sufficiently different so as to have observationally interesting corrections to the inflationary scenario of structure formation? Preliminary results: Yes!

5. Summary

• Quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Repulsive force rises and dies *very* quickly but makes dramatic changes to classical dynamics. (Origin: Planck scale non-locality of quantum Einstein's equations.)

New paradigm: Physics does not end at singularities.

Quantum space-times may be vastly larger than Einstein's.

• Long standing questions I began with have been answered. Challenge to background independent theories: Detailed recovery of classical GR at low curvatures/densities (Green and Unruh). Met in cosmological models. Singularities analyzed are of direct cosmological interest.

• Detailed analysis in specific models but taken together with the BKL conjecture on the nature of space-like, strong curvature singularities in general relativity, the LQC results suggest that all these singularities may be resolved by the quantum geometry effects of LQG.
(Recall the history in classical GR).

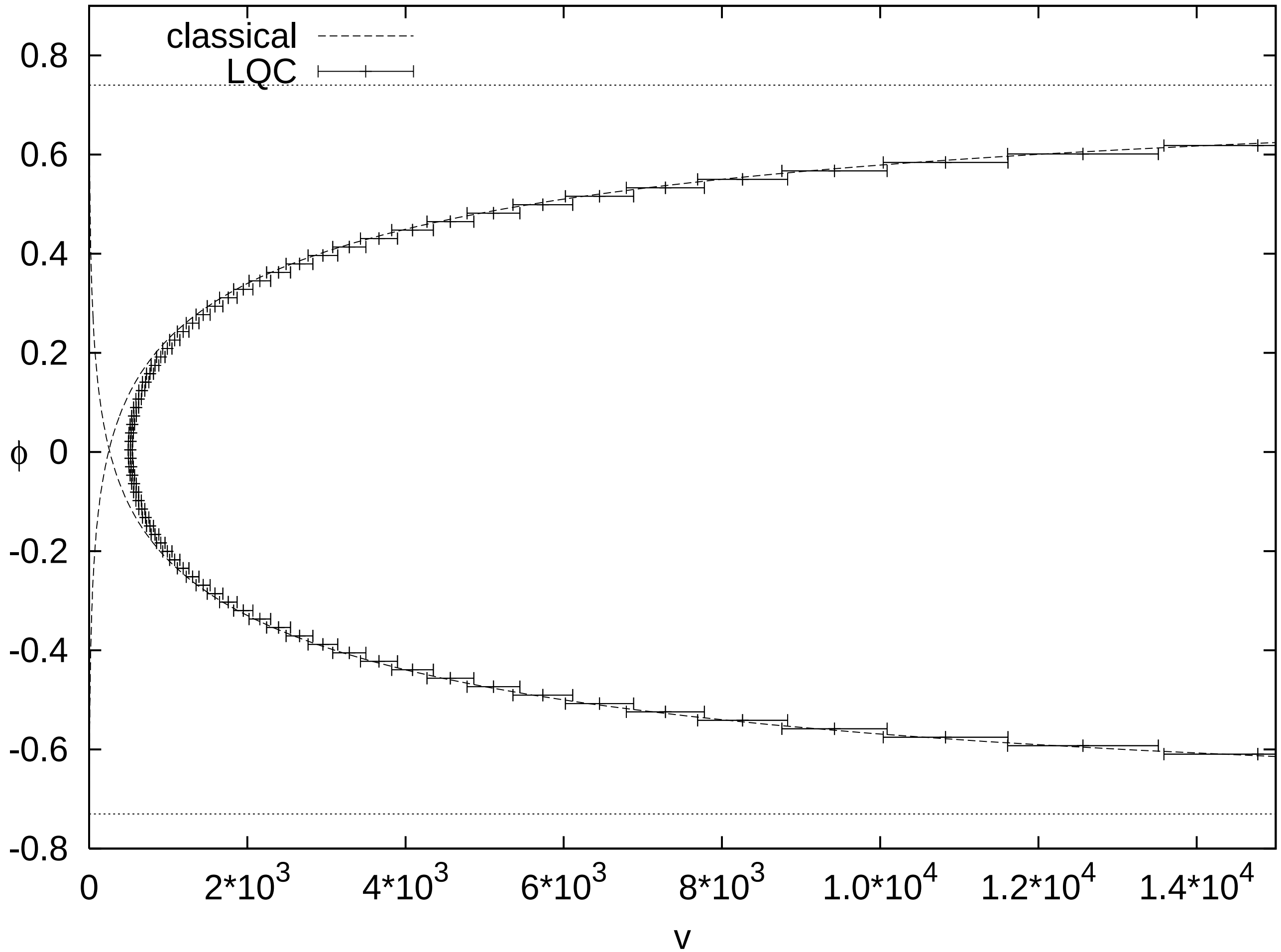
4. Summary (contd)

• Loop quantum cosmology also provides input into full quantum gravity; both conceptual and technical insights into dynamics. Hamiltonian Loop Quantum Gravity (Borja, Diaz, Garay, Livine; Laddha, Varadarajan); Spin foams: Path integral approach based on quantum geometry (AA, Campiglia, Henderson, Nelson; Bianchi, Rovelli, Vidotto).

• Frontier: Inhomogeneous perturbations, Phenomenology. First results have already appeared. Growing exchange between cosmologists and the LQC community. Hopefully this will provide the transition from the Friedmann-Lemaître stage to Nucleosynthesis-CMB stage.

• So, what replaces the big bang in LQG? Broadly, Lemaître's "Phoenix Universe". Since the advent of GR, leading thinkers (Eddington, Lemaître, Tolman, Gamow, Zanstra, Hoyle, Bondi, Gold, Sciama, Dicke, Sakharov, Weinberg, ...) have expressed hopes and philosophical preferences. Now, because the singularity is resolved and LQC equations provide a deterministic evolution from the pre-big-bang to the post-big-bang branch, what happens is not a philosophical preference but depends on quantum Einstein's equations and the cosmological parameters of today's universe.

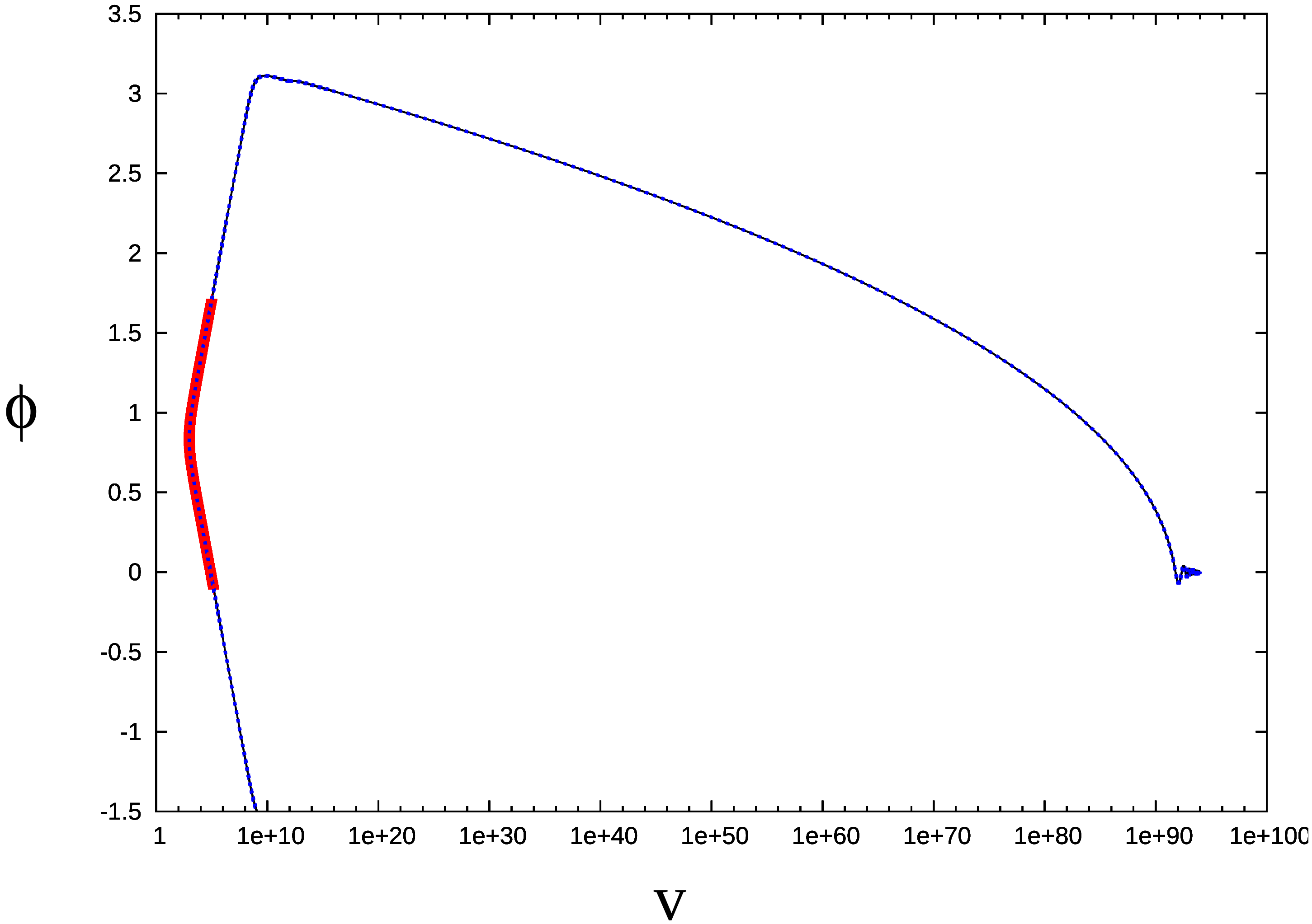
k=0 Model with Positive Λ



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(AA, Pawlowski)

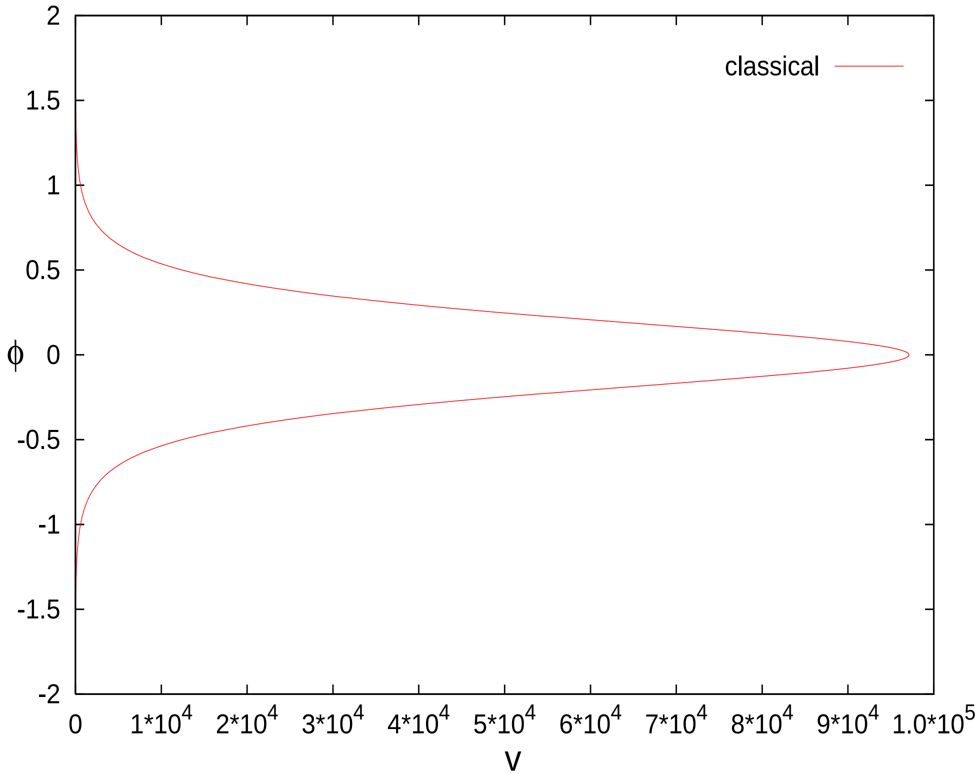
Inflation



Expectations values and dispersions of $\hat{V}|_\phi$ for a massive inflaton ϕ with phenomenologically preferred parameters (AA, Pawlowski, Singh).

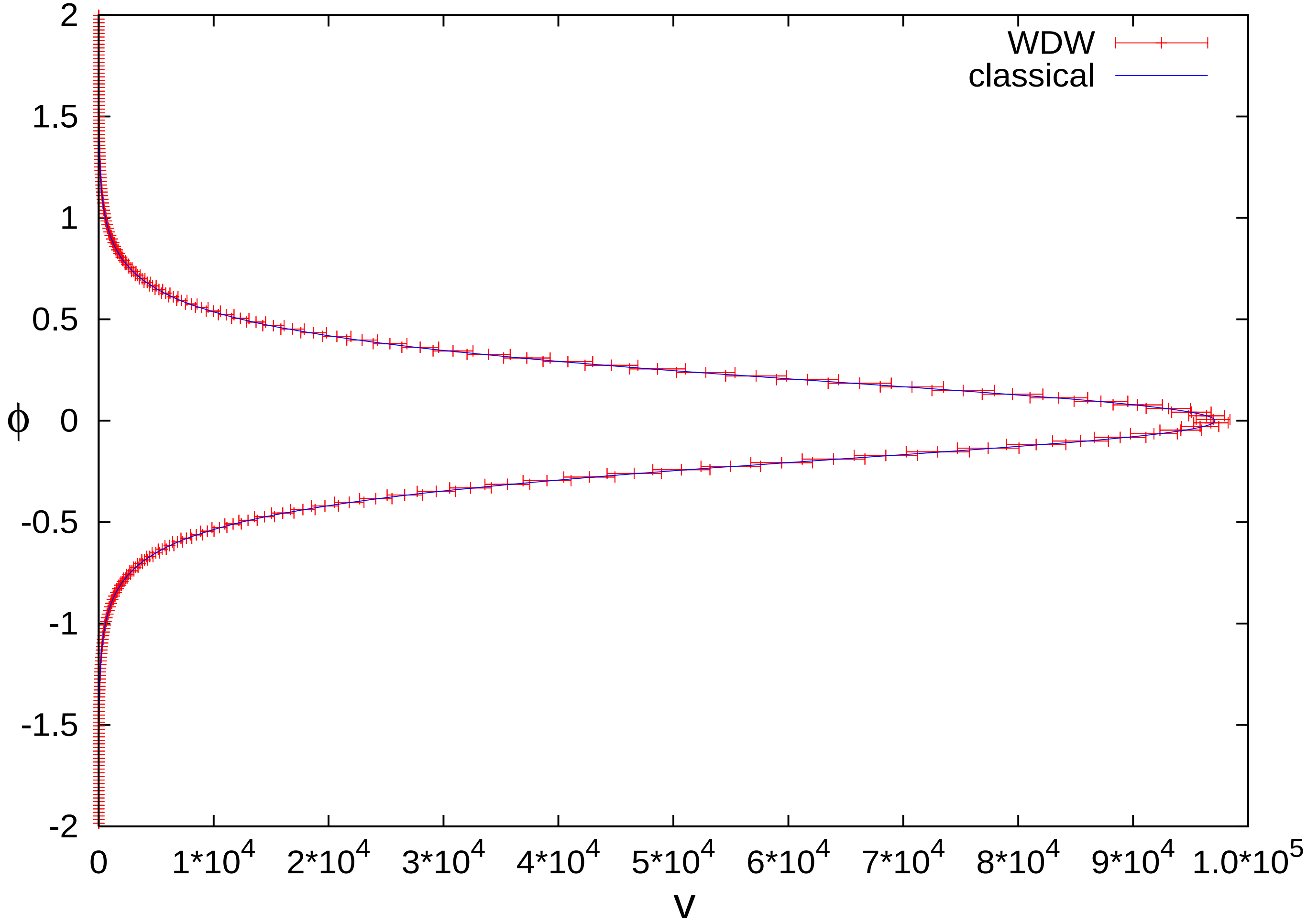
The k=1 Closed Model: Bouncing/Phoenix Universes.

Another Example: $k = 1$ FLRW model with a massless scalar field ϕ. Instructive because again every classical solution is singular; scale factor not a good global clock; More stringent tests because of the classical re-collapse. (Lemaître, Tolman, Sakharov, Dicke,...)



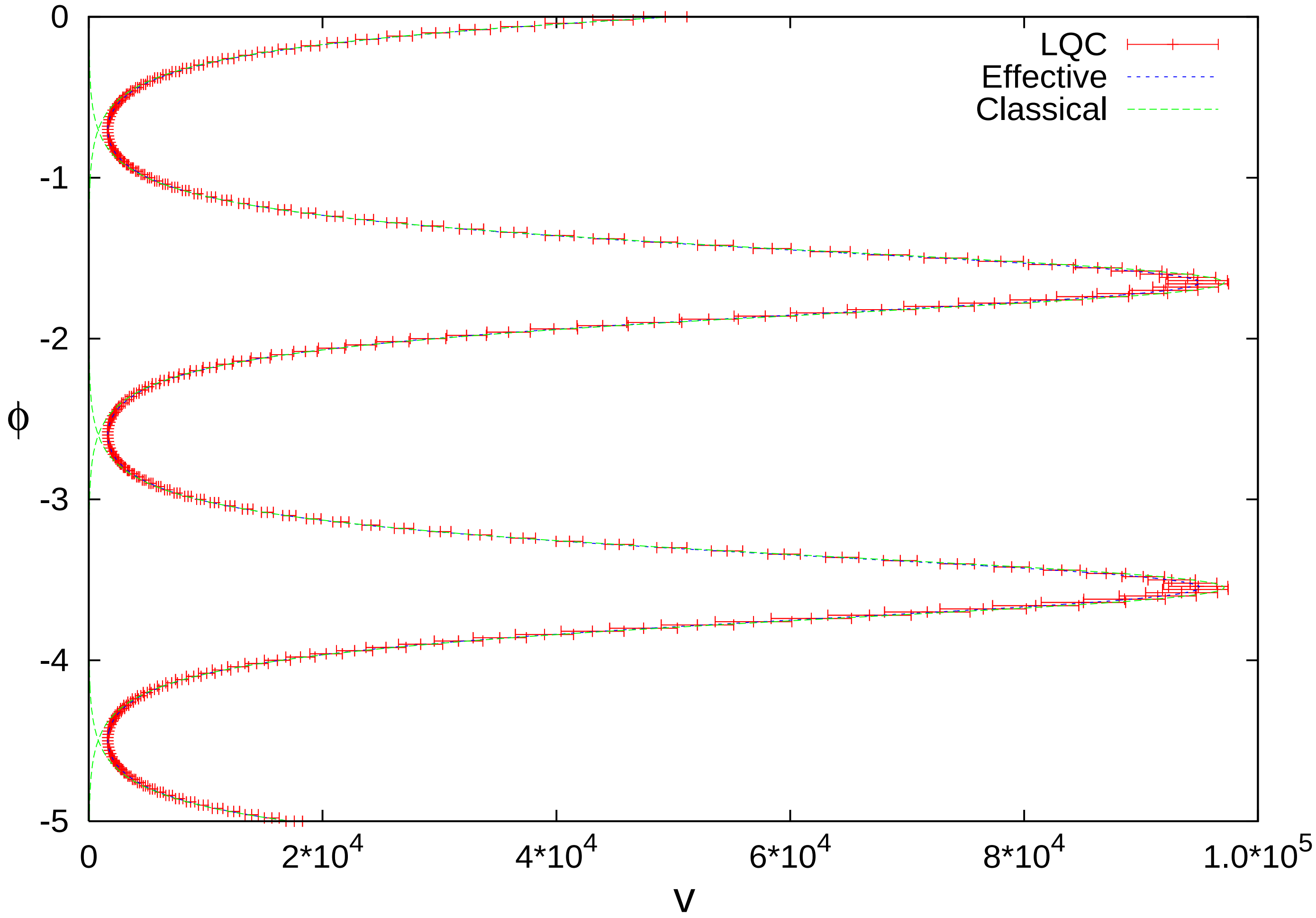
Classical Solutions

k=1 Model: WDW Theory



Expectations values and dispersions of $\hat{V}|_\phi$.

k=1 Model: LQC



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(AA, Pawlowski, Singh, Vandersloot)

Merits and Limitations of QC

One's first reaction: Symmetry reduction gives only toy models! Full theory much richer and much more complicated. But examples can be powerful.

- Full QED versus Dirac's hydrogen atom.
- Singularity Theorems versus first discoveries in simple models.
- BKL behavior: homogeneous Bianchi models.

Do *not* imply that behavior found in examples is necessarily generic. Rather, they can reveal important aspects of the full theory and should not be dismissed a priori.

One can work one's way up by considering more and more complicated cases. (e.g. recent work of the Madrid group on Gowdy models which have infinite degrees of freedom). At each step, models provide important physical checks well beyond formal mathematics. Can have strong lessons for the full theory.