The black hole information paradox

Lecture 3

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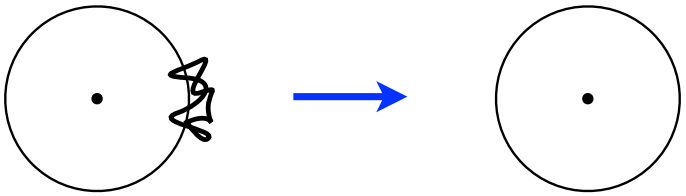
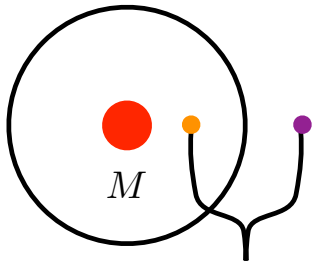
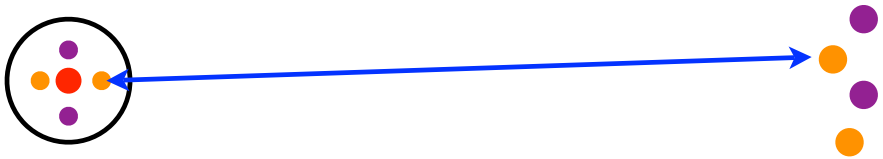
The information paradox is a combination of two observations:

(1) The no-hair 'theorems' tell us the black hole tends to quickly settle down to a state where the region around the horizon is vacuum



(2) The vacuum creates entangled pairs by the Hawking process

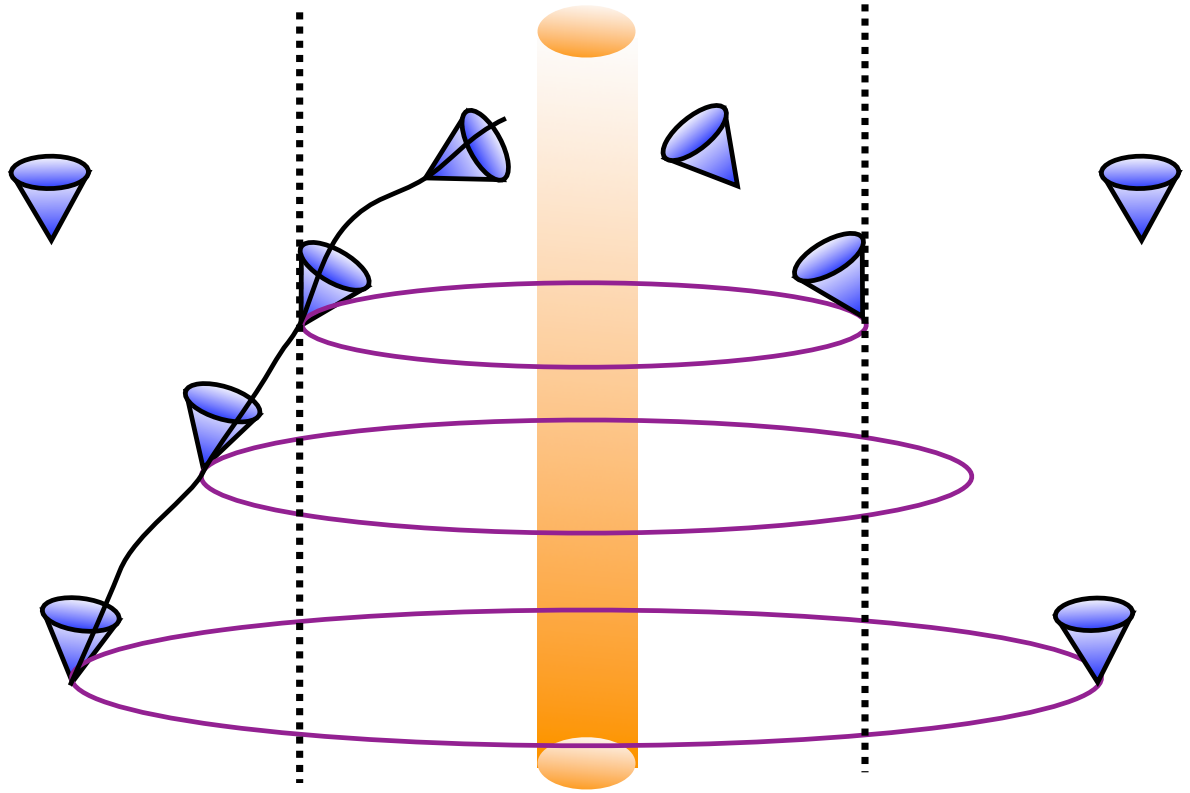




The energy of the hole is now in the radiation

A massless (or planck mass) remnant is left

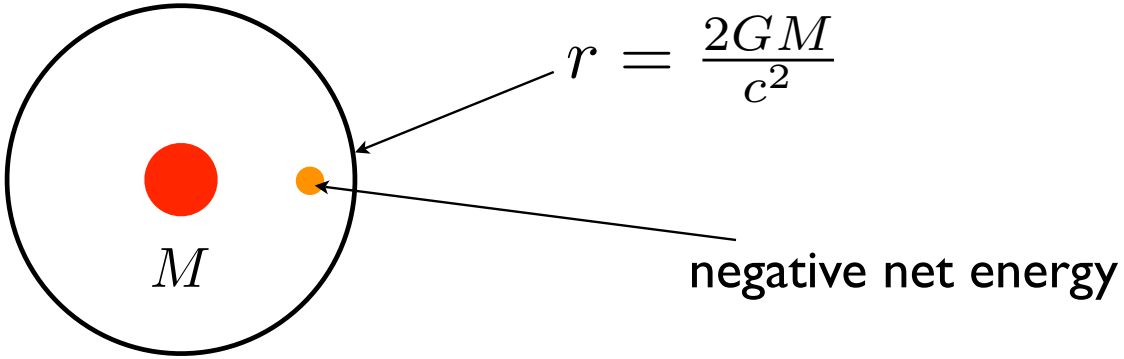
It is hard to bypass the no-hair theorem because of the geometry of the black hole



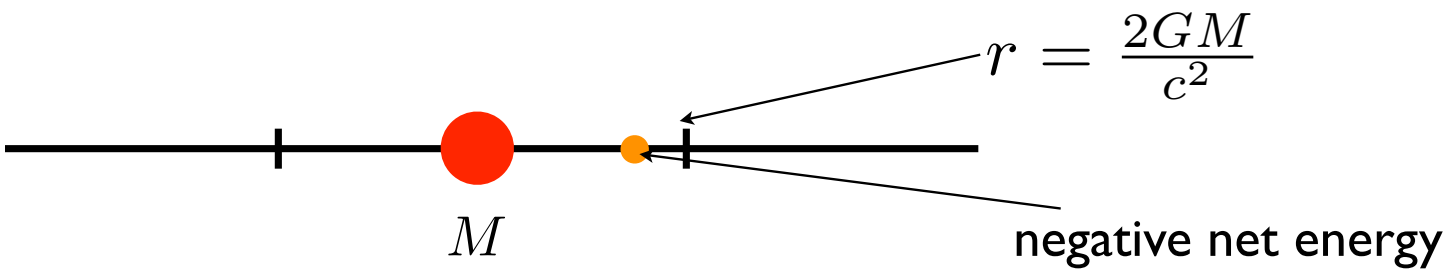
How do you make a structure at the horizon that does not ‘fall in’?

The fuzzball construction

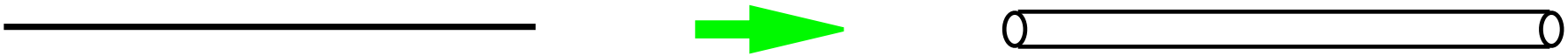
We live in 3 space and 1 time dimension. Recall the black hole ...



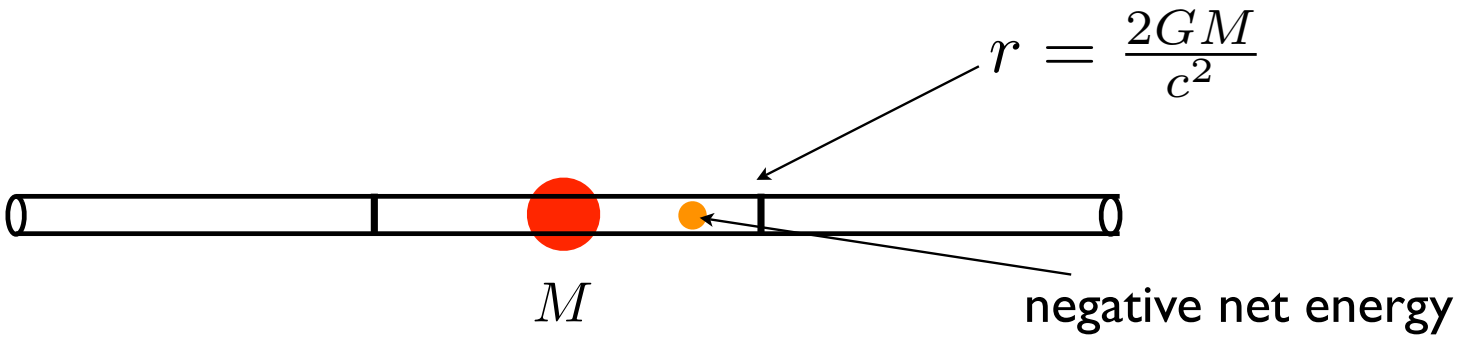
Let us draw just one space direction for simplicity



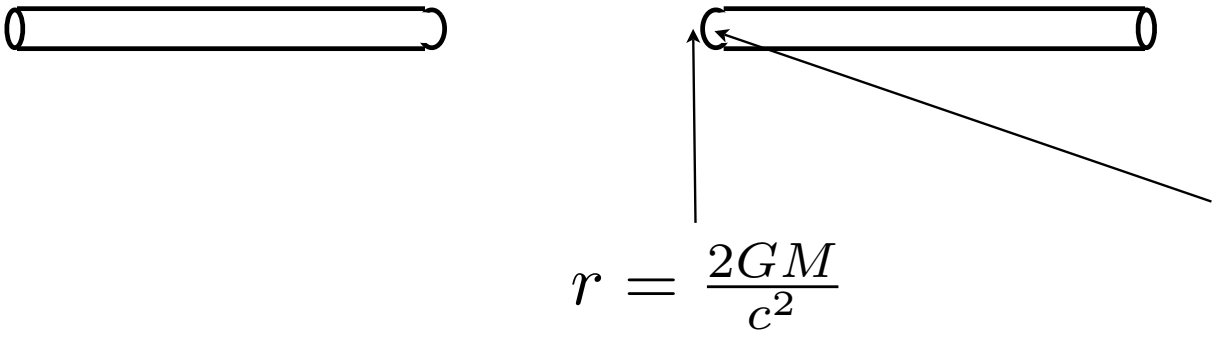
Now suppose there was an extra dimension (e.g., string theory has 6 extra dimensions)



People have thought of extra dimensions for a long time, but they seemed to have no particular significance for the black hole problem



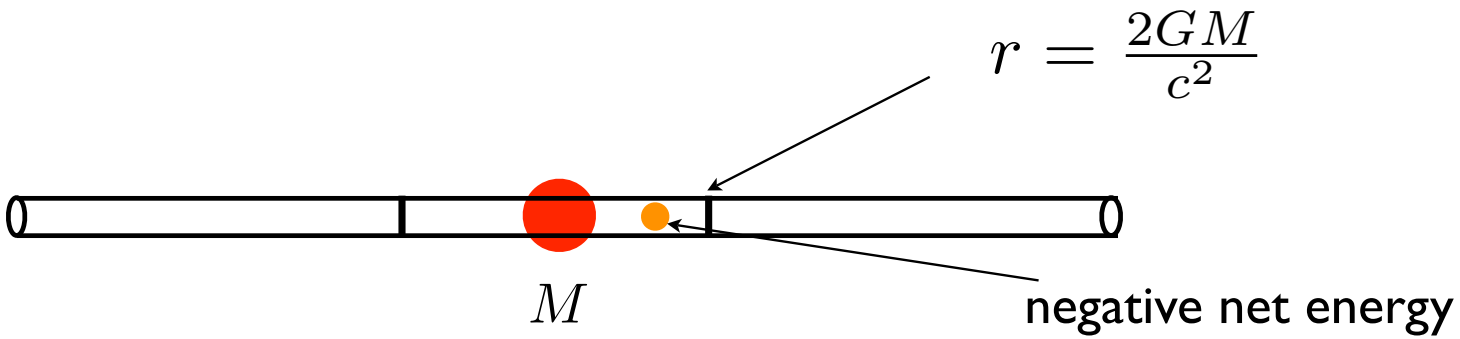
But there is a completely different structure possible with compact dimensions ...



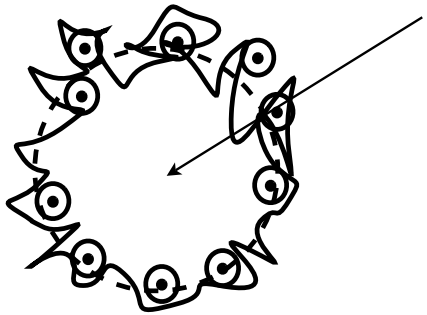
No place to put particles with net negative energy

The mass M is captured by the energy in the curved manifold

Contrast this with



1-dimension



not part of spacetime

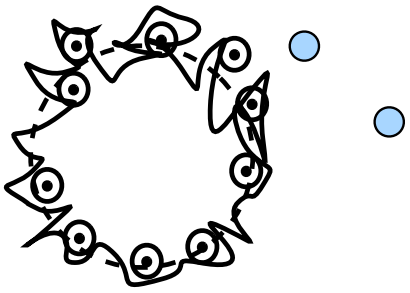
Many dimensions

"Fuzzball"

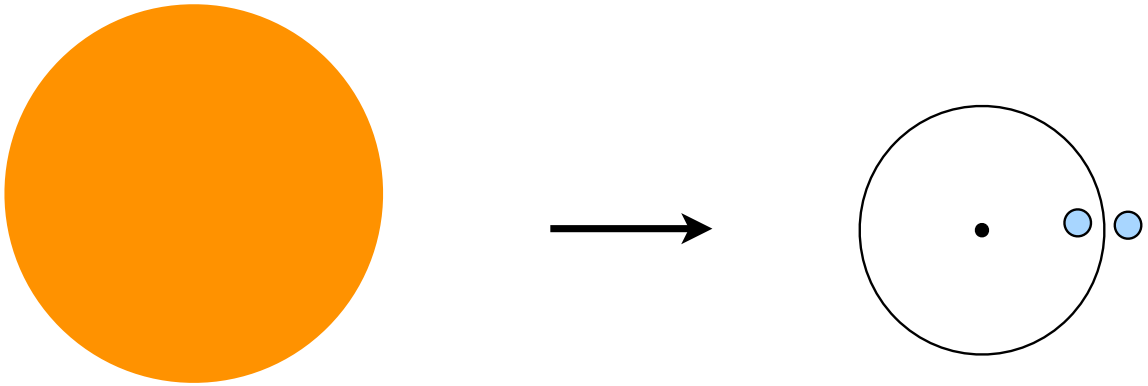
The stuff at the horizon does not fall in because the stuff is actually a topology that provides to a smooth end to space

How do fuzzballs form ?

The fuzzball construction seems to be the only correct solution to the paradox ...



But if a star collapses, then the physics looks quite classical, and so one seems to make the usual black hole with a smooth horizon ...



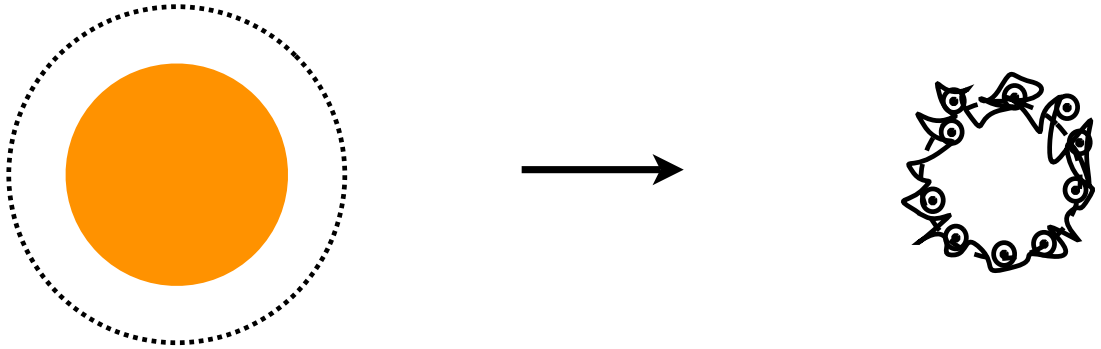
In 1972, Bekenstein taught us that black holes have an entropy

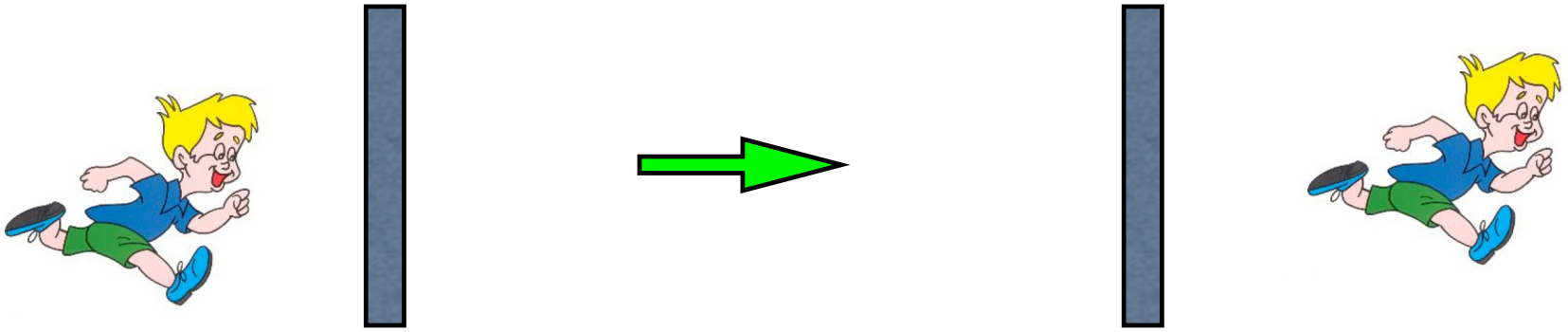
$$S = \frac{c^3}{\hbar}\frac{A}{4G} \sim \frac{A}{l_p^2}$$

This means that a solar mass black hole has $\sim 10^{10^{144}}$ states

This is far larger than the number of states of normal matter with the same energy

Consider the amplitude for the collapsing star to tunnel into one of the fuzzball states



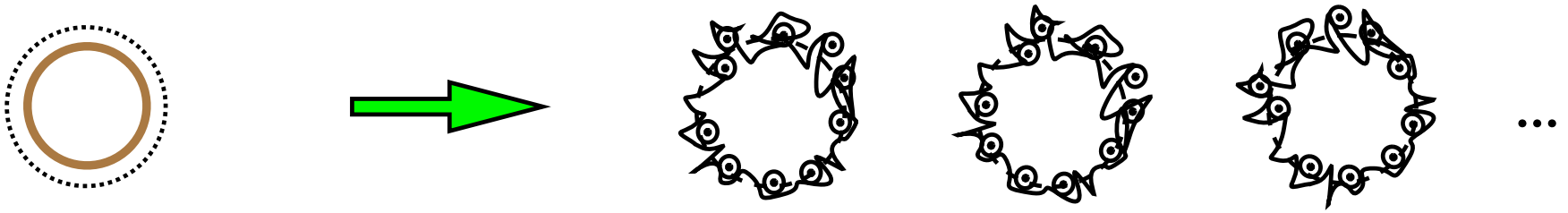


There is always a small probability that an object can tunnel ...

But this probability is usually ignorable for a macroscopic object ...

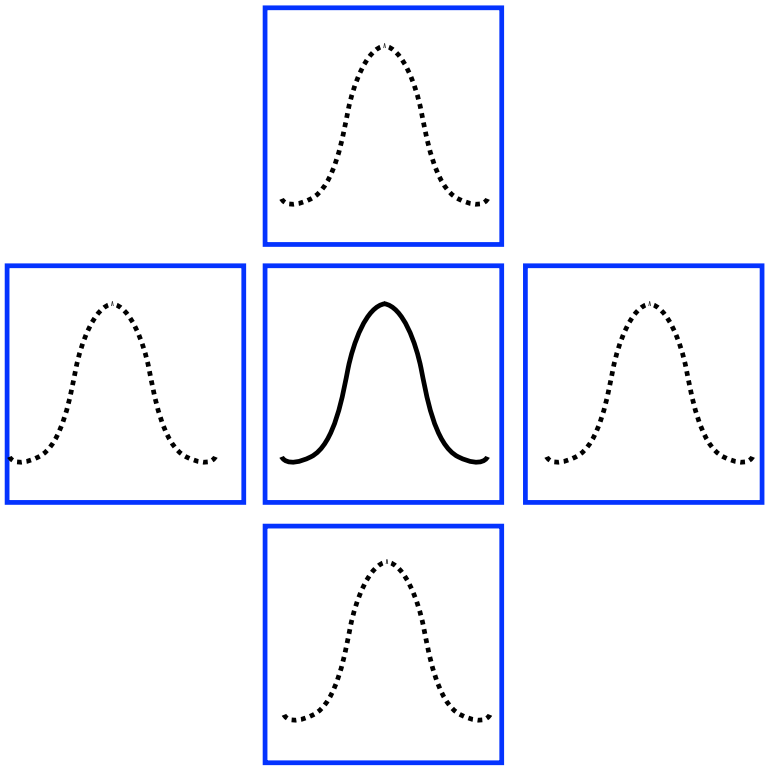
Is there something special about a black hole ?

A black hole has an enormous entropy ... so we can tunnel into a large number of possible states ...



Toy model

Small amplitude to tunnel to a neighboring well, but
there are a correspondingly large number of adjacent wells



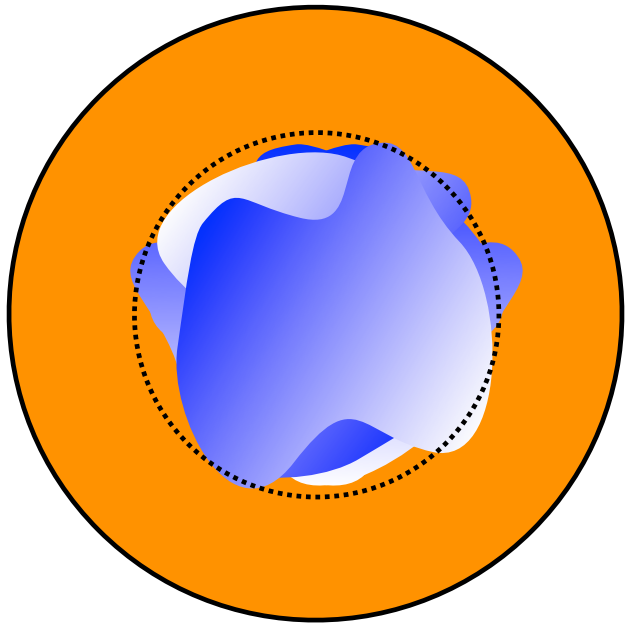
In a time of order unity, the wavefunction in the central well becomes a linear combination of states in all wells

This suggests that the entire black hole is a very quantum object ...

The hole is like one atom ... the electron cannot be localized within the atom, but spreads all over the atom ...

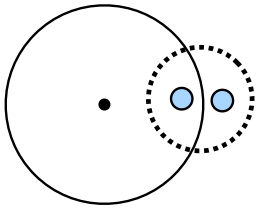
The atom was quantum because it was small ...

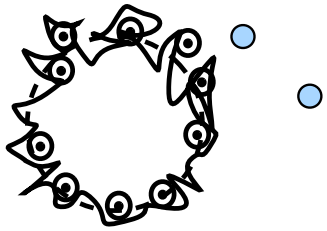
The black hole is big, but it has a very large phase space of states ...



Complementarity

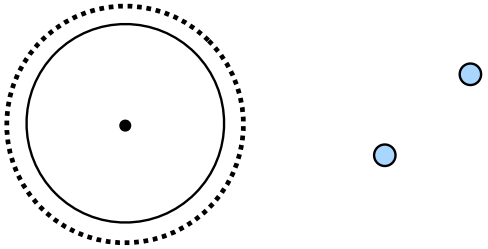
We want to get information out, not get the growing entanglement ...



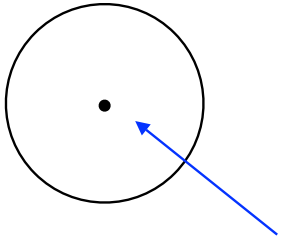


Suppose we dont have a construction of hair (no fuzzballs in our theory) ...

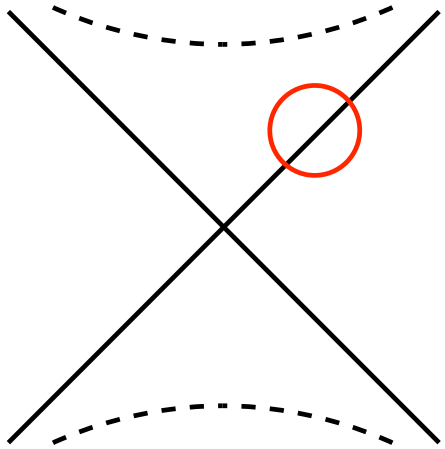
Then we postulate "new physics": The state depends on who is looking at it



For the purpose of the outside observer, information bounces off the horizon ... the body behaves just like a piece of coal

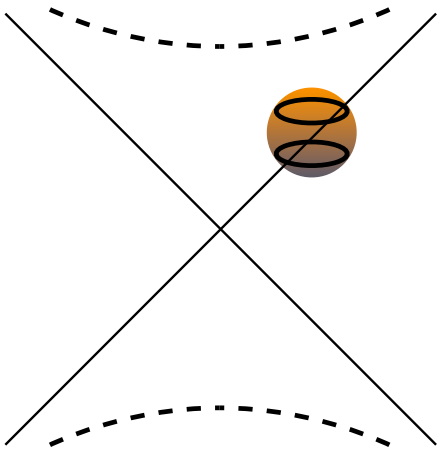


For the purpose of an infalling observer, observer, the black hole behaves like the standard Schwarzschild metric

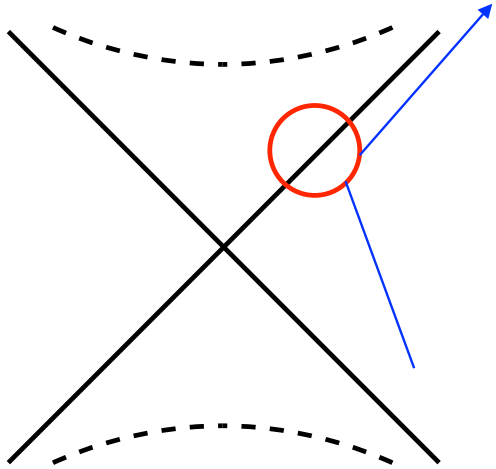


Consider a vacuum fluctuation loop of a scalar particle

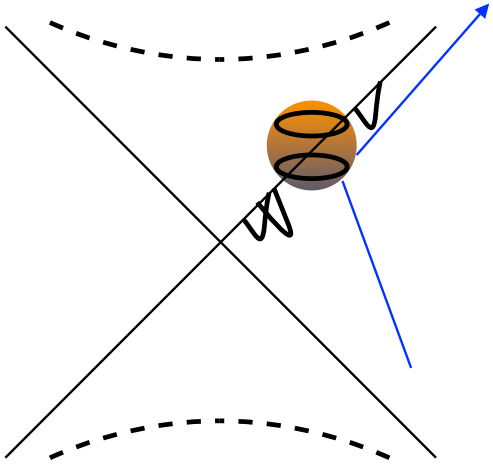
In the Schwarzschild frame, it looks as if a particle came out and fell back into the horizon



If we do this with a string loop, then it looks like strings emerge from the horizon and fall in ... at any time we will see a gas of strings with endpoints on the horizon

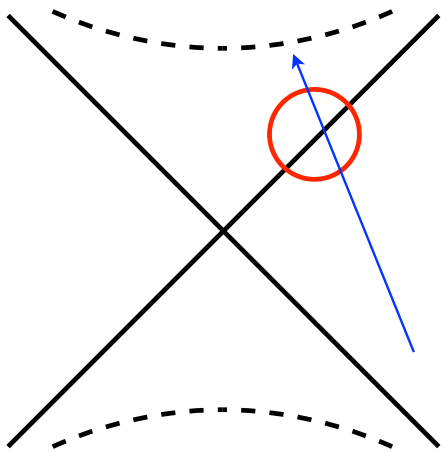


An incoming particle would scatter off this loop and return to infinity ....



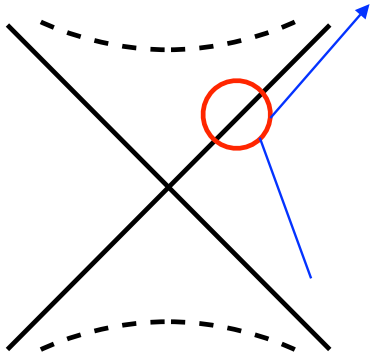
In string theory, there is a gas of strings which will catch the infalling quantum, thermalize it, and re-radiate the energy ...

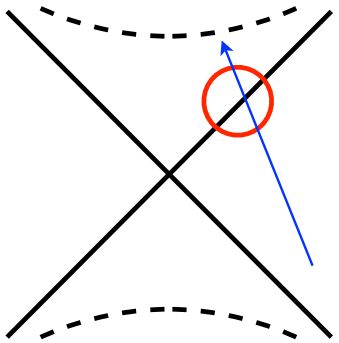
This makes the stretched horizon. An outside observer therefore never sees anything fall into the hole



But vacuum loops are not supposed to scatter an incoming particle .. their effect is supposed to cancel out ... ??

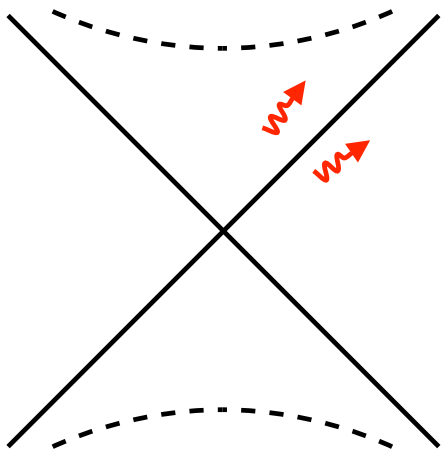
picture 1





picture 2

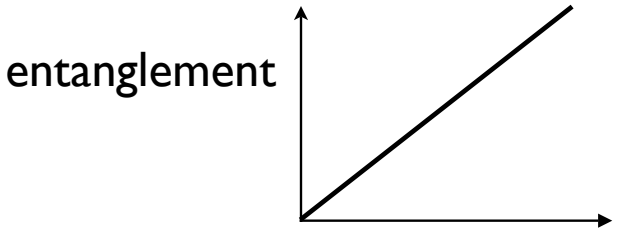
The two pictures are consistent because there is not enough time to measure the particle outside and also check its state inside ... (complementarity)

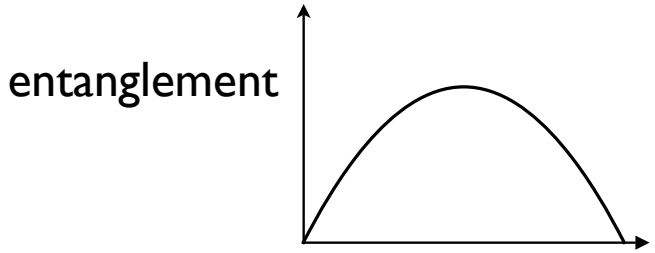


But what happens to Hawking's pair creation process? Doesn't it create the growing entanglement etc ... ??

Hawking 1975: Vacuum at the horizon leads to growing entanglement

Hawking 1975: (Equivalent statement) If you want entanglement to decrease, you cannot have vacuum at the horizon (FIREWALL)





AMPS used this to say: One cannot have complementarity ... in the picture where there is a smooth horizon we will get growing entanglement ...

Hawking 1975: If you want information out, then horizon cannot be smooth (FIREWALL)

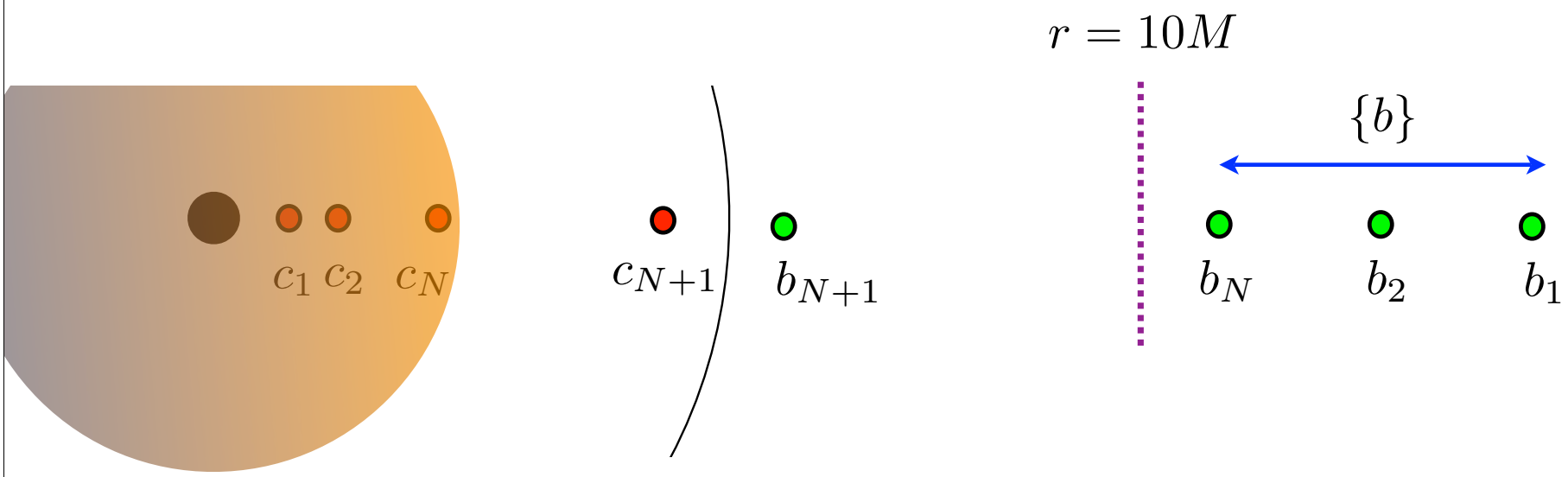
Hawking argument cannot be invalidated by small corrections (SDM: 0909.1038)

$$S_{N+1} > S_N + \ln 2 - (\epsilon_1 + \epsilon_2)$$

AMPS: Use this mathematical setup to say that you cannot have 'complementarity' (smooth horizon in sone description)

(Unfortunately, most people confused the AMPS argument against complementarity with Hawking's original argument ... so people think that AMPS showed that unitarity requires a firewall. But this is actually Hawking's work.)

The AMPS argument: in the frame where infall us smooth:



$S(b_{N+1}, c_{N+1}) = 0$

$S_{N+1} = S(\{b\} + b_{N+1})$

$S_N = S(\{b\} + b_{N+1} + c_{N+1})$

$S(b_{N+1}) = \ln 2$

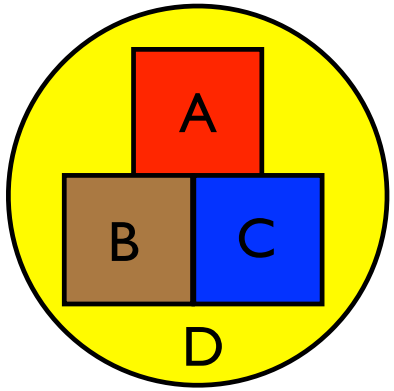
Strong subadditivity requires

$$S(A+B) + S(B+C) \geq S(B) + S(A+B+C)$$

$$S(\{b\} + b_{N+1}) + S(b_{N+1} + c_{N+1}) \geq S(b_{N+1}) + S(\{b\} + b_{N+1} + c_{N+1})$$

$S_{N+1} \geq \ln 2 + S_N$ (Entanglement keeps rising)

(i) The strong subadditivity of quantum entanglement entropy



$$S(A+B)+S(B+C) \geq S(A)+S(C)$$

We take

$$A=\{b\}\ , \qquad B=b_{N+1}\ , \qquad C=c_{N+1}$$

We recall $\quad S_N = S(\{b\})\ , \qquad S_{N+1}=S(\{b\}+b_{N+1})$

$$S(b_{N+1},c_{N+1})<\epsilon_1\ , \qquad S(c_{N+1})>\ln 2-\epsilon_2$$

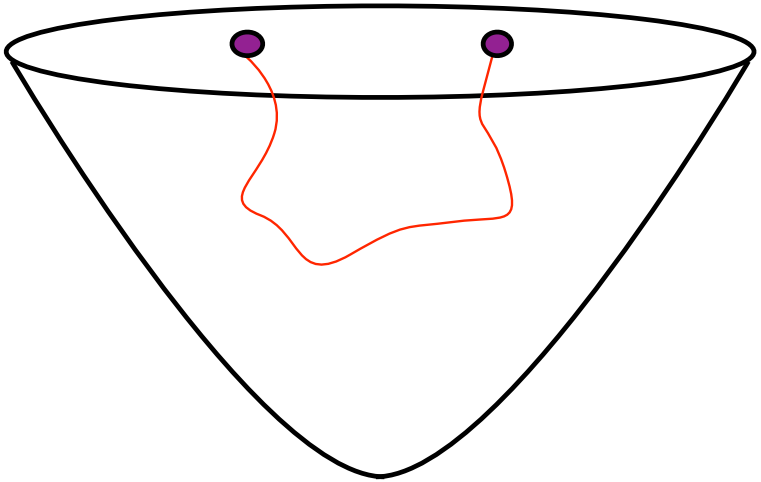
Then we get

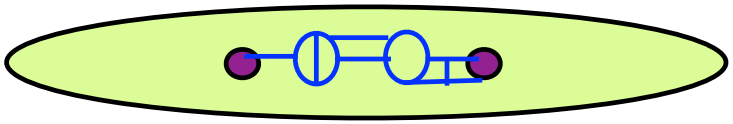
$$S(\{b\}+b_{N+1})+S(b_{N+1}+c_{N+1}) \geq S(\{b\}+S(c_{N+1})$$

$$\boxed{S_{N+1} > S_N + \ln 2 - (\epsilon_1+\epsilon_2)}$$

Fuzzball complementarity

The AdS/CFT correspondence





We can compute a 2-point function by summing over paths in this curved spacetime

We can get the same value by doing a 2-point function in a complicated field theory

In the simplest examples, the spacetime is anti-de-Sitter space (AdS), and the field theory is a conformal field theory (CFT)

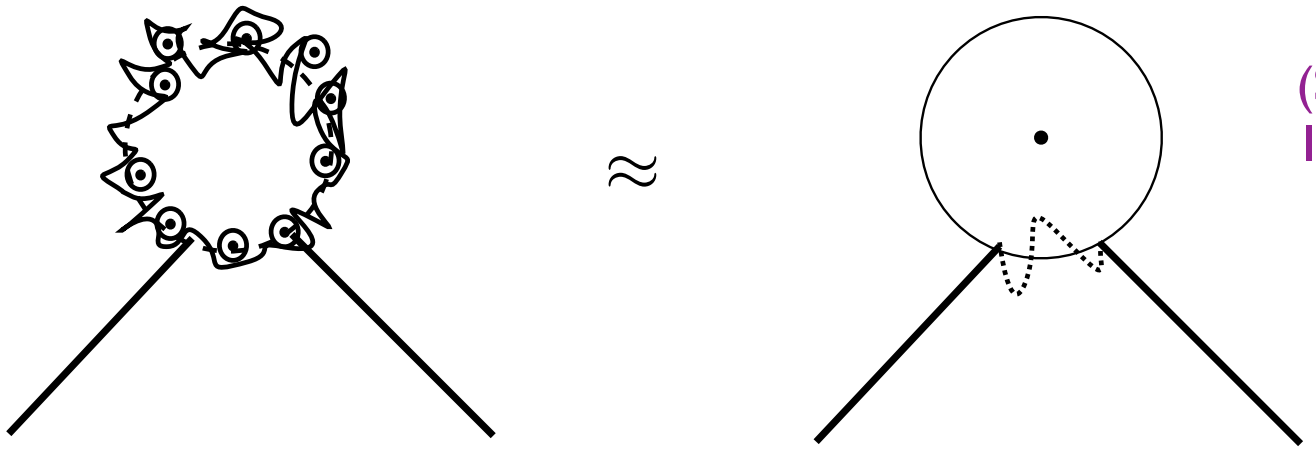
So this equality is called the AdS/CFT correspondence (Maldacena 1997)

Disturb the fuzzball surface with a probe

The disturbance will spread along the surface of the fuzzball

Pick it up at some other point with a second probe ⟶ Green's function

Fuzzball complementarity conjecture: This Green's function can be obtained to a good approximation by using the traditional black hole metric



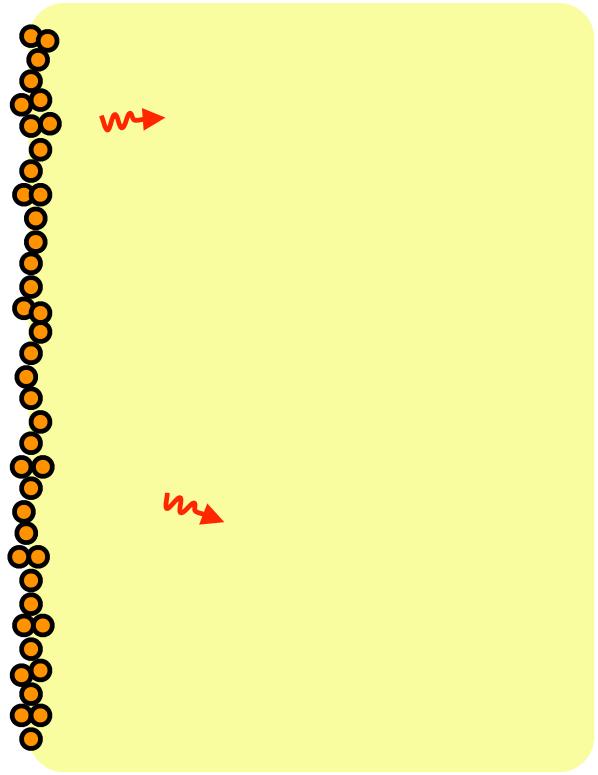
Fuzzball complementarity

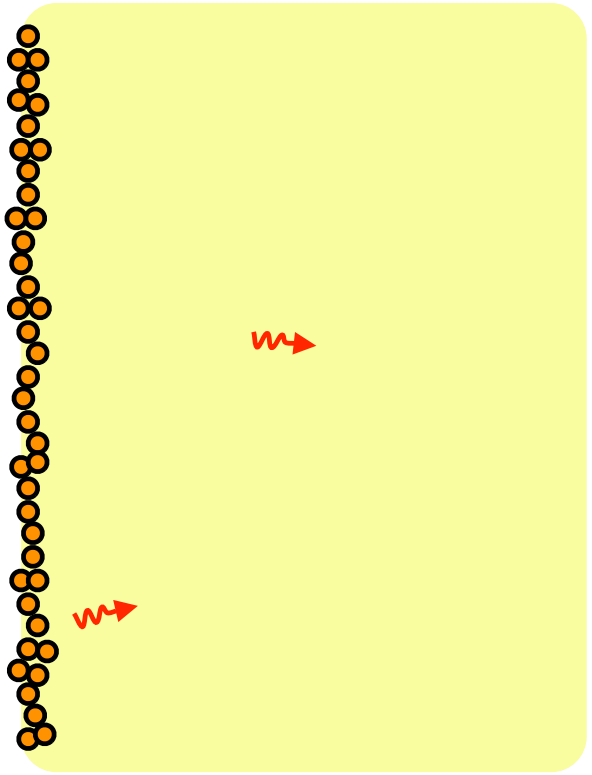
(SDM+Plumberg: 1101.4899)

The fuzzball surface is different for different microstates

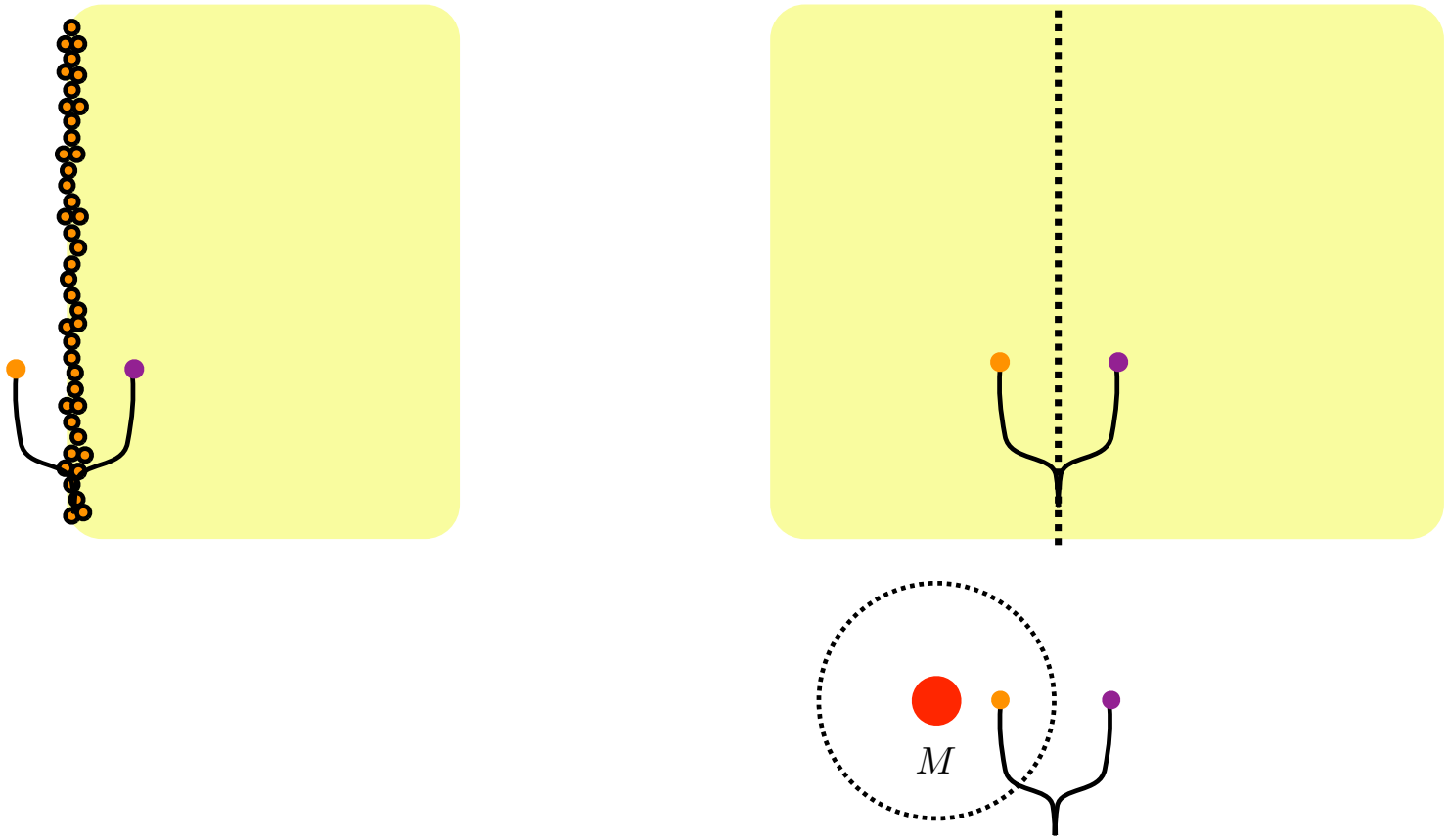
This different microstates radiate differently, just like different pieces of coal

So there is no information problem ....

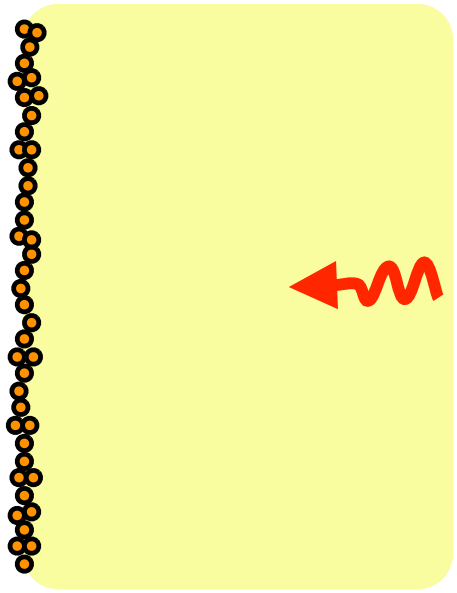




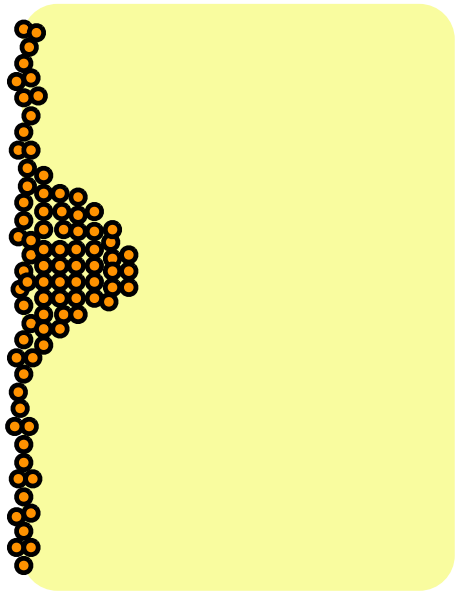
In particular, there are no vacuum fluctuations straddling the horizon, since the interior region does not even exist



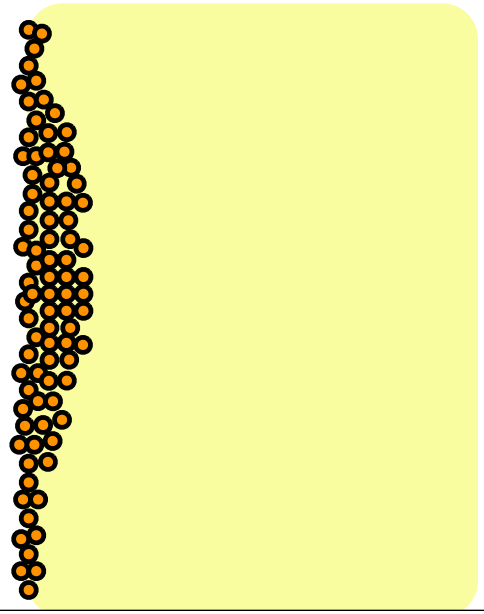
So we do not have the Hawking pair production problem



A incident quantum
with $E \gg T$

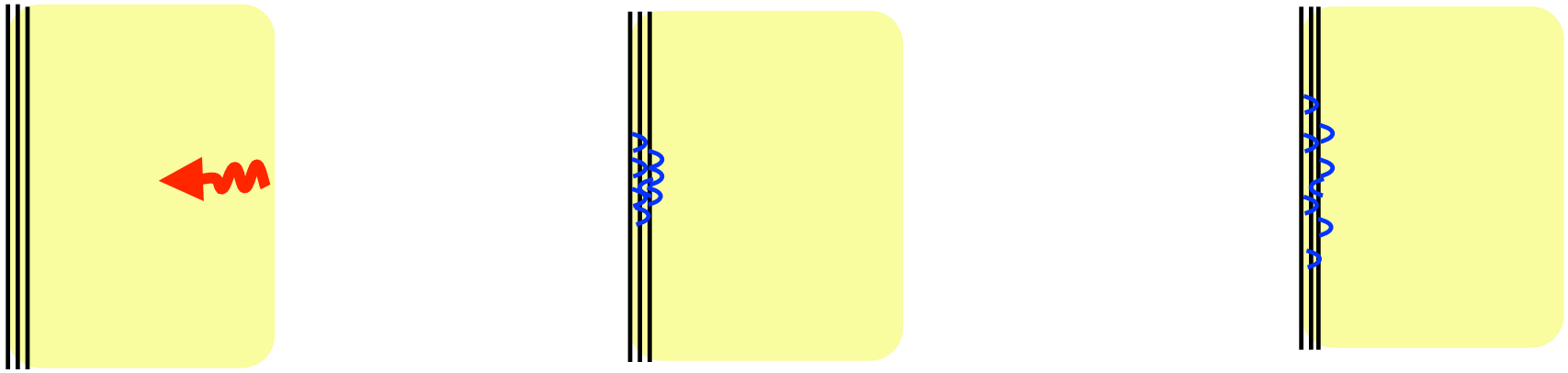


The quantum creates a large
disturbance on the fuzzball surface

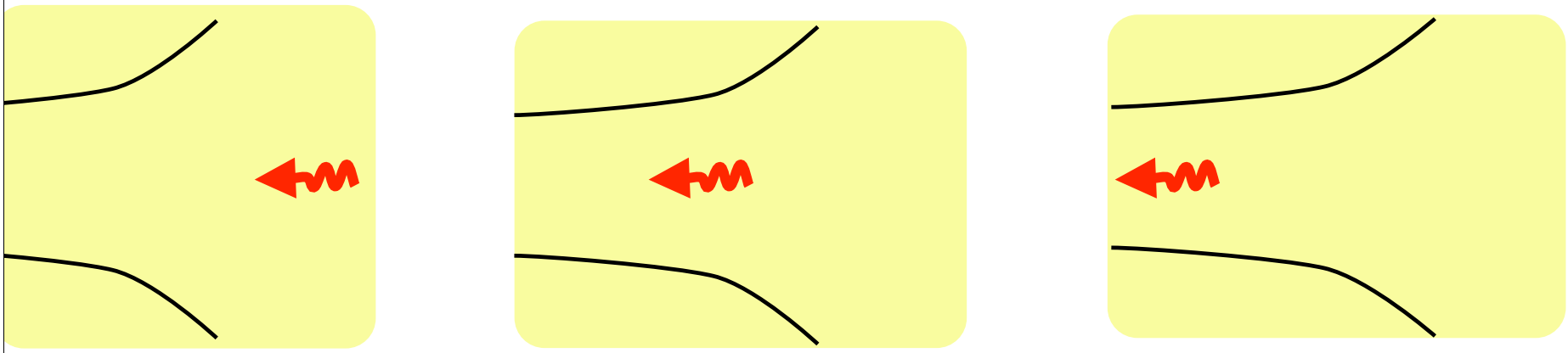


The shape of this disturbance, and its subsequent evolution, are approximately independent of the initial configuration of the fuzzball

Now we can borrow the underlying idea of AdS/CFT duality ...

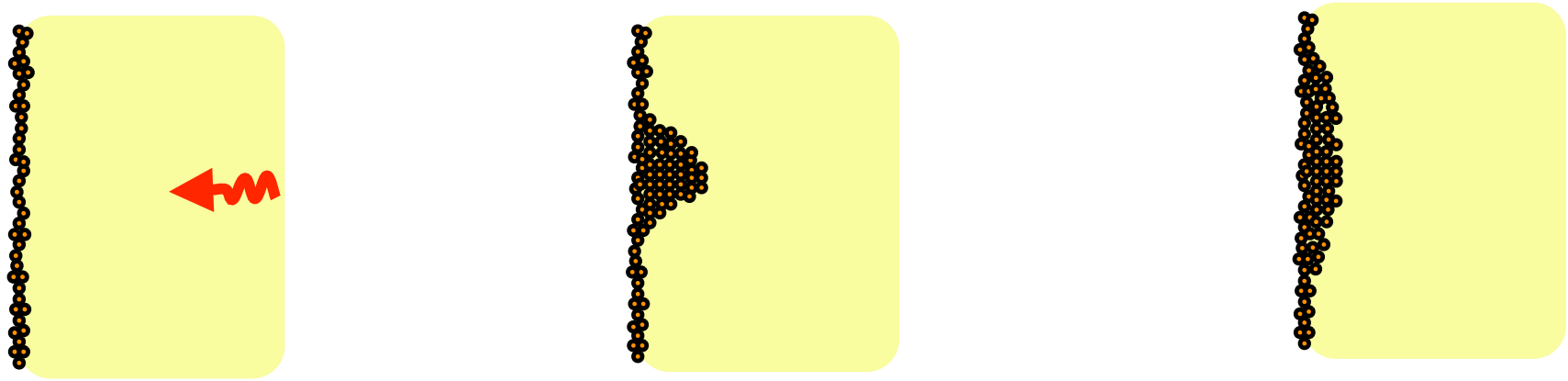


The D-brane picture .....

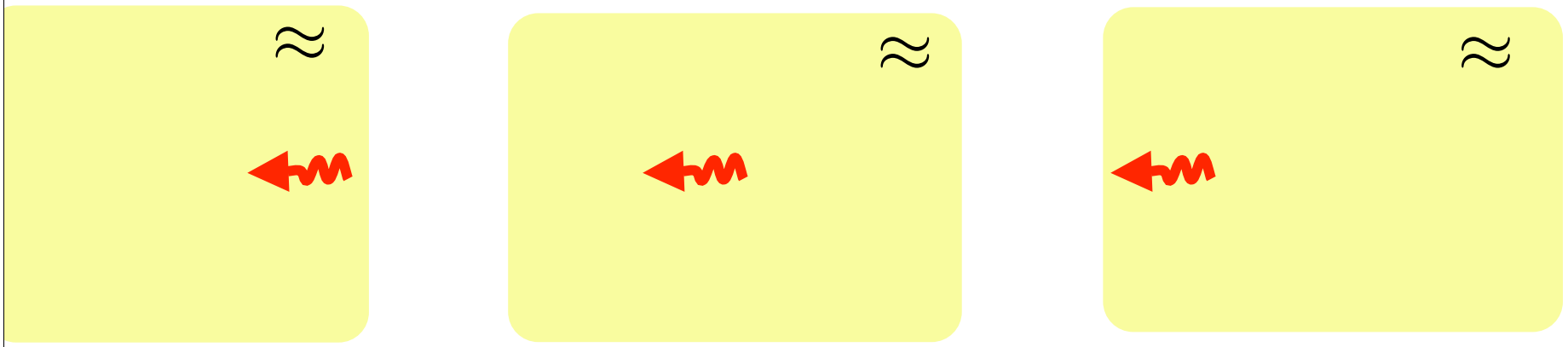


The gravity picture ...

If we have a fuzzball surface ...



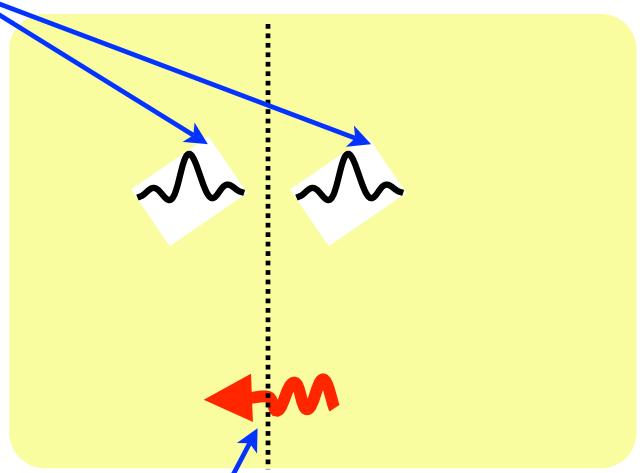
The fuzzball picture (exact) ...



A picture with infall through the horizon (approximate) ...

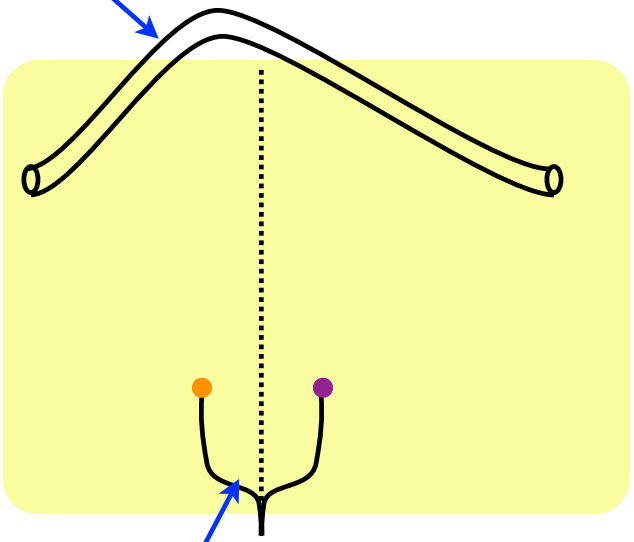
The AdS/CFT + small corrections approach:

Quantum modes entangled the same way as in the vacuum



Nothing seen at the horizon as you fall in

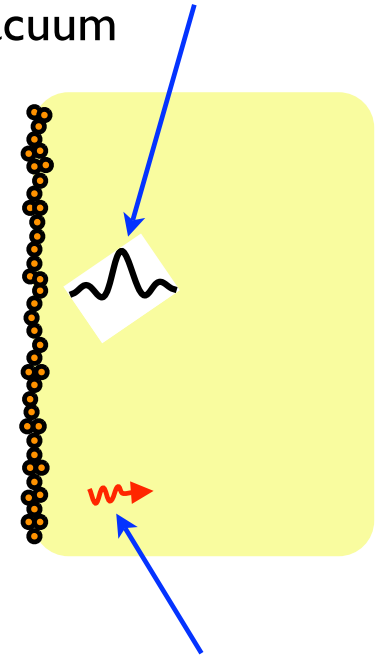
You need a wormhole (or other new effect) to remove the entanglement



Then you get the creation of enangled pairs

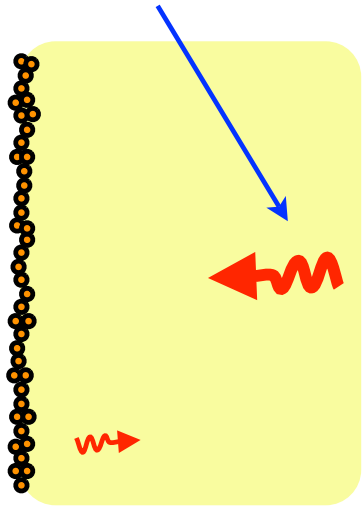
The Fuzzball approach:

Quantum modes
NOT entangled the
same way as
in the vacuum



Unitary radiation, no entanglement problem

Infalling object cannot
'go through' fuzzball surface

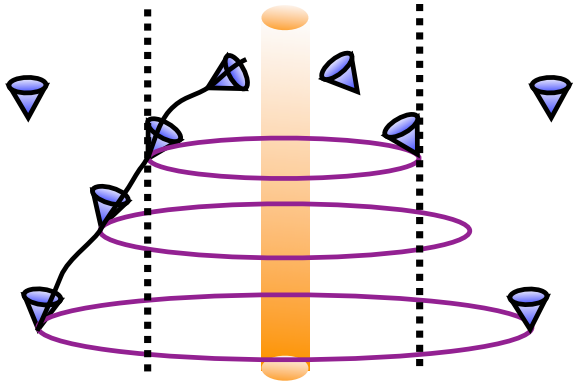


Complementary description is approximate, with correction

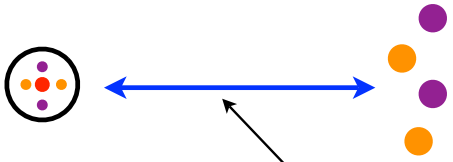
$$\left(\frac{E}{T}\right)^{-\frac{1}{D-2}}$$

Summary

(A) It is very hard to change the structure of a hole (the no-hair theorem)



(B) With this structure, we get the production of entangled pairs, leading to the entanglement problem



highly entangled

OR

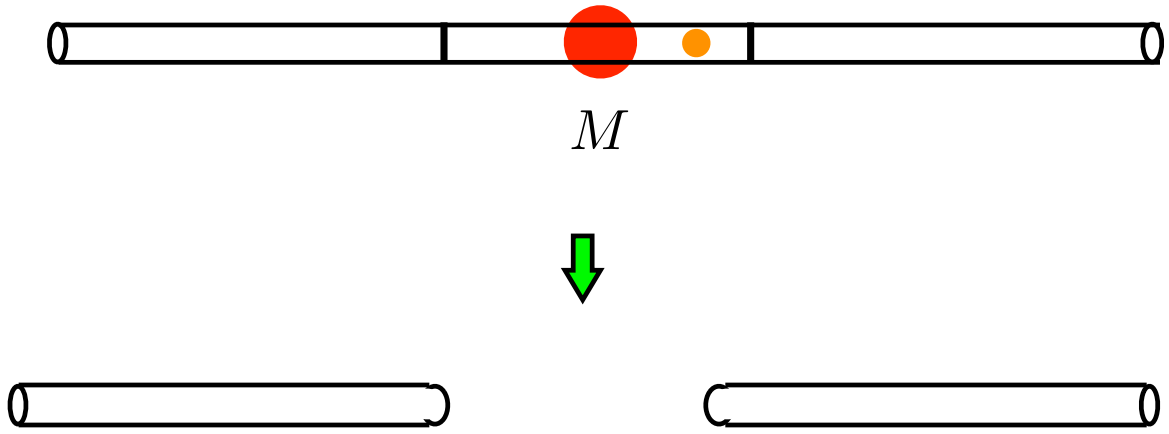
vacuum

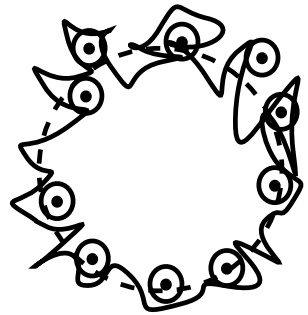
does not have a state by itself

(C) Small corrections to Hawking's leading order computation do not solve the problem .. (so Maldacena 2001, Hawking 2004 were mistaken)

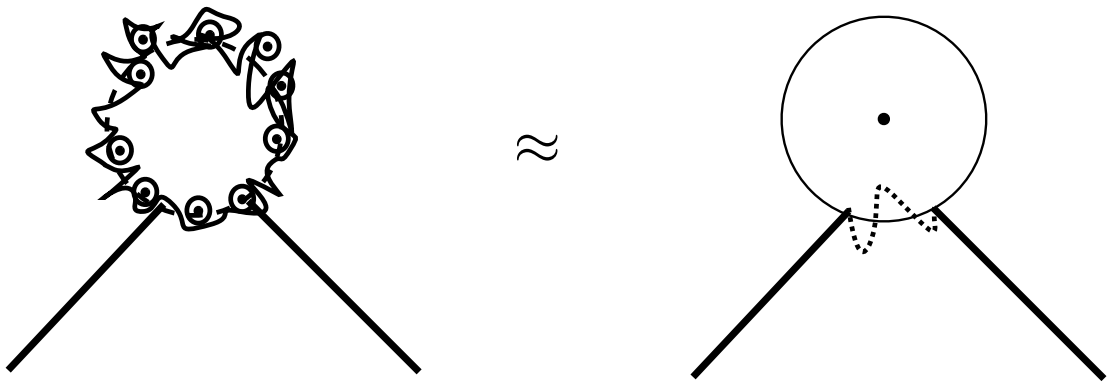
$$S_{N+1} > S_N + \ln 2 - (\epsilon_1 + \epsilon_2)$$

(D) The information puzzle is solved in string theory because the horizon does not form ...





(E) AMPS argued that complementarity was not possible, but they attacked the wrong definition of complementarity: fuzzball complementarity (which works only as a high energy approximation) is not ruled out by their argument



(F) It appears that the fuzzball picture gives a complete consistent theory of the quantum dynamics of black holes