A Question of Hierarchy

Matter Effects with Atmospheric Neutrinos

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Phys. Rev. D71, 013001(2005)

Outline of the talk

Bruno Pontecorvo

- Evolution of neutrino mass
- Pontecorvo and neutrino oscillations
- Experiments: Status
- Confirming oscillation
- Question of hierarchy
- Experiments: Future

Evolution of neutrino mass

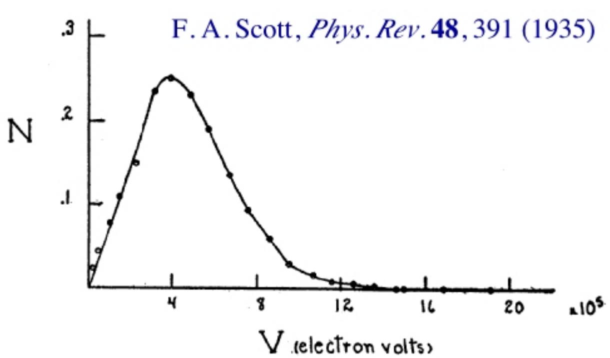


FIG. 5. Energy distribution curve of the beta-rays.

- **1930** W. Pauli– Neutrino is a light neutral particle with mass less than electron mass. Fermi and Perren (**1934**) proposed the measurement of neutrino mass from β-spectrum end point.
- **1949**–Hanna and Pontecorvo, Curran, Angus and Cockroft.

$$m_\nu \leq 500\ eV$$

- **1956** Reines et al–Discovery of the neutrino through $\bar{\nu_e} + p \rightarrow e^+ + n$ using neutrinos from fission reactor. Around this time

$$m_\nu \leq 200\ eV$$

Massless neutrino: Origins

- Following Lee and Yang suggestion, Wu et al(**1957**) found Large (maximal !) parity violation in β-decay ...
- This led to two-component neutrino theory (Pauli, Landau, Lee and Yang, Salam **1957**)–maximal violation of parity is ensured with $m_\nu = 0$

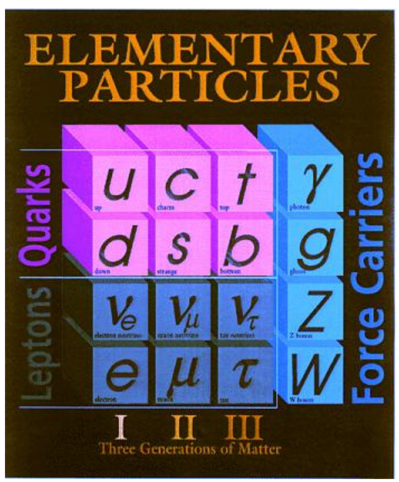
$$i\gamma_\mu \partial_\mu \nu_{L,R} - m_\nu \nu_{R,L} = 0 \quad \Rightarrow \quad i\gamma_\mu \partial_\mu \nu_{L,R} = 0 \; (Weil)$$

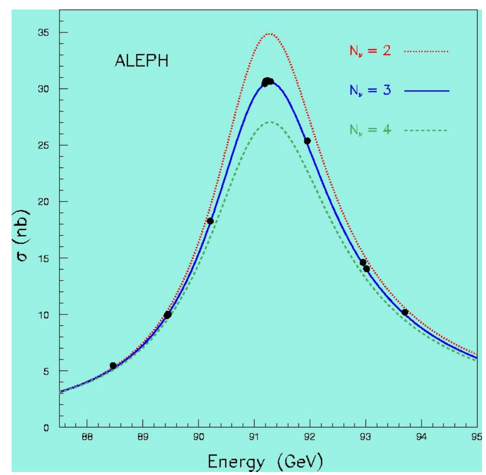
- Neutrino can either be ν_L or ν_R- Goldhaber et al (**1958**) found the helicity to be -1 ± 0.3, neutrino is left-handed!
- Lone dissenting voice – Pontecorvo (**1957**)

  If $m_\nu \neq 0$ – neutrino oscillations can preserve hadron-lepton symmetry with an analogue of $K^0 \leftrightarrow \bar{K}^0$. (No quarks yet)

Massless neutrino: Continued

- V-A theory (1958): The Hamiltonian involves only left-handed components of all fields. PV is not confined to Neutrinos alone- generic property of all weak interactions. (Sudarshan and Marshak, Feynman and Gell-Mann, **1958**)
- The Standard Model (SM) is built on the assumption of mass-less two component neutrino (Glashow, Salam and Weinberg, **1967**)





The Standard Model lists particles, baryons and leptons, and their interactions. Three active Neutrinos ν_e, ν_μ, ν_τ.

Pontecorvo and his legacy

- Bruno Pontecarvo (1957) believed that there must be some symmetry between hadrons and leptons.
- Thus there must be a leptonic analogue to the famous $K^0 \leftrightarrow \bar{K}^0$ oscillations.
- Natural candidate for this is the neutrino oscillation since neutrino was the only neutral lepton known.

  ...there exists the possibility of real neutrino to antineutrino transitions in vacuum provided lepton charge is not conserved...

- Raymond Davis (1958) was looking at reactor antineutrinos and searching for production of Argon in $\bar{\nu}_e +^{37} Cl \rightarrow e^- +^{37} Ar$ which is allowed only if lepton number is not conserved.

Neutrino Oscillation: Primer

A rumour that Davis had seen some events set Pontecorvo on his mission. "...a beam of neutral leptons consisting mainly of antineutrinos when emitted from a nuclear reactor, will consist at some distance R from the reactor of half neutrinos and half antineutrinos...."

For simplicity consider two types of neutrinos, ν_a, ν_b: At time $t = 0$.

$$
\begin{aligned}
|\nu_a(0)\rangle &= \cos\theta|\nu_1(0)\rangle + \sin\theta|\nu_2(0)\rangle \\
|\nu_b(0)\rangle &= -\sin\theta|\nu_1(0)\rangle + \cos\theta|\nu_2(0)\rangle
\end{aligned}
$$

where ν_l is the propagating state with mass m_l and momentum p. At time t,

$$
\begin{aligned}
|\nu_a(t)\rangle &= \cos\theta|\nu_1(t)\rangle + \sin\theta|\nu_2(t)\rangle, \\
&= \cos\theta\, e^{-iE_1 t}|\nu_1(0)\rangle + \sin\theta\, e^{-iE_2 t}\nu_2(0)\rangle
\end{aligned}
$$

where $E_i = (p^2 + m_i^2)^{1/2} \approx p + \frac{m_i^2}{2p}; \quad m_i \ll E_i$

Oscillation Probability

$$
\begin{aligned}
P_{ab} &= |\langle \nu_a(0)|\nu_b(t)\rangle|^2 = \sin^2 2\theta \;\; \sin^2\left(\frac{\delta m^2\, t}{4p}\right) \\
&= \sin^2 2\theta \;\; \sin^2\left(\frac{\delta m^2\, L}{4E}\right) \\
P_{aa} &= 1 - P_{ab} \\
\delta m^2 &= m_2^2 - m_1^2.
\end{aligned}
$$

Thus, if neutrinos have mass and mix, they can oscillate.

When Pontecorvo proposed this idea there was only one neutrino ν_e known to exist. His idea was " ... there exists the possibility of real neutrino $\leftrightarrow$ antineutrino transitions in vacuum...". He also stressed that the oscillation length R may be very large but will certainly occur on an astronomical scale.

Possible only for Majorana type neutrinos.

- Pontecorvo (1967) generalised the idea of neutrino oscillations to $\nu_e \leftrightarrow \nu_\mu$ after ν_μ was well established. (Flavour oscillations)
- Around this time Davis had just begun the Solar neutrino experiment through

$$\nu_e +^{37} Cl \rightarrow e^- +^{37} Ar$$

with a view to confirm the standard solar model of John Bahcall and a direct proof of energy production mechanism in stars.

- **Pontecorvo pointed out: "... From an observational point of view the ideal object is the Sun. If the oscillation length is smaller than the radius of the Sun..the only effect on the earth's surface would be that the flux of observable sun neutrinos must be two times smaller than the total neutrino flux..."**

Pontecorvo and SNO

- At the time Pontecorvo proposed the idea of depletion of solar neutrinos he thought “ ... thermo nuclear reactions in the sun and its central temperatures are not known well enough to permit a comparison of the expected and observed solar neutrino intensities”

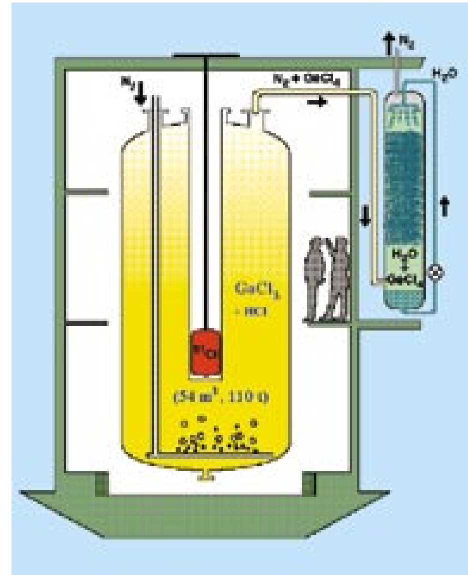
  He could not anticipate SNO result since neutral current weak interactions had not yet been observed.

- But in 1988 when SNO was going through difficulties, Pontecorvo strongly supported the experiment as the only experiment that can nail the oscillation hypothesis as well as confirm the Standard Solar Model. Further he pointed out that for historical reasons it could only be done in Canada.

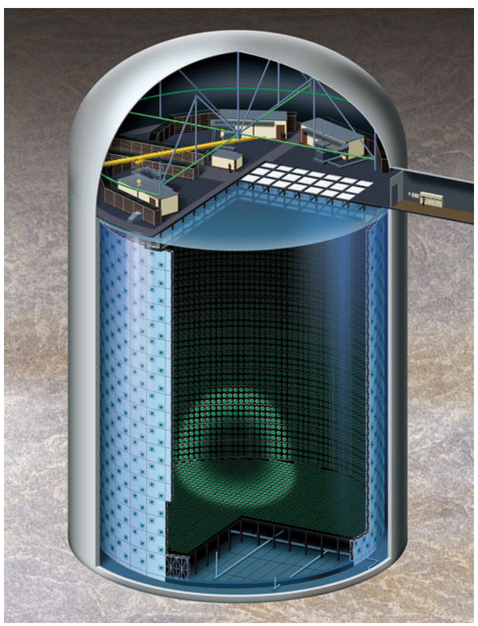
- Pontecorvo did not live to see the confirmation of his path breaking ideas – he died in 1993.

Evidence for neutrino mass and oscillations

Homestake



Gallex



SK

SNO

Evidence for Neutrino Oscillations: From Super-K, SNO, Kamland and K2K using Solar and Atmospheric Neutrinos

Experimental Status

- The Homestake Chlorine experiment by Davis and collaborators first observed a deficit in the observed solar neutrino flux. Confirmed by Gallex.
- The Super-Kamiokande real-time water Cerenkov experiment proved that the observed neutrinos indeed originated in the Sun.
- The SNO heavy water experiment provided the very important corroboration that the electron neutrino flux is depleted while the total solar neutrino flux is consistent with theory.
- The Super-Kamiokande also showed that atmospheric muon neutrinos (and anti-neutrinos) were depleted; especially those ν_μ that had travelled a large path-length through the Earth before they were observed in the detector. No depletion in electron neutrinos.

Parameters of 3ν framework

The ν_e, ν_μ and ν_τ flavours do not have definite masses:

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i \ .$$

where ν_1, ν_2 and ν_3 have well-defined masses: m_1, m_2 and m_3, some are non-zero. $U(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP})$ is the mixing matrix.

Experiments tell us:

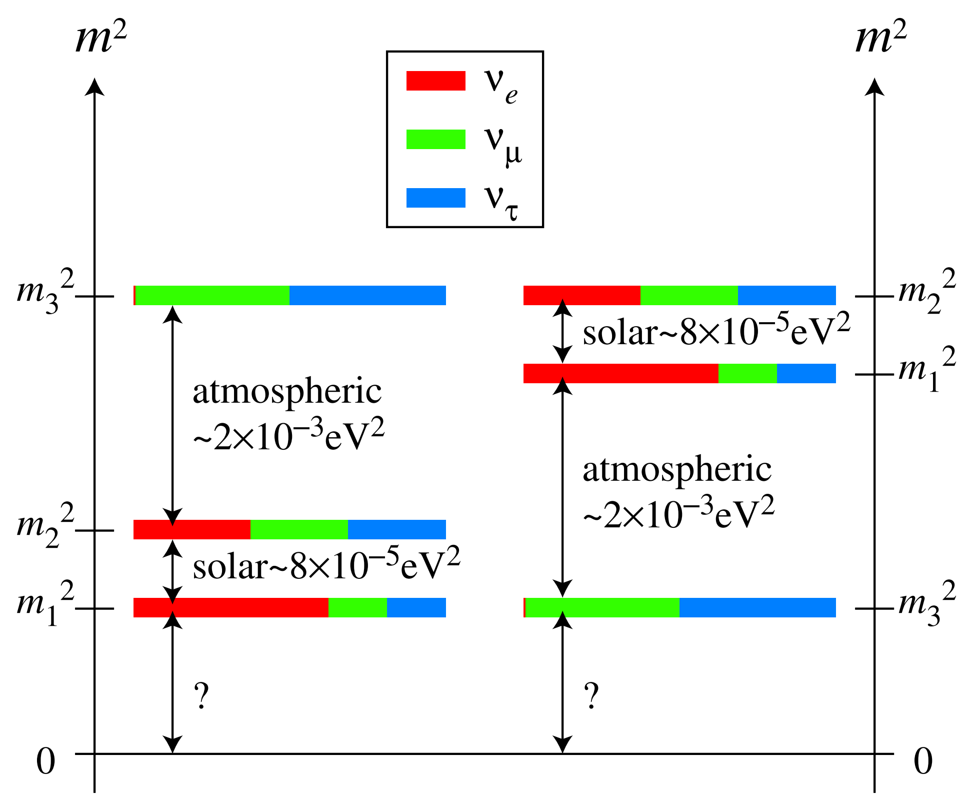
- $\delta m^2_{21} = m^2_2 - m^2_1 \sim 8 \times 10^{-5}$ eV^2.
- $|\delta m^2_{32}| = |m^2_3 - m^2_2| \sim 2 \times 10^{-3}$ eV^2.
- Mass scale $m \leq 2.2$ eV from direct limits. (KATRIN in future can go down to 0.3 eV).
- Cosmology (incl. WMAP) constrains $\sum_i m_i < 0.7$–2 eV.

Mass Hierarchy ?

Recall $\sqrt{\delta m^2_{21}} \sim 0.01$ eV, $\sqrt{|\delta m^2_{32}|} \sim 0.05$ eV, $\sum_i m_i < 0.7$–2 eV.

- Degenerate hierarchy : $m_1 \sim m_2 \sim m_3 \sim 0.2$ eV .
- Normal hierarchy : $m_1 < m_2 \ll m_3$.
- Inverted hierarchy : $m_3 \ll m_1 < m_2$.

- $\sin^2\theta_{12} \sim 0.30$
- $\sin^2\theta_{23} \sim 0.50$
- $\sin^2\theta_{13} \leq 0.047 (3\sigma)$



The APS multi-divisional neutrino study

Executive summary: Ref: physics/0411216

What should we learn from future expts:

1. Searches for neutrinoless double beta decay
2. determination of the neutrino mass hierarchy.
3. measurement of the mixing angle θ_{13}
4. CP violation and origin of matter
5. extra neutrinos (sterile)

An inverted mass hierarchy may be interpreted to mean that leptons obey a new (only slightly broken symmetry) $L_e - L_\mu - L_\tau$ which would raise doubts about quark-lepton symmetry, a fundamental ingredient of GUTs such as SO(10). A normal mass hierarchy is expected in generic see-saw models including most SO(10) GUTs.

The mass hierarchy–Direct or Inverted ?

- No observable in vacuum can distinguish the two cases–sensitive only to $|m_3^2 - m_2^2|$.
- Need to distinguish the effect of oscillation on ν_μ and $\bar{\nu}_\mu$ – Matter Effect– sensitive to the sign
- Need charge identification to separate neutrino and antineutrino events in the detector –Magnetic Field

$$\nu_\mu \longrightarrow \mu^- \; ; \;\; \bar{\nu}_\mu \longrightarrow \mu^+$$

The following combination can in principle solve the question

- Source : Atmospheric Neutrinos, $\nu_\mu, \bar{\nu}_\mu$
- Detector: Magnetised Iron Calorimeter (ICAL)
- Earth : Source of Matter Effect

But what is the observable?

The Asymmetry

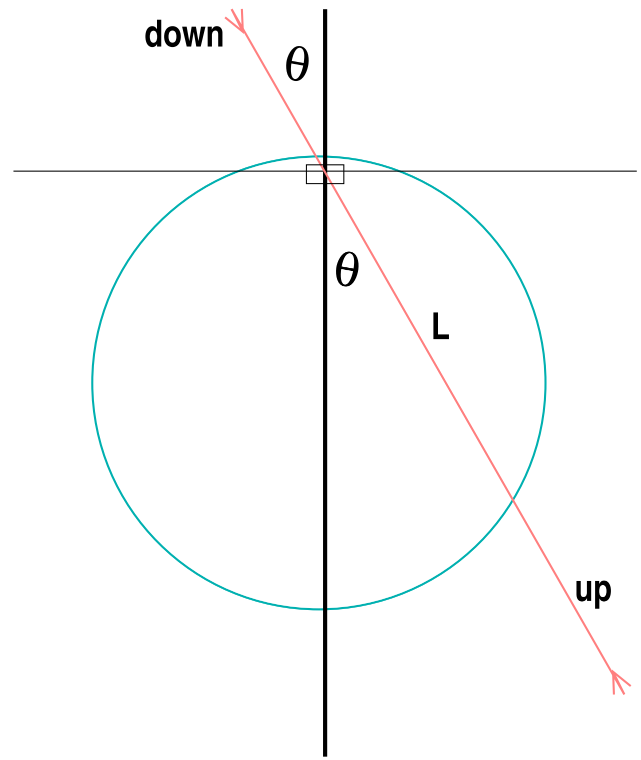
- Define $U(D)$ as up (down)– going ν_μ events
- Define $\bar{U}(\bar{D})$ as up (down)– going $\bar{\nu}_\mu$ events.
- Define $R = U/D; \quad \bar{R} = \bar{U}/\bar{D}$,
- It turns out that $R \sim P_{\mu\mu}$ is simply the ν_μ survival probability.

Two possible observables:

- Double Ratio: $r = \frac{(U/D)_\nu}{(\bar{U}/\bar{D})_{\bar{\nu}}}$
- The Asymmetry:

$$\mathcal{A} = (U/D)_\nu - (\bar{U}/\bar{D})_{\bar{\nu}}$$

Asymmetry is the preferred observable.



Earth matter effect

The matter effect mainly occurs in the θ_{13} parameter:

$$(\sin\ 2\theta_{13})_m = \frac{(\sin\ 2\theta_{13})}{\sqrt{[\cos 2\theta_{13} - (A/\delta m^2_{32})]^2 + (\sin 2\theta_{13})^2}}$$

where

$$A = 7.6 \times 10^{-5}\ \rho\ E\ eV^2 \quad \delta m^2_{32} = m^2_3 - m^2_2$$

ρ = earth density (gms/cc)
E = neutrino energy in GeV.

Let $\eta = A/\delta m^2_{32}$

- Neutrinos (A)$\Leftrightarrow$ antineutrinos ($-A$).
- The matter effect depends only on η and not on A and δm^2_{32} separately (to a very good approximation).

Probabilities in matter

Hierarchy discriminator: Earth matter, difference in interactions between ν and $\overline{\nu}$.

$$
\begin{aligned}
P^m_{\mu\mu}(A,\delta) &\approx P^{(2)}_{\mu\mu} - \sin^2\theta_{13} \times \\
&\left[\frac{A}{\delta - A}T_1 + \left(\frac{\delta}{\delta - A}\right)^2 \left(T_2 \sin^2[(\delta - A)x] + T_3\right)\right] , \\
\overline{P}^m_{\mu\mu}(A,\delta) &\approx P^{(2)}_{\mu\mu} - \sin^2\theta_{13} \times \\
&\left[-\frac{A}{\delta + A}T_1 + \left(\frac{\delta}{\delta + A}\right)^2 \left(T_2 \sin^2[(\delta + A)x] + T_3\right)\right] ,
\end{aligned}
$$

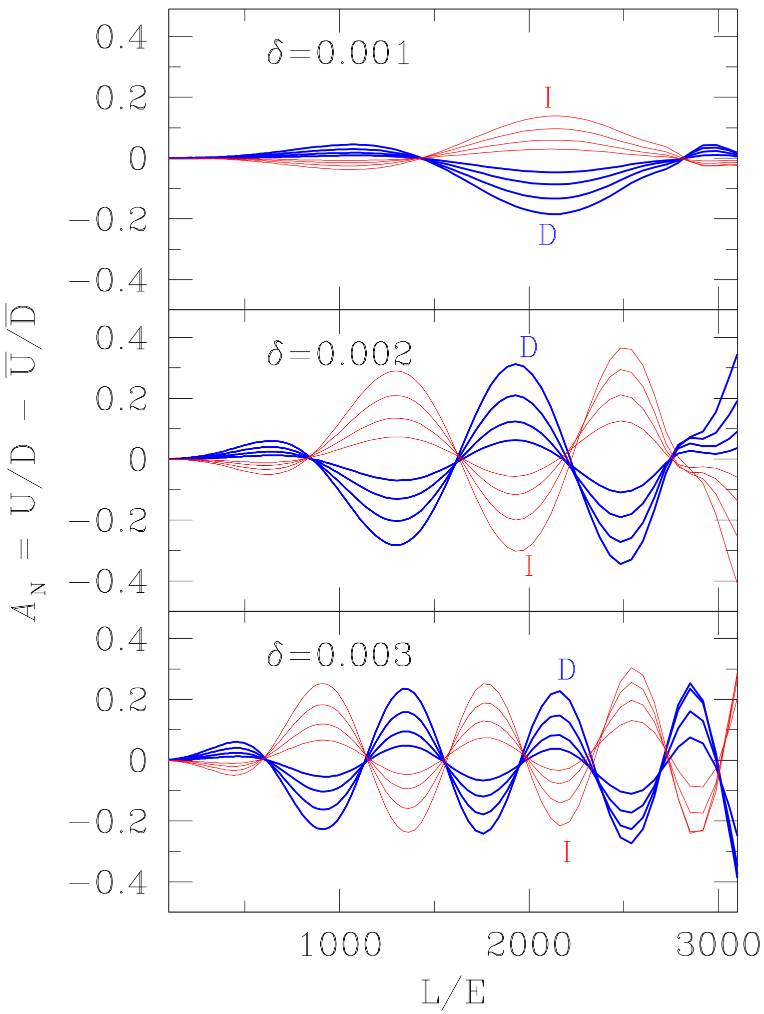
$A \propto \rho E$. **Changes sign between neutrinos and anti-neutrinos.**

Consequently the asymmetry obeys

$$\mathcal{A}(A,\delta) = -\mathcal{A}(A,-\delta) = -\mathcal{A}(-A,\delta) = \mathcal{A}(-A,-\delta)$$

A: The difference asymmetry

Asymmetry as a function of θ_{13} and L(km) / E(GeV)



$\delta \equiv \delta m^2_{32}$

Hence sensitive to the mass ordering (red vs blue) of the 2–3 states; however, needs large exposures of about 500–1000 kton-years.

How to realise this experimentally?

ICAL at INO : Proposal

The choice of detector

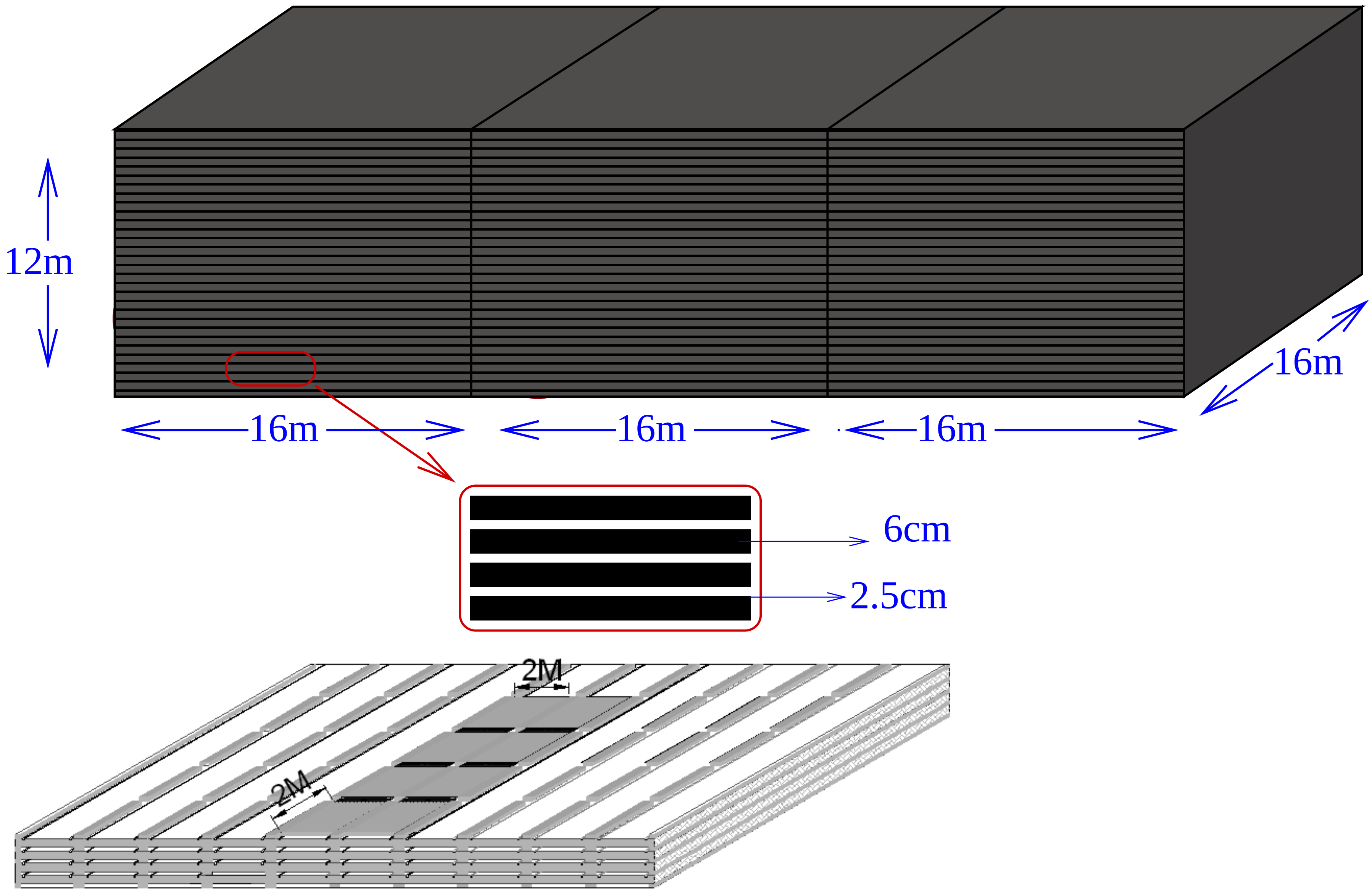
The detector should have the following features:

- Large target mass: 30 kton, 50 kton, 100 kton . . .
- Good tracking and energy resolution
- Good directionality; hence nano-second time resolution for up/down discrimination
- Good charge resolution
- Ease of construction (modular)

Use (magnetised) iron as target mass and RPC as active detector element. Similar to MONOLITH.

Note: Is sensitive mainly to muons

The ICAL detector

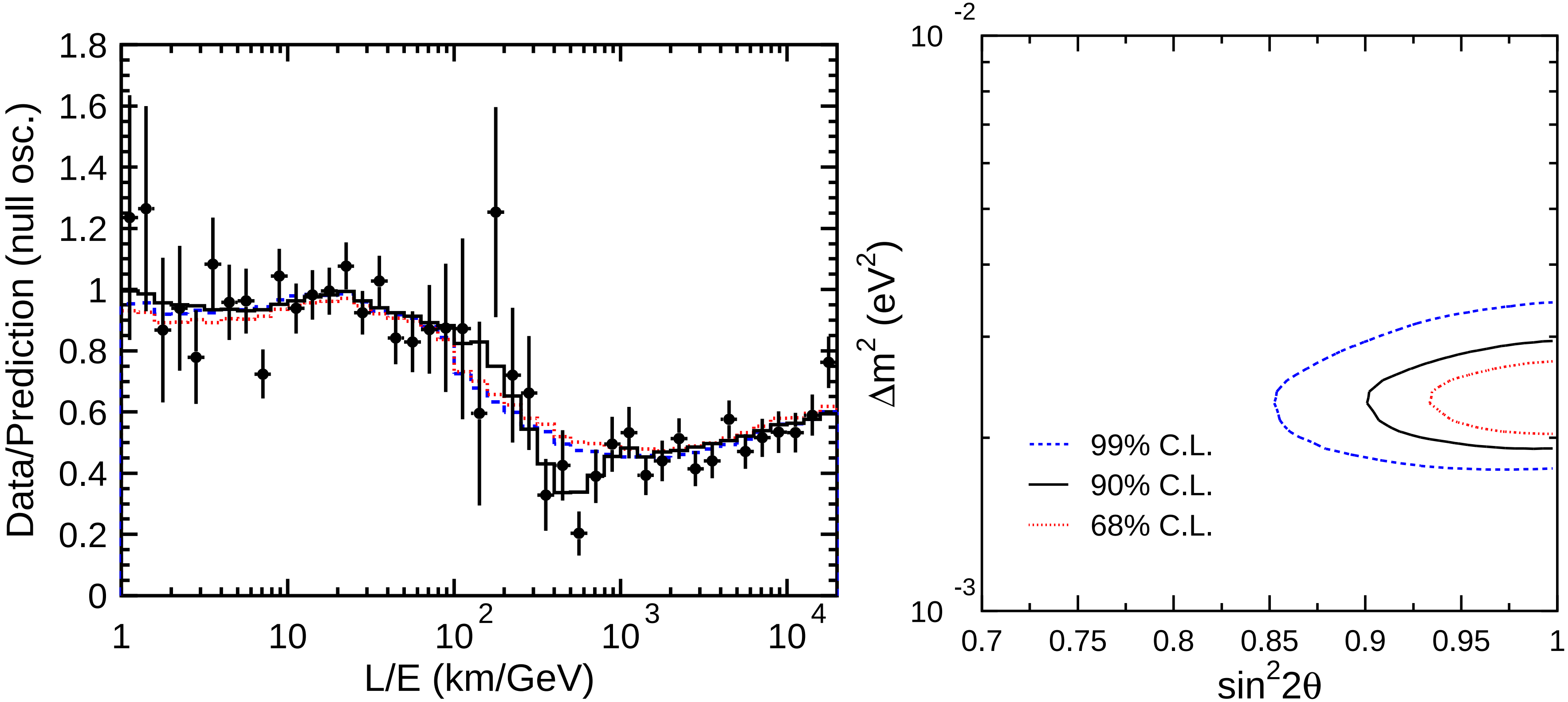


Specifications of the ICAL detector

ICAL	
No. of modules	3
Module dimension	16 m $\times$ 16 m $\times$ 12 m
Detector dimension	48 m $\times$ 16 m $\times$ 12 m
No. of layers	140
Iron plate thickness	$\sim$ 6 cm
Gap for RPC trays	2.5 cm
Magnetic field	1.3 Tesla
RPC	
Readout strip width	3 cm
No. of RPC units/Road/Layer	8
No. of Roads/Layer/Module	8
No. of RPC units/Layer	192
Total no. of RPC units	$\sim$ 27000
No. of electronic readout channels	3.6 $\times 10^6$

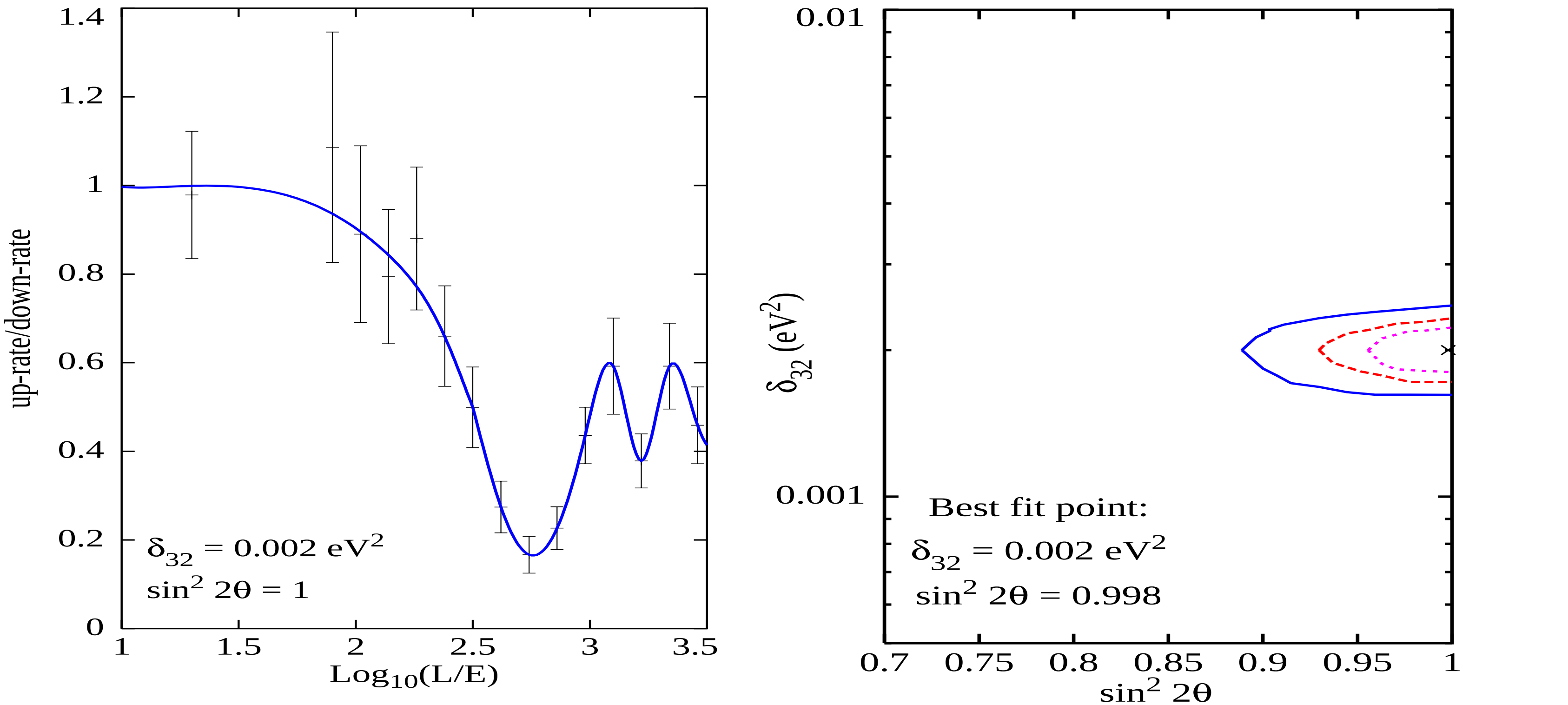
Oscillation and hierarchy with Magnetised ICAL

The difficulty . . . and the hope



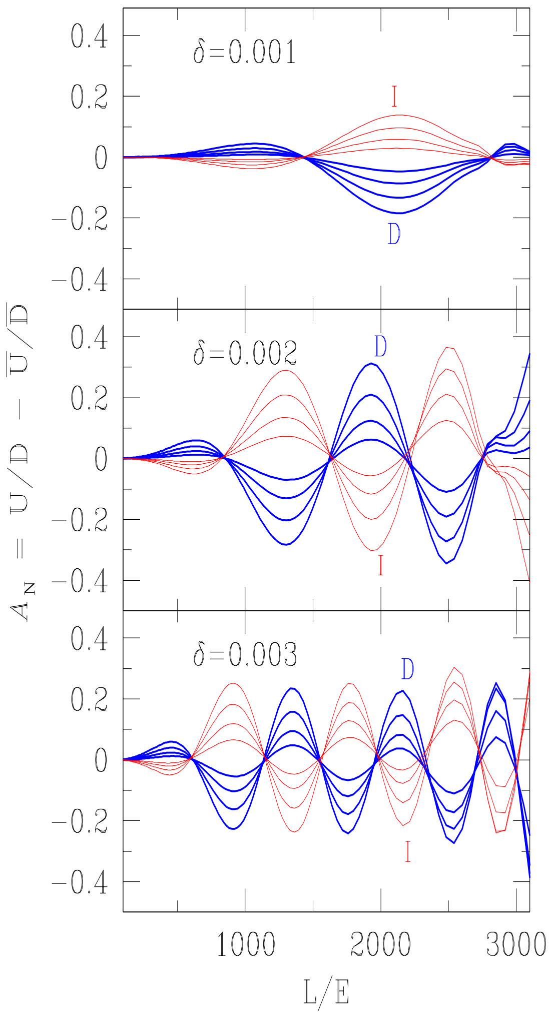
- $\Delta m^2 = 2.4 \times 10^{-3}$ eV^2 ; $\sin^2 2\theta = 1.0$.
- Decay, decoherence, disfavoured at more than 3σ

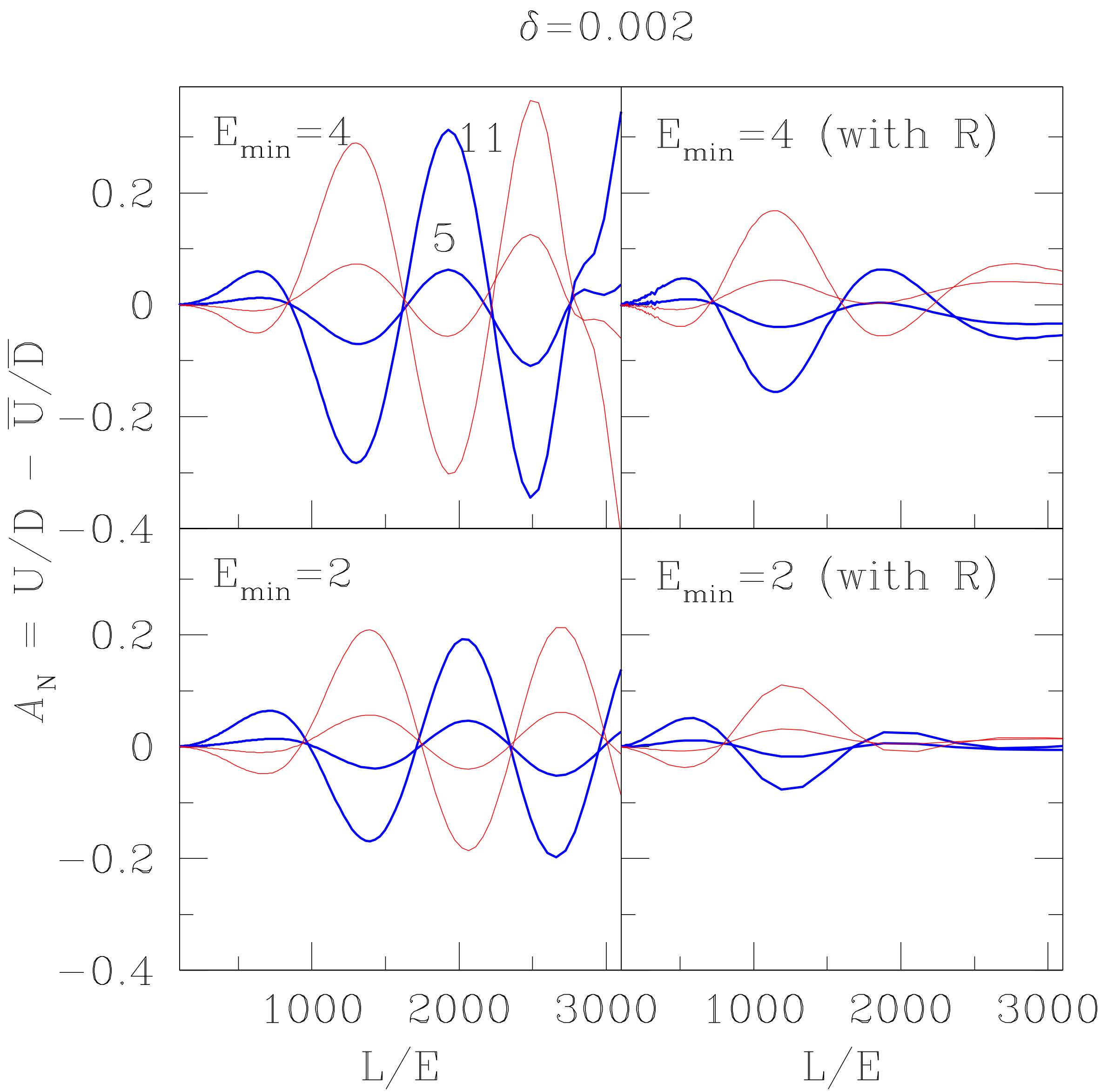
Y. Ashie et al., Super-K Collab., Phys.Rev.Lett. 93 (2004) 101801 [hep-ex/0404034]



Simulation with ICAL detector, assuming 50% efficiency in L/E reconstruction

Physics vs Simulation





Asymmetric sum

To improve the result, we try summing over several envelopes:

$$\mathcal{A}_n^H = \sum_i^n (-1)^i \mathcal{A}^{H,i}$$

where

$$H = D, I$$

The quantity

$$\Delta\mathcal{A}_n = \mathcal{A}_n^D - \mathcal{A}_n^I$$

with an error σ indicates the probability of the observed assigned to the correct hierarchy.

10% Resolution– kton-year exposure

Exp.	$\Delta\mathcal{A}_2$ (CL %)	$\Delta\mathcal{A}_3$ (CL %)
	$\theta_{13} = 7^\circ$	
480	$0.185 \pm 0.167 \quad (1.1\,\sigma,\ 72.9\%)$	$0.232 \pm 0.220 \quad (1.1\,\sigma,\ 72.9\%)$
800	$0.185 \pm 0.130 \quad (1.4\,\sigma,\ 83.8\%)$	$0.232 \pm 0.170 \quad (1.4\,\sigma,\ 83.8\%)$
1120	$0.185 \pm 0.110 \quad (1.7\,\sigma,\ 91.1\%)$	$0.232 \pm 0.144 \quad (1.6\,\sigma,\ 89.0\%)$
	$\theta_{13} = 9^\circ$	
480	$0.287 \pm 0.165 \quad (1.7\,\sigma,\ 91.1\%)$	$0.384 \pm 0.220 \quad (1.8\,\sigma,\ 92.8\%)$
800	$0.287 \pm 0.128 \quad (2.2\,\sigma,\ 97.2\%)$	$0.384 \pm 0.170 \quad (2.3\,\sigma,\ 97.9\%)$
1120	$0.287 \pm 0.108 \quad (2.7\,\sigma,\ 99.3\%)$	$0.384 \pm 0.144 \quad (2.7\,\sigma,\ 99.3\%)$
	$\theta_{13} = 11^\circ$	
480	$0.399 \pm 0.163 \quad (2.4\,\sigma,\ 98.4\%)$	$0.565 \pm 0.221 \quad (2.6\,\sigma,\ 99.1\%)$
800	$0.399 \pm 0.126 \quad (3.2\,\sigma,\ 99.9\%)$	$0.565 \pm 0.171 \quad (3.3\,\sigma,\ 99.9\%)$
1120	$0.399 \pm 0.107 \quad (3.7\,\sigma,\ 99.98\%)$	$0.565 \pm 0.144 \quad (3.9\,\sigma,\ 99.99\%)$
	$\theta_{13} = 13^\circ$	
480	$0.515 \pm 0.161 \quad (3.2\,\sigma,\ 99.9\%)$	$0.758 \pm 0.223 \quad (3.4\,\sigma,\ 99.9\%)$
800	$0.515 \pm 0.125 \quad (4.1\,\sigma,\ 99.996\%)$	$0.758 \pm 0.173 \quad (4.4\,\sigma,\ 99.999\%)$
1120	$0.515 \pm 0.102 \quad (4.9\,\sigma,\ 99.9999\%)$	$0.758 \pm 0.146 \quad (5.2\,\sigma,\ 100\%)$

Where are we then?

- Better than 2σ results for atmospheric neutrinos with modest detector resolution and about 800 kton-years of data taking.
- In contrast, looking for other oscillation parameters may require only about 150 kton years of data taking (former is 3–5 times larger data sample; sheer statistical requirement).
- Caveat: There is a major unknown: the mixing angle θ_{13}. If it is as large as 0.1 or more, then we can be very optimistic (MINOS will give this info soon).
- If this is small: $\sin^2 2\theta_{13} < 0.05$, then there is no hope of making this measurement in a meaningful time-frame.
- ICAL during stage II with neutrino beam can in principle answer the question– but way into the future.

Bottom line: Still a very difficult experiment.