RAJAJI SYMPOSIUM

Festschrift on the occasion of the 65th birthday of

G. Rajasekaran

on 22nd February 2001

Editors:

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FOREWORD

Professor G. Rajasekaran, or simply Rajaji to his many students and admirers, completed sixty five years on 22 February 2001. His active career as a theoretical physicist spans more than four decades so far. His career is marked by the breadth of his research interests, clarity of thought and expression, insight into subtleties and elegant exposition of these as evidenced in his research publications and conference talks. These have earned him a highly respectable place in the community of theoretical and experimental physicists.

When the organising committee of the Rajaji Symposium decided to hold a one-day Symposium in his honour, the response from the community of theoretical physicists was not only spontaneous but also overwhelming. Many of his students, collaborators and colleagues gathered on 22 February 2001 to pay their tributes to him and presented contributions. This is the subject matter of this IMSc REPORT.

Rajaji's research work covers a wide range, starting from hyper-nuclei, S-matrix theory, quantum field theory (in particular gauge theories), high energy physics phenomenology, neutrino physics, gravity and quantum statistics. He has delivered individual and series of lectures on his work in all leading research centres in India and abroad. His lectures have inspired beginners to embark upon their research career and often helped experts to explore new streams of thought. As an example, mention must be made of his lectures on gauge theory, delivered at the Saha Institute of Nuclear Physics, Calcutta, which emphasised the importance of gauge theories in particle physics as early as ... This was an year before the proof of renormalisability established the electroweak theory as one of the corner-stones of particle physics. With similar insight, of late, he has been enthusiastically supporting the idea of an India-based Neutrino Observatory.

"Age cannot wither him, nor custom stale his infinite variety" of research interests.

The Organising Committee is thankful to the Director of the Institute of Mathematical Sciences for his active support in conducting the one-day symposium in Rajaji's honour. The editors of this report also thank him for his enthusiastic support in bringing out the proceedings in the form of an IMSc REPORT.

The Editors.

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Chapter 1

About Rajaji

Rajaji was born on 21 February 1936 at Kamudi in Ramanathapuram district of Tamil Nadu. He had his early education in Kamudi and then in Madurai and went on to study in Madras Christian College from where he obtained B.Sc.(Honours) with top honours.

He later joined the Training School of Atomic Energy Establishment in 1957. After spending three years at the training school and TIFR, he went on to do his Ph.D at the Univesity of Chicago under the supervison of Professor Richard Dalitz.

On his return to India Rajaji spent many years at the Tata Institute of Fundamental Research in Bombay before moving to Madras where he is currently based. In Madras Rajaji was first associated with the University of Madras and later with the Institute of Mathematical Sciences where is presently a distinguished Professor. During his long and eventful career he has held many distinguished visiting positions at various Institutions both in India and abroad. Rajaji is a fellow of the Indian Academy of Sciences, Bangalore and fellow of the Indian National Science Academy, New Delhi. He is also a fellow of the National Academy of Sciences, Allahabad.

Rajaji's research interests cover a broad spectrum of theoretical Physics—in particular, quantum field theory and high energy Physics. Highlights of his research are included along with his list of publications at the end of the Proceedings.

In addition, Rajaji is a recipient of many awards and honours for his contribution to research, including

- Meghnad Saha Award for research in Theoretical Sciences (1987).
- Federation of Indian Council for Commerce and Industries Award for Physical Sciences including Mathematics (1990).
- SN Bose Medal of the INSA (1998).

Rajaji has served and continues to serve on the apex bodies of many institutions. He has also served on the Editorial boards of some research Journals. His latest passion is his work towards setting up an underground laboratory for neutrino physics—the India-based Neutrino Observatory (INO). He is a passionate advocate of the ethical uses of science and technology. As a member of the group Indian Scientists Against Nuclear Weapons (ISANW), he has strongly and openly voiced his concerns about the misuse of Science for military adventurism.

Chapter 2

Inaugural Address

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Opening Words

I am very much privileged to have known Prof. G. Rajasekaran for nearly four decades. He has been for me, like for several of you, a role model. My first encounter with him was when both of us appeared at the orientation program at the University of Chicago during the last week of September 1961. That was the occasion to determine whether, as entering students, we were ready to start on graduate level courses and how soon we would be prepared to take the dreaded 'Chicago basics' — a comprehensive examination after clearing which one gets confirmed as a candidate for Ph.D. at the University of Chicago, Rajasekaran was head and shoulders above all of us and was naturally asked to take the test at the first opportunity, viz., at the end of the same quarter. Rajasekaran cleared the basics and got on with his research with Prof. R.H. Dalitz. He was the inspiration for all of us then at Chicago and to his contemporaries in India when we returned and started working in India.

Dr. Rajasekaran got back to TIFR in 1964 and has been a *Karma Yogi* in his Physics career ever since. When you go through his list of publications, you will see the broad base he covered in Particle Physics - both formal and phenomenological - in the preceding four decades. He was aware of the importance of gauge theory, well before it became recognised as a new paradigm that will metamorphose into the 'Standard Model' of all interactions. He has contributed to various facets of it, as indeed several of you (most of the speakers in this symposium being either his collaborators or those who have benefited by his insight) will demonstrate in the course of this symposium. He has been one of the main pillars of the contemporary Particle Physics activity in India and has been a living example of one who can do Physics at the internationally competitive level with firm roots here in India. It is a matter of pride that he started from a rather ordinary high school in an obscure corner of the country and progressed with the only asset of a deep commitment to reach such level that continually inspired many who came in contact with him. There are several national initiatives, Dr. Rajasekaran has taken part and he gave his special insight and the

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very best attention to each of them. He is among the few, who consciously tried to promote 'Pramana' - as a quality Physics journal from India, by sending some of his best papers for publication in it, notwithstanding the result that it meant a limited circulation and publicity for his work. (Unfortunately, unlike today there were no electronic preprint archives yet and SLAC - PPF (Preprints in Particles and Fields) lists could play only a limited role). He has been one of the moving spirits behind the highly successful sequence of SERC Schools in Theoretical High Energy Physics, a process by which we have been able to get pedagogic expositions on recent developments to the research scholars, particularly in many different University departments. Professor Rajasekaran's lectures at SERC Schools, UGC Schools, etc., have been starting point of research for several youngsters in our country and indeed, as is pointed out by Dr. Lalith Sehgal of several physicists elsewhere.

Even though, Dr. Rajasekaran and I have been close to each other in many Physics pursuits, there is only one paper that we wrote together and it remained unpublished. I will take this opportunity to talk briefly on this work and also remark how since then, the ideas contained therein, have developed.

Is Colour Broken by Monopole?

Spontaneous breaking of gauge theories produce monopoles, if in the process the sub group H of the original symmetry group G is such that there are non-contractable loops in H that are contractable in G. The topological classification $\pi_1(H)$ characterises the 'magnetic' charges of the monopole. When $G = SO(3)$ or $SU(2)$ and $H = U(1)$, we get 't Hooft-Polyakov monopole solution, which is a finite energy solitonic configuration of the gauge fields and Higgs scalars. Dokos and Tomoras found monopole solutions when $G = SU(5)$ is spontaneously broken down to $H = \mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{em}}$. In these solutions the stability is a consequence of topology, brought about by the linking of the orientation of the internal symmetry space and the direction of the physical space. The angular momentum operator gets the expression:

$$\vec{J} = i(\vec{r} \times \vec{\nabla}) + \vec{T} \ ,$$

where $\vec{T}$ is the 'isospin' generator. In Dokos-Tomoras monopole this was observed to be the $SU(2)$ embedded in $SU(5)$ in such a way that the fundamental representation has one entry in one of the three $\mathrm{SU}(3)_c$ directions and the other in one of the $\mathrm{SU}(2)_L$ entries. Thus

$$T_3 = 1/2 diag(0, 0, 1, -1, 0) \ ,$$

and the generator of $U(1)_{EM}$ is chosen to be

$$Q_{\mathrm{em}} = diag(-1/3, -1/3, -1/3, 1, 0) \ .$$

We observed (in the unpublished paper written in 1982) that when $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{em}}$ is further spontaneously broken to $\mathrm{U}(1)_{HN}$, then, the generator for $\mathrm{U}(1)_{HN}$, call it Q_{HN}, turns out to be

$$Q_{HN} = Q_{em} + Y_c \ ,$$

where $Y_c = diag(1/3, 1/3, -2/3, 0, 0)$ is the colour hypercharge so that, the Han-Nambu electromagnetic generator (responsible for integer charged quarks) is

$$Q_{HN} = diag(0, 0, -1, 1, 0) = -2T_3 \ .$$

This suggests the possibility that colour breaking is signalled by the presence of monopole. This was an attractive possibility since at that time the integer charged quark model (and the electromagnetic current having a colour octet component that is dynamically suppressed) was an option not precluded by the experimental results on the Deep Inelastic Scattering structure functions. Q_{HN} implies for the charges on the fermions to be $Q_{d_3} = -1, Q_{d_1} = Q_{d_2} = 0$; (The average d-quark charge being $-1/3$ as in the fractional charged quark models) $Q_{e^+} = 1, Q_\nu = 0$. Further for the $\{10\}$ representation of SU(5), we have $Q_{u_1} = Q_{u_2} = 1, Q_{u_3} = 0$.

Monopoles induce baryon number violation brought about by Rubakov - Callan effect, which is another manifestation of the mixing of helicity of fermion and the charge represented by the states of T_3. The Dokos-Tomoras monopole has a core made up of condensates of $(u_1 u_2 d_3 e^-)$ and is responsible for the helicity conserving baryon number violating processes, such as $\bar{d_3} \to e^+, u_1 \to \bar{u_2}$, etc. The fermions that form the condensate are noticed to be precisely those that remain charged in the broken colour version of theory.

Our main motivation for colour being broken by monopole was to address an embarrassment of 'Witten Charge' for the coloured monopole. In the presence of strong CP violation, signalled by the θ-term in the QCD Lagrangian, monopoles carry a fractional electric charge $e\theta/2\pi$. Such a term was known to be consistent with Dirac–Schwinger quantisation $e_1 g_2 - e_2 g_1 = n/2$. A dyon (n_1, m_1) has electric charge $(n_1 + m_1\theta/2\pi)$ units of e and magnetic charge m_1 units of $g(\equiv 1/2e)$. While this fractional electric charge for magnetically charged states is an amusing possibility to be confirmed by experiment, this turns out to be an oddity for the monopole such as Dokos-Tomoras state that carries chromomagnetic charge as well. We expect a fractional chromo-electric charge as well, but $\mathrm{SU}(3)_c$ being compact, there is no room for such a possibility. This will mean that local colour group is a symmetry, but global colour is undefined. Indeed integer charged quark model, generated by Q_{HN} avoids fractional coloured charges.

There is another brand new perception possible as illustrated by T. Vachaspati (Phys. Rev. Letters **76**, 188 (1996)), who observed that Dokos-Tomoras monopole is characterised by paths expressed by $\exp(isQ_m)$: $Q_m = 2T_3, s \in [0, 2\pi]$ and

$$Q_m = Q_1 + Q_2 + Q_3 \ ,$$

with the monopole generator made up as a sum of the generators $Q_1(\mathrm{of} U(1)$, $Q_2(\mathrm{of} SU(2)_L$ and $Q_3(\mathrm{of} SU(3)_c)$:

$$Q_1 = \mathrm{diag}(1/3, 1/3, 1/3, -1/2, -1/2), \qquad Y \mathrm{of} U(1) \ ,$$

$$Q_2 = -\mathrm{diag}(0, 0, 0, 1/2, -1/2), \qquad -t_3 \mathrm{of} SU(2)_L \ ,$$

$$Q_3 = \mathrm{diag}(-1/3, -1/3, 2/3, 0, 0), \qquad \lambda_8 \mathrm{of} SU(3)_c \ .$$

Accordingly the fundamental SU(5) monopole has a magnetic charge $m = 1$ and is a $\mathrm{SU}(2)_L$ doublet and $\mathrm{SU}(3)_c$ triplet. Multiply charged monopole carries for nQ_m, magnetic U(1) charge of $n_1 = n$ units, $\mathrm{SU}(2)_L$ magnetic charge of $n_2 = n(mod\ 2)$ units and $SU(3)_c$ magnetic charge of $n_3 = n(mod\ 3)$ units. He draws up a list of stable monopoles as follows:

n	n_1	n_2	n_3	$SU(3)_c$	$SU(2)_L$	state
1	1	1	1	$\{3\}$	$t = 1/2$	$(\tilde{u}, \tilde{d})_L$
2	2	0	2	$\{3^*\}$	$t = 0$	$\tilde{\bar{d}}_R$
3	3	1	0	$\{1\}$	$t = 1/2$	$(\tilde{\nu}, \tilde{e}^-)_R$
4	4	0	1	$\{3\}$	$t = 0$	$\tilde{\bar{u}}_R$
5	[unstable; state decays to $n = 2$ and $n = 3$ states]					
6	6	0	0	$\{1\}$	$t = 0$	e_R

All higher monopoles are unstable. We observe the correspondence that the magnetic charge of the stable monopoles are same as the electric charges of the first generation of fermions of the standard model. Does this indicate that quarks and leptons are 'monopoles' (or rather solitonic states) of the dual SU(5) theory, call it $\tilde{SU}(5)$. Spectrum of such a theory will consist of gauge fields (gluons, $W^{\pm}$, Z and γ) and quarks and leptons as the monopole configuration of $\tilde{\mathrm{SU}}(5)$. In order that these monopoles are fermions, we may incorporate Witten Effect and assign $\theta = \pi$, which will imply additional magnetic charge of $m\theta/2\pi = m \times 1/2$, which is indeed permitted. There is no need for fermionic matter fields in such a theory!

Concluding Words

Let me close this address with an analogy. I recall a few years ago Professor Helmut Fritzche, who was at that time chairman of the Physics Department of University of Chicago was visiting at IIT Kanpur. He said, when he got back to Chicago, he will be taking part in a pleasant function to felicitate Professor S. Chandrasekhar on his 75th Birthday and note his formal retirement. He added that 'anyway it does not matter; all these years Chandra was working furiously and taking home a salary and now he will do the same and take his pension instead'. I guess it will be similar with Rajaji who will draw his last salary on February 28, 2001 and start working from the next day as INSA Honorary Scientist. Let me join all of you in wishing him all the best in this phase of his career. Further let me finally add a special personal thanks to Rajaji for all that I have learnt from him and all that I will continue to learn.

Chapter 3

1968-1973: Some Reminiscences

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It is a pleasure to be here and to be able to congratulate Rajaji on his 65th birthday. We will hear a great deal at this Symposium about his contributions to physics, and his qualities as a scientist, teacher and colleague. I should like to express my own admiration for his devotion to the cause of particle physics in India, and for the integrity that characterises his scientific work and his dealings with human beings.

For the purpose of this talk, I will go back to a period about 30 years ago, when I began my scientific career and made Rajasekaran's acquaintance for the first time. I will begin the story in March 1968 when I joined the Tata Institute of Fundamental Research, Bombay, as a Visiting Member immediately after finishing my Ph.D. in the U.S.. The Institute was a wonderful place to work in. There was a thrill in being associated with such a magnificent building (Fig. 3.1) with its marble interior, air-conditioned rooms and the view of the Arabian sea. Even more impressive was the array of bright and talented people assembled in the theory group: T. Das, N. Mukunda, P. Divakaran, G. Rajasekaran, K. V. L. Sarma, V. Gupta, V. Singh, S. M. Roy, D. Sankaranarayan, J. Pasupathy, L. K. Pandit, and many others. There was an atmosphere of free and open interaction. I had only to step out of my office and wander into any of the adjacent rooms to discuss an idea or some new paper or preprint, or just to indulge in some light-hearted banter. It was in this kind of informal atmosphere that I came to know Rajasekaran. There were two things that struck me about him. First, he seemed to be well-versed in all of the theoretical methods used in particle physics in those days: field theory, dispersion relations, current algebra, symmetry principles. At the same time, he was remarkably open to unconventional and speculative ideas, and a great person to discuss with.

My own interests in those years were in the decays of the K meson and especially in the phenomenon of CP violation. I was convinced that this remarkable system held further secrets that would come to light if one explored its rare decay modes. So I immersed myself in a study of rare decay channels such as $K \to l\bar{l}$, $K \to \gamma\gamma$ etc., analysing the ways in which discrete symmetries could be tested.

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In the beginning of 1971, it became clear that my term in TIFR was coming to an end, and that I had to move on. I had been in touch with a couple of universities in India but had received no clear signals. So I decided to apply for a post-doctoral position abroad. Now, 1971 was a disastrous year for people looking for positions in the U.S.A.. In the aftermath of the Vietnam war, funds in American universities had suddenly disappeared, and the number of openings had declined precipitously. Keeping this in view, I decided to inquire in a few places in Europe as well. Browsing among papers in the TIFR library one day, I came across some lecture notes on K mesons and CP violation written by an experimentalist by the name Helmut Faissner in a place called Aachen in Germany. These notes were written in a lively style, and the author appeared both knowledgeable and enthusiastic about the subject. I had also noticed the name Aachen on some experimental papers coming out of CERN. I addressed an application to Faissner. The response was immediate and positive. Yes, they had a position, and I could come for two years. A longer stay might be possible and even desirable. They would endeavour to pay my travel expenses. It was a generous and friendly offer, and I accepted.

Two interesting things happened during that summer of 1971. An experiment at Berkeley had looked for the decay $K_L \to 2\mu$ and had announced an upper limit that violated a lower bound expected from unitarity [1]. This bound had been calculated by me as part of my Ph.D. thesis at Carnegie-Mellon, and I had written a paper on the subject in my first year at TIFR. The Berkeley result was causing considerable consternation, and reinforced my belief that research in K-decays was a worthwhile pursuit. A second thing that happened shortly before I was to leave for Germany was the announcement by Rajasekaran of a series of lectures on "Yang-Mills Fields and Theory of Weak Interactions". I had no idea of this subject, but knowing Rajasekaran's pedagogical ability, I looked forward to hearing him. Unfortunately, I was so bogged down in the formalities of my trip (air-tickets, visas, income-tax clearance and so on) that I could not attend those lectures. Just before leaving TIFR, however, I acquired a copy of the lecture notes [2] and took it along to Germany.

On my arrival in Aachen in September 1971, I found an environment very different from that in TIFR. My host Institute (the III. Physikalisches Institut of the Rheinisch-Westfälische Technische Hochschule) was located in a building (Fig. 3.2) that was far less imposing than the Tata Institute. It was actually an abandoned Philips factory, converted into an Institute of Physics. Unlike the theory group in Bombay, the Institute in Aachen consisted almost entirely of experimentalists, including people like H. Faissner, K. Schultze, J. von Krogh, J. Morfin, H. Reithler, E. Radermacher, K. Eggert, A. Böhm, F. Hasert, H. Weerts and D. Lanske. Finally, much to my discomfiture, everybody spoke German, and I could not follow what was being said.

The situation was not so desperate, however. It turned out that Faissner himself spoke an elegant and fluent English, complete with touches of humour. He was a flamboyant, dynamic figure, with a passion for physics, and he took me into confidence immediately. It became clear that my role in the Institute was to provide theoretical support and advice to the experimental group. I also discovered, to my pleasure, that the younger members of the team, when questioned about what they were doing, were glad to switch to English. Indeed they seemed flattered that a theorist showed interest in their work, and I was able to establish a friendly and fruitful communication with them.

During my first months in Aachen, I plunged into a study of rare K-decays, motivated partly by the $K_L \to 2\mu$ puzzle, and produced a couple of papers. However, some time in the beginning of 1972, Faissner met me in the corridor and said, "You know, we appreciate the work you are doing on K mesons, but it looks as if in the near future all of our Institute's energies are going to be devoted to neutrino physics. How would it be if you got interested in neutrinos?"

Following this conversation, I started to read a couple of review articles on neutrino physics. At about the same time, I noticed a poster announcing the Neutrino '72 Conference in Balatonfüred, Hungary, to take place in June. A glance at the programme showed that they were also planning to discuss the $K_L \to 2\mu$ problem. I decided to go there.

The Neutrino '72 Conference was the very first conference I attended and turned out to be an important event in my scientific life. There was a whole galaxy of famous people there - R. Feynman, T. D. Lee, R. Marshak, V. Weisskopf, B. Pontecorvo, Y. Zeldovich, V. Gribov, V. Telegdi, F. Reines, R. Davis, J. Bahcall, B. Barish, D. Cline, C. Baltay and many others (Fig. 3.3). I listened to all the talks with great interest. But the person who made the strongest impression on me was Feynman. He gave two wonderful lectures titled "What Neutrinos can tell us about Partons", and after listening to them I had the exhilarating sense of having understood everything. (That was an illusion , of course. Feynman was a magician, and magicians can create illusions.)

I returned to Aachen in an elated state, eager to start working on a neutrino problem. That summer of 1972, there was a strange rumble going through the world of particle physics. Someone came from Hamburg and said he had heard about an important breakthrough in the theory of weak interactions, triggered by a graduate student in Utrecht by the name Gerardus 't Hooft. (Utrecht is across the border from Aachen, a hundered miles away.) Some experimentalists returning from CERN said there was a sudden interest in looking for neutral currents in the Gargamelle experiment, and mentioned a model due to Weinberg. At that point I remembered the lecture notes of Rajasekaran that I had brought along from Bombay. Reading these notes, I found a lucid account of what is now called the gauge principle, including a discussion of Weinberg's paper of 1967. (Remarkably, Rajaji's lectures were given *before* the appearance of 't Hooft's work.) With these lectures as a guide, I spent the summer reading the original papers of Glashow, Salam, Weinberg and Higgs as well as the "charm" paper of Glashow, Iliopoulos and Maiani. I was able to distil from these papers the essential physical idea of the new theories, especially their prediction for neutral currents.

As soon as the winter semester began in October 1972, I announced a series of four lectures under the title "Unified Theories of Weak and Electromagnetic Interactions". They were intended mainly for experimentalists in Faissner's team working in the Gargamelle neutrino experiment. A group of about 15 people gathered in a small room and listened with rapt attention as I went through the basic ideas of gauge invariance, spontaneous symmetry breaking, Higgs mechanism, and the Weinberg model, concluding with the prediction of neutral currents. One could sense a tension in the atmosphere as these lectures progressed, the mood wavering between hope and optimism on one side, doubt and scepticism on the other.

Six weeks after these lectures, Jorge Morfin came into my room holding a photograph (Fig. 3.4) and said, "I think we have a Weinberg event". He wanted to show me this picture before he rushed off to an urgent meeting of the group called by Faissner. The photograph showed the track of a single electron close to the forward direction, bending in the magnetic field of the detector and producing a characteristic shower. The energy of the electron was $E \sim 300\,\mathrm{MeV}$, and its angle $\theta \sim 2^\circ$. The product $E\theta^2$ satisfied the condition $E\theta^2 < 2m_e$, the kinematical requirement for neutrino-electron scattering. This "Aachen event" was the first observation of the neutral current process $\overline{\nu}_\mu + e^- \to \overline{\nu}_\mu + e^-$.

As soon as the meeting of the experimentalists was over, Faissner came to my room and said, "I want you to write up those lectures immediately, and I want every member of the group to have a copy in his hands.". And so, shortly before the Christmas break, I started to formulate my lectures, and early in the new year produced a report of about 30 pages [3]. Since the report was meant only for internal distribution, just 50 copies were made. I sent a handful of copies to the principal libraries such as CERN, SLAC and NAL. A few weeks later, I received a telex from the CERN librarian, which said: "Could you please send us 85 additional copies of your report ..."

During that spring of 1973, there was mounting excitement that CERN-Gargamelle might be on the verge of announcing neutral currents. The Aachen event was, at the time, the only candidate for $\overline{\nu}_\mu + e^- \to \overline{\nu}_\mu + e^-$, a process with a tiny cross section. On the other hand semileptonic events of the type $\nu_\mu + \mathcal{N} \to \nu_\mu +$ hadrons were expected to be more abundant, even though distinguishing them from backround was more difficult. I saw here an opportunity to apply the parton model ideas I had learnt from Feynman's lectures at Balaton. I calculated the neutral and charged current cross sections and obtained predictions for the ratios $R = \sigma(\nu \to \nu)/\sigma(\nu \to \mu^-)$ and $\overline{R} = \sigma(\overline{\nu} \to \overline{\nu})/\sigma(\overline{\nu} \to \mu^+)$. The model was sufficiently detailed that one could allow for effects of experimental cuts, neutrino energy spectra, etc.. These calculations were reported in a paper submitted to Nuclear Physics B in June 1973 [4].

A few weeks later, at summer conferences in Bonn and Aix-en-Provence, the Gargamelle collaboration announced their discovery of semileptonic neutral currents. A comparison of their data with my calculations showed that the results were compatible with the Weinberg model for $\sin^2\theta \sim 0.3$.

It was clear at that moment that a new era in particle physics was opening up, and that I was privileged to be a witness. I also sensed that neutrino physics was now going to be my principal domain of activity, and that I might end up staying in Aachen much longer than I had foreseen. And when I think back to those days, I also think of the lecture notes of Rajaji that I carried along on my journey from Bombay to a destination called Aachen, that was destined to become my home away from home.

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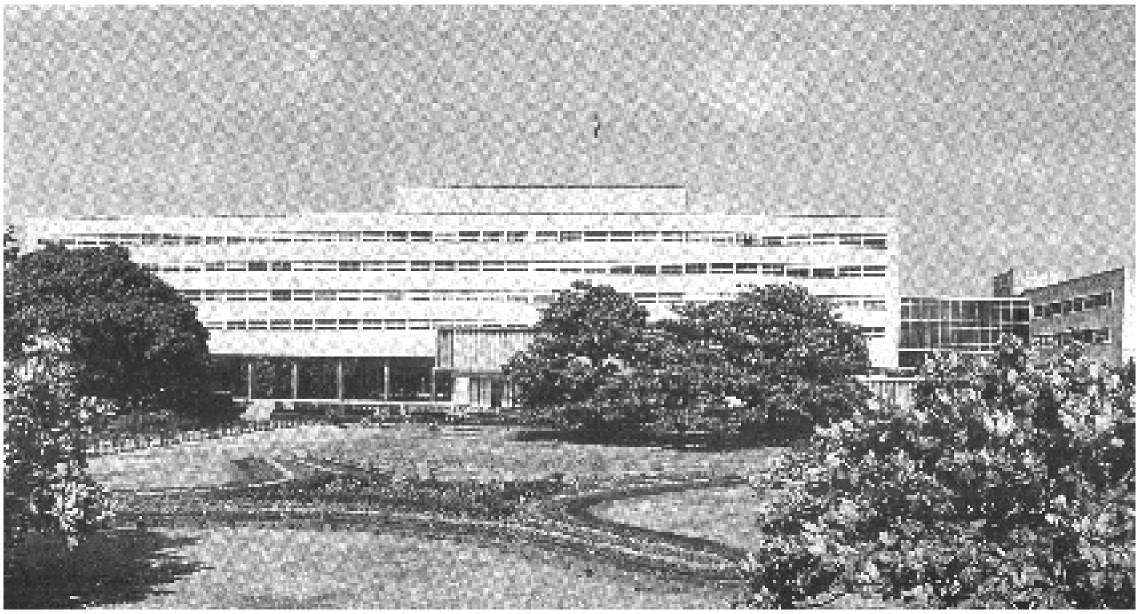

Figure 3.1: View of the Tata Institute of Fundamental Research, Bombay.

Figure 3.2: View of the III. Physikalisches Institut, RWTH Aachen, as it looked on my arrival in September 1971.

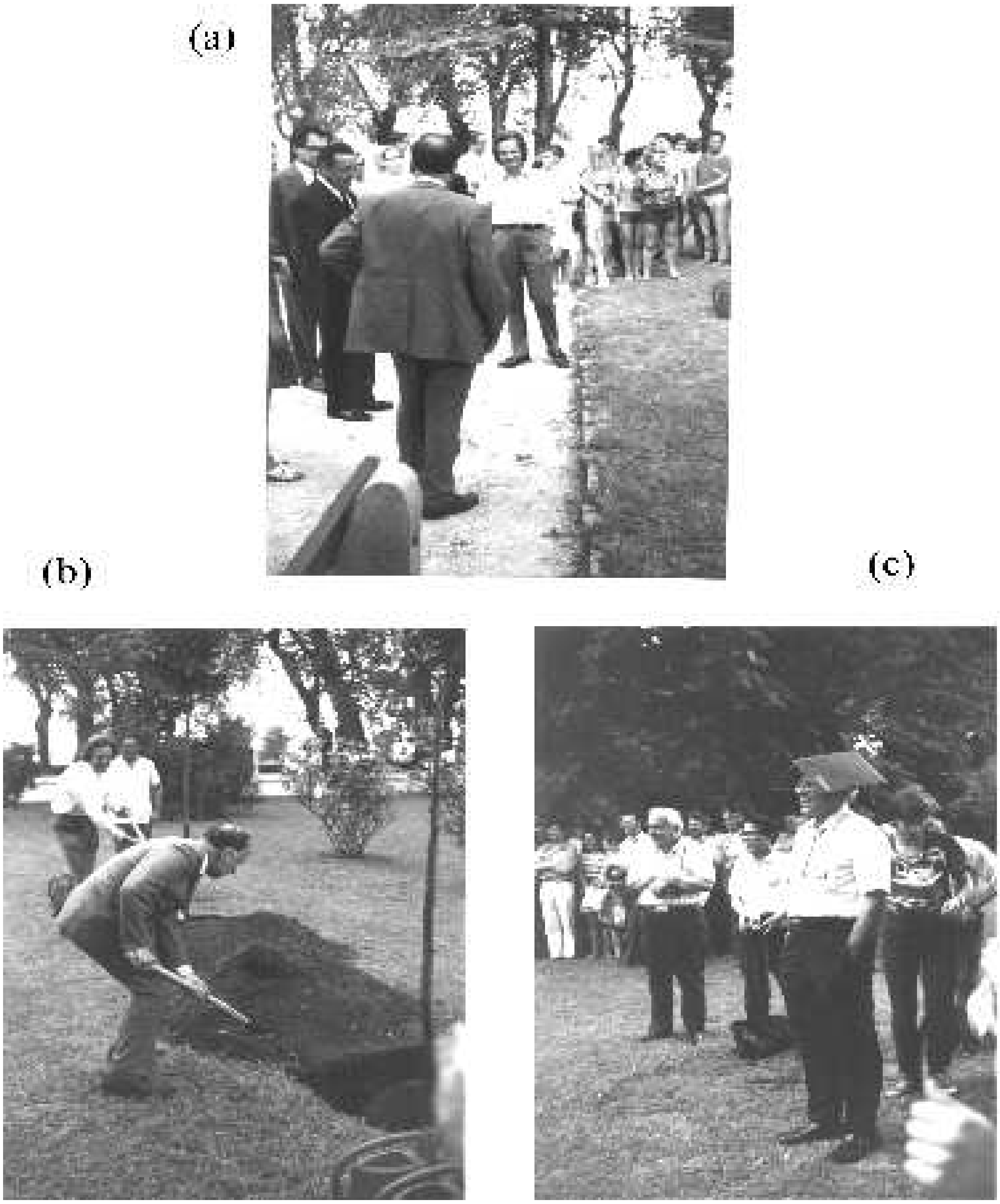

Figure 3.3: Tree-planting ceremony in the Tagore gardens of Balatonfüred (pictures taken during Neutrino '72 Conference). (a) and (b) show Feynman and Pontecorvo, (c) shows Reines and Cowan.

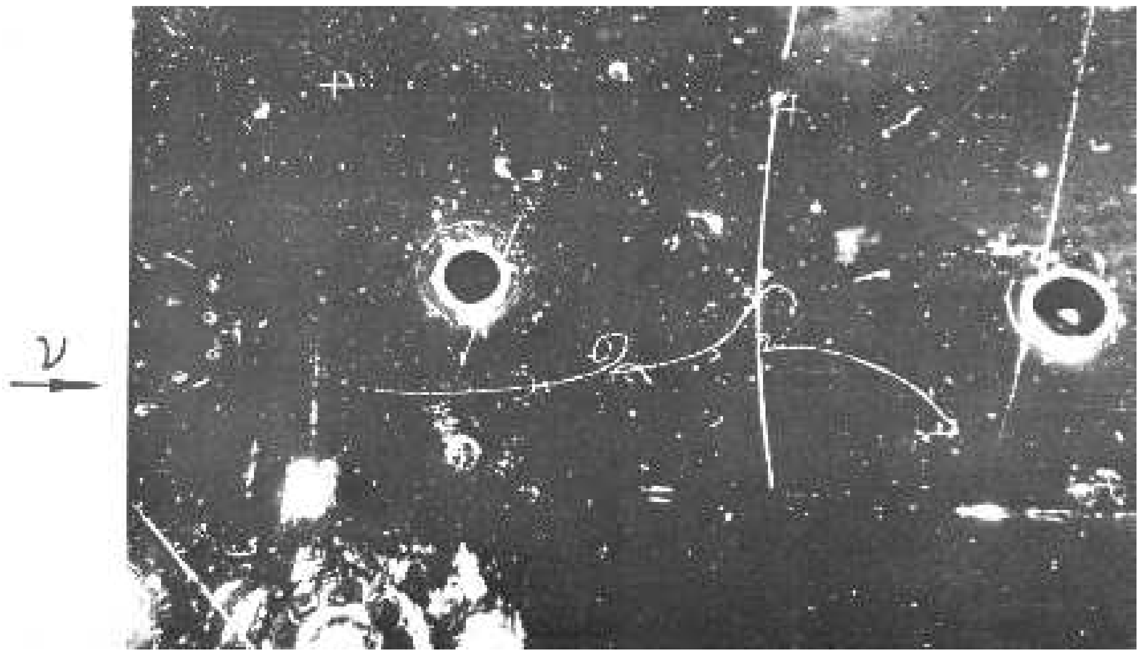

Figure 3.4: The "Aachen event" found shortly before Christmas 1972. This was the first observation of the reaction $\overline{\nu}_\mu + e^- \to \overline{\nu}_\mu + e^-$.

Chapter 4

Composite Fields and Fermion Masses (Variations on the Standard Model)

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A singular feature of the standard model is the way it treats the right handed fermions $\{f_R\}$. All of them transform trivially under SU(2): there are no R-fermion currents coupling to the SU(2) gauge fields. This was of course done to make certain that the charged currents remained L-chiral. The two main functions served by $\{f_R\}$, namely fermion mass generation and ensuring that the strong and the electromagnetic currents become pure V, are not critically dependent on all f_R being SU(2)-trivial. One is then provoked to ask:

Q1. If all fermions were massless, would they still have R-projections ?

A non-zero femion mass is itself a manifestation of gauge symmetry breaking *via* Yukawa couplings to the Higgs fields H with $\langle H^0 \rangle \neq 0$. So we can be more daring and sharpen Q1 to

Q2. Is it possible that $\{f_R\}$ themselves, not just m_f, are generated by gauge symmetry breaking ?

Let us call the standard gauge model, with all $\{f_R\}$ banished, strongly chiral-symmetric as compared to the corresponding weakly chiral-symmetric version in which f_R are present but $m_f = 0$. Then Q2 asks really whether the violation of gauge invariance and the breaking of strong chiral invariance have the same origin.

Related to this is a more specific question highlighted by the discovery of the depletion of the atmospheric neutrino flux. The interpretation of this effect as due to neutrino family mixings/oscillations shows that at least one of the neutrinos is massive, with $m_\nu \geq 10^{-1}$ eV. Given also the upper limit on the mass of the neutrinos emited in β -decay, $m_\nu \leq$ 2–3 eV, one can ask

Q3. Why is the mass of at least one neutrino so small ?

I would like to stress that this question is meaningful in the framework of the standard model in that introducing (SU(2) singlet) $\{\nu_R\}$ into the model changes none of the currents (they couple to no gauge field at all) but can give arbitrary masses to all neutrinos as the Higgs couplings can be arbitrarily chosen.

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A popular way of accommmodating (some) small ν masses is to assume that neutrinos are Majorana fermions, with the see-saw mechanism at work. What if they turn out to be Dirac fermions?

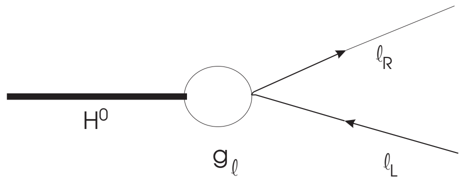
I address these questions here by suggesting some speculative variations on the standard model, restricted only to leptons and of one family (moving from the mildly speculative to the wildly speculative as we proceed), which leave the established low energy properties of the model unchanged. Some of these ideas arose in discussions with Rajasekaran [1] and so it is doubly apropriate to talk about them in this gettogether honouring him, appended to a meeting on neutrino physics.

The starting point is our poor understanding of the dynamics of (unbroken) nonabelian gauge theory at low energy scales μ due to the growth of the effective gauge coupling as $\mu \to 0$. We suppose, dealing with Q3 first, that the gauge dynamics allows the formation of a scalar composite field having the transformation properties of the Higgs fields H. The quantum numbers allow this and no specific assumption about the constituents of H need be made. It is then a trivial but very general remark that the absence of a ν_R current forbids exactly any Yukawa coupling of ν to H since ν_R couples to none of the possible constituents of H, and hence that $m_\nu = 0$.

In all cases of occurrence of unusually small values of physical parameters, a natural first step is to identify a limit in which they vanish. In the neutrino mass problem, we have found such a limit: the limit of the exact standard model with composite Higgs. Note that the charged lepton masses are nonzero (and in principle computable) in this limit ("allowed"),

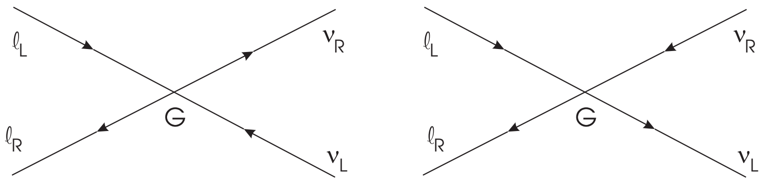
$$m_\ell \approx g_\ell \langle H^0 \rangle \ ,$$

via the effective Yukawa vertex,



where g_ℓ is an effective coupling constant determined dynamically by the vertex which has a form factor with a characteristic length scale $\Lambda \approx 100$ GeV, the only scale available.

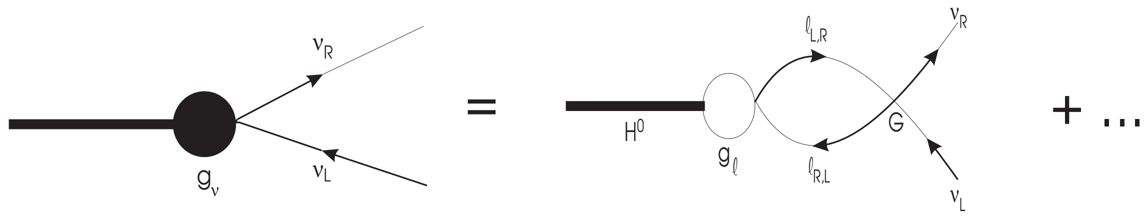
The only way now to give the neutrino a nonzero mass is to invoke forces beyond the standard model. At this stage it is not very illuminating to construct specific models for doing so, though it can be done readily enough [1]. Instead, parametrise the nonstandard effects by effective 4-fermion vertices $\overline{\nu}_R \nu_L \overline{\ell}_R \ell_L$ and $\overline{\nu}_L \nu_R \overline{\ell}_R \ell_L$:



They generate an effective $H^0 \overline{\nu}_R \nu_L$ vertex by,

To the lowest order in G we get,

$$g_\nu \approx g_L G \int^\Lambda \frac{\mathrm{d}^4 p}{(\not{p})^2} \approx g_\ell G \, \Lambda^2 \ ,$$



generating a "first-forbidden" mass,

$$m_\nu \approx m_\ell G\,\Lambda^2 \ ,$$

for the neutrino. (There can also be an order G correction to m_ℓ). We can use this formula to estimate G and hence the energy scale M at which nonstandard physics becomes operative:

$$M \approx G^{-\frac{1}{2}} \approx 100\sqrt{m_\ell/m_\nu}\ \text{GeV} \ .$$

A lower (upper) bound on m_ν results in an upper (lower) bound on M. If $m_\nu \geq 10^{-1}$ eV, then $M \leq 10^5$–10^6 GeV and if m_ν is bounded above by an eV or so, then $M \geq 10^4$–10^5 GeV (these estimates have to be adjusted for variations of m_ν across families), squeezing the range of M very tightly indeed. This is quite a dramatic result; the ratio of the scales characterising standard and nonstandard physics is bounded roughly as

$$100 \leq M/\Lambda \leq 10{,}000 \ .$$

There is no *a priori* reason why m_ℓ and m_ν should be so finely correlated as to make it come out so right. Indeed if the wide mass gap between ν and ℓ were to begin to close, the separation between the standard and the beyond-the-standard regimes will rapidly blur. It is definitely possible to argue from this alone that the idea of the composite Higgs field is a persuasive one.

I now turn to the question Q2, briefly. As a first step, let us reverse the logic of the first part of this talk and suppose that the Higgs fields are elementary; can f_R then be generated dynamically? The simplest option [2] is to build f_R as composites of f_L and H, bound by the low mass scale guage dynamics. Specifically, take

$$f_R \approx \sigma_\mu f_L \partial_\mu H \ ,$$

where $\sigma_0 = 1$ and σ_1, σ_2, σ_3 are Pauli matrices, ignoring all the refinements required to make sense of the right side as a local quantum field. The helicity is right by construction. Under SU(2), f_R transforms as $\underline{2} \otimes \underline{2} = \underline{3} \oplus \underline{1}$; so it is necessary to choose $\underline{1} \approx \nu_L H^0 - \ell_L H^+$ as ν_R and to assume that the $\underline{3}$ does not bind. Moreover, we can bind f_L and the conjugate Higgs H^* to produce $\ell_R \approx \nu_L H^{*-} - \ell_L H^{*0}$. The electric charges come out just right. There are no f_R currents coupling to $W^\pm$ and the electromagnetic current is guaranteed pure V.

At this book-keeping level, the idea can be extended quite naturally to quarks (with pure V colour currents) and to any number of families [2].

We have thus two partial but complementary approaches to understanding deviations from full symmetry, namely gauge + chiral, of the unbroken standard model: starting with elementary Higgs fields transforming as $\underline{2}$ under SU(2), we get a qualitatively correct picture of f_R as $f_L H$ composites and, starting with elementary f_R (and a composite H), we have a semi-quantitative picture of neutrinos, $m_\nu = 0$. The really interesting question now is whether they can be combined. Very naively, can we demand that the composite structures $H \sim \overline{f}_L f_R$ (or $\overline{f}_R f_L$) and $f_R \sim \sigma_\mu f_L \partial_\mu H$ hold simultaneously? Clearly, the quantum number book-keeping will remain valid. For instance, the neutrino will remain massless even if f_R is dynamically generated as above since there is no ν_R current. To make dynamical sense, however, we have to do the dynamics. A possible approach is to start with a strongly chiral SU(2) $\times$ U(1) gauge theory and to generate both f_R and H as manifestations of dynamical symmetry breaking *a la* Nambu-Jona-Lasinio. Appropriate non-perturbative amplitudes should then signal both a mass gap in the fermion propagator and scalar "collective modes" breaking gauge invariance. This is of course a nontrivial task. But the reward for success will be to rid the model, at least in principle, of all parameters except the gauge couplings and the symmetry breaking scale.

The hospitality of the Institute of Mathematical Sciences, Chennai, where this work was done and of the School of Physics of the University of Hyderabad, where it was written up, is gratefully acknowledged.

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Chapter 5

Proof of absence of spooky action at a distance in quantum correlations

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Preface: Tribute to a teacher

I first met Rajaji during my undergraduate days, at a summer training program at the Indian Institute of Technology, Madras. I had come from a college in Kerala, and was being overwhelmed by the ambience - I was meeting live researchers for the first time. Then an 'examiner' came to check whether we were doing fine with our projects. He did not look like an academic. In fact he did look like an investigating officer. We were a bit worried. But the first interaction changed all that. The softness, the concern of a real teacher, and the friendliness. I am really happy to be able to talk about some work that has caught his attention, and has benefitted from his encouragement.

Quantum correlations

A physical flaw in local realism

The present interpretations of quantum measurement of entangled multiparticle systems invoke and support the notion of nonlocal state reduction by an unknown, spooky, action at a distance. There are two reasons for this. The standard quantum mechanical description of multi-particle correlations uses the inseparable entangled wavefunction that describes the total state of all the particles in a single entity. Then a measurement on one of the particles instantaneously changes the entire wavefunction and changes the states of the other correlated particles. The second reason is the failure of the local hidden variable theories to reproduce the experimentally observed quantum correlations, and this failure is described by the Bell's theorem [1]. There has been a universal acceptance that this is due to nonlocality, rather than just due to the inadequacy of the notion of the physical reality used.

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However, decades of debates and contemplation have not brought out the physical nature of the nonlocal influences that make the quantum correlations possible. Quantum nonlocality even violates the spirit of relativity, though not its signalling features. This suggests that perhaps some deep and fundamental aspect is overlooked. In fact, a closer look at the Einstein-Podolsky-Rosen (EPR) argument [2] and the proof of the Bell's theorem reveals a fundamental physical flaw in the way the concept of physical reality is used. This is clearly evident in the proof of the Bell's theorem [1], as the inability of the formalism to account for any possible phases coherences at source encoding the *prior* correlations. In effect, the correlation function used in Bell's theorem as well as in the standard local realistic theories completely ignores the wave nature associated with microscopic particles.

The two distinct assumptions that go into the Bell's theorem are that of locality and the EPR physical reality [1]. Consider the measurement of the spin components on the two particles in two different directions **a** and **b**. The result of such a measurement is two-valued in any direction. If A and B denote the outcomes $+1$ or -1, and **a** and **b** denote the settings of the analyzer or the measurement apparatus for the first particle and the second particle respectively, then the statement of locality is that

$$A(\mathbf{a},\mathbf{h}_1)=\pm 1, \quad B(\mathbf{b},\mathbf{h}_2)=\pm 1 \tag{5.1}$$

The outcome for the first particle is decided by the local setting **a** for the first analyzer, and by some hidden variable set $\mathbf{h}_1$. For the second particle the outcome is decided by the local setting **b** and local hidden variables $\mathbf{h}_2$. The correlation of the *outcomes* is

$$P(\mathbf{a},\mathbf{b})=\frac{1}{N}\sum(A_iB_i) \tag{5.2}$$

The task of the theory is to calculate this function starting from suitable basic ingredients of the theory. Bell chose to calculate this correlation by multiplying $A(\mathbf{a},\mathbf{h}_1)$ and $B(\mathbf{b},\mathbf{h}_2)$ and integrating over the distribution of the hidden variable $\mathbf{h}$, since the assumption of realism demanded that the outcomes were "there" even before the measurement was made. The Bell correlation is then $\int d\mathbf{h}\rho(\mathbf{h})A(\mathbf{a},\mathbf{h}_1)B(\mathbf{b},\mathbf{h}_2)$, where $\int d\mathbf{h}\rho(\mathbf{h})=1$. The essence of Bell's theorem is that the function $P(\mathbf{a},\mathbf{b})$ has distinctly different dependences on the relative angle between the analyzers for a local hidden variable description and for quantum mechanics.

The crucial aspect to note is that the Bell correlation function has no way of incorporating the concept of phase or phase coherence since it uses the measured outcomes. The essence of even single particle quantum behaviour is the inherent wave aspect and this is missed out in the local realistic formulation. Having identified such a serious physical flaw, the next question to ask is whether it is possible to reproduce the correct correlation functions if the concept of phase and phase coherence at source are incorporated in the formalism, keeping the locality assumption intact. This has been shown to possible [3, 4]. There are local theories with a definite reality for phases (a definite, though random, value for the phase before a measurement) that do not violate the Bell's theorem.

Locality and separability of probability

Most of the local realists use separability of probabilities as a criterion for locality. This is fundamentally flawed. If $P(a)$ and $P(b)$ are the local probabilities, the joint probability $P(a,b)=P(a)\cdot P(b)$, if they are separable. The joint probability for measurements on an entangled system is not separable, and therefore local realists insist that there is nonlocality.

But separability of probability as a criterion for locality is not valid when there are wavelike aspects, even when there is locality. The well known example is that of the Hanbury Brown-Twiss correlations [5]. The intensity of light from a source is measured by two detectors and correlated. The average of the intensity-intensity correlation (joint probability) is not the product of average intensities in each detector (local probabilities)! There is an interference pattern as a function of the separation between the two detectors. There is no nonlocality. In fact, the effect is present classically. The Brown-Twiss correlations do not violate the Bell's inequality since classical wave aspects are involved, but this feature changes when one goes to the complex 'wave' aspect of quantum systems. But already at the level of classical waves, it is evident that separability is not a good criterion for locality [4].

The correct correlation from locality and phase coherence

To illustrate the main aspects, let us consider the maximally entangled state

$$\Psi_S = \frac{1}{\sqrt{2}}\{|1,-1\rangle - |-1,1\rangle\} \tag{5.3}$$

where the state $|1,-1\rangle$ is short form for $|1\rangle_A|-1\rangle_B$, and represents an eigenvalue of $+1$ for the first particle and -1 for the second particle *if measured* in any particular direction. Ψ_S is inherently nonlocal.

An experiment in which each particle is analyzed by orienting the analyzers at various angles θ_1 and θ_2 is considered next. The correlation $P(\mathbf{a},\mathbf{b}) = \frac{1}{N}\sum(A_iB_i)$ denotes the average of the quantity (*number of detections in coincidence* $-$ *number of detections in anticoincidence*), where 'coincidence' denotes both particles showing same outcome and 'anticoincidence' denotes those with opposite outcomes.

We postulate that the correlation at source (conservation law) is encoded in the relative value, or the difference, of two internal variables ϕ_1 and ϕ_2 associated with the two particles. The value of ϕ can vary from particle to particle, being random, but the relative phase between the two particles in all correlated pairs is constant. Denoting $\theta_1 - \phi_1 = \theta$ and $\theta_2 - \phi_2 = \theta'$, we have $\theta - \theta' = \theta_1 - \theta_2 + \phi_o$.

We specify the *local probability amplitudes as a complex number, whose square gives the probability of transmission.* These are $C_A = \frac{1}{\sqrt{2}}\exp(i\theta s)$ for measurements at analyzer A, and for particle B is $C_B = \frac{1}{\sqrt{2}}\exp(i\theta' s)$ at analyzer B. In these expressions, the quantity s is the spin (in units of $\hbar$) of the particle. The locality assumption is strictly enforced since the two complex functions depend only on local variables and on an internal variable determined at source and then individually carried by the particles without any subsequent communication of any sort.

The probabilities for the outcomes of measurements at each end are now correctly reproduced, for *any angle of orientation.* These probabilities are $\mathrm{Re}(C_1C_1^*) = \mathrm{Re}(C_2C_2^*) = \frac{1}{2}$. The amplitude correlation for an outcome of either $(++)$ or $(--)$ of two maximally entangled particles is

$$\begin{aligned} U(\theta,\theta') &= 2\mathrm{Re}(C_1C_2^*) = \mathrm{Re}\{\exp \mathrm{i}s(\theta-\theta')\} \\ &= \cos\{s(\theta-\theta')\} = \cos\{s(\theta_1-\theta_2) + s\phi_o\}. \end{aligned} \tag{5.4}$$

This, in contrast to the Bell correlation, carries the crucial wave aspects.

We rewrite this as $U(\theta_1, \theta_2, \phi_o)$ since all references to the individual values of the 'hidden variable' ϕ has dropped out. The square of $U(\theta_1, \theta_2, \phi_o)$ is the probability for coincidence detection of the two particles through the analyzers kept at angles θ_1 and θ_2.

Since $U^2(\theta_1, \theta_2, \phi_o)$ is the probability for a coincidence detection ($++$ or $--$), the quantity $(1 - U^2(\theta_1, \theta_2, \phi_o))$ is the probability for an anticoincidence (events of the type $+-$ and $-+$). Then

$$P(\mathbf{a}, \mathbf{b}) = U^2(\theta_1, \theta_2, \phi_o) - (1 - U^2(\theta_1, \theta_2, \phi_o)) = 2U^2(\theta_1, \theta_2, \phi_o) - 1 \tag{5.5}$$

For the case of the singlet state breaking up into two spin 1/2 particles, we set $\phi_o = \pi$. Then the probability for joint detection through two Stern-Gerlach analyzers oriented at relative angle $\theta_1 - \theta_2$ is

$$U^2(\theta_1, \theta_2, \phi_o) = \sin^2(\frac{1}{2}(\theta_1 - \theta_2)) \tag{5.6}$$

$$P(\mathbf{a}, \mathbf{b}) = -\cos(\theta_1 - \theta_2) = -\mathbf{a} \cdot \mathbf{b} \tag{5.7}$$

This is identical to the quantum mechanical predictions obtained from the singlet entangled wave-function and the Pauli spin operators.

Similar exercise can be carried out for other entangled states. We have reproduced the correct quantum correlations in a local theory [3]. The most striking implication is that there is no spooky state reduction at a distance.

Proof of absence of spooky action at a distance

Now we point out the proof from experiments that there is no state reduction at a distance due to partial measurements on entangled multiparticle systems. A state reduction in quantum systems is characterized by two essential aspects. a) The state assumes a definite value for an observable or a set of observables. b) The dispersion in any noncommuting observable change so as to satisfy the uncertainty principle. In classical systems the second aspect is irrelevant. Clearly, if a measurement on one particle reduces the state of the companion particle then the dispersion associated with the noncommuting observable should also change as a result of the measurement [3, 4]. For example, if the position of the first particle is measured to be x_1 with spread Δx_1, the second particle's position can be predicted with good certainty using the conservation law. If this amounts to a true state reduction to a definite position x_2 with uncertainty Δx_2, then its momentum should spread out to satisfy $\Delta p_2 \geq \hbar/\Delta x_2$. This does not happen, since if it did, signal locality could be violated – we could send signals faster than light. This becomes possible since $\Delta x_1 \sim \Delta x_2$ can be made as small as one wishes and then Δp_2 will become larger than any initial spread of the original state. This feature is not observable in experiments with spin, since it is a bounded variable. Remarkably, K. Popper had proposed [6] such an experiment, two years before the EPR argument. Recently this experiment was performed [7] and, consistent with our assertion based on signal locality, no additional spread of momentum of the second particle was seen. This clearly proves that there is no state reduction at distance. It also shows that terms like quantum teleportation are inappropriate. What is seen in those experiments are prior correlations encoded in the initial phase coherence.

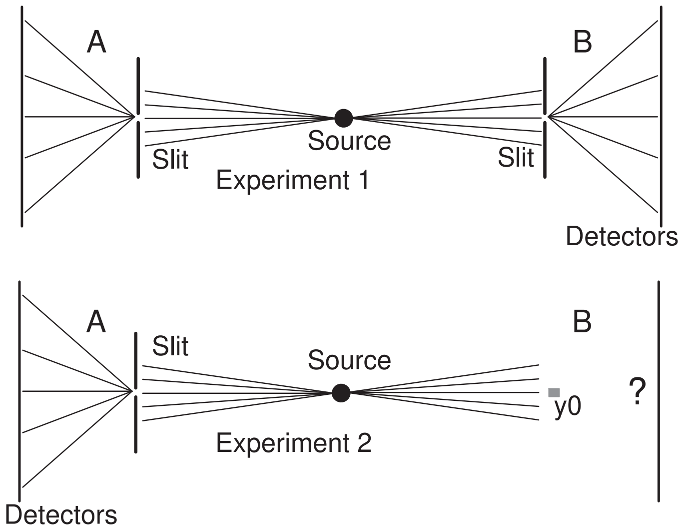

Figure 5.1: Scheme of Popper's experiment. The upper figure is a measurement of position (localization within a slit) on both particles of the entangled system. The dispersion after the state reduction obeys the uncertainty principle. The lower figure depicts measurement on one of the particles. If the second particle 'localizes' due to the measurement on first, its momentum should disperse exactly as in the upper figure. This does not happen.

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Chapter 6

Are Accelerators Needed for High-Energy Physics?

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Introduction

When the organizers of the Rajaji Seminar invited to me speak here, I accepted at once with pleasure, but also with some bemusement. I couldn't figure out why I was being invited. I am not a student of Prof. Rajasekaran's, I am not a 'grand-student' of Prof. Rajasekaran's, I am not a particle theorist, I am not a high-energy physicist, and I am not even that low-energy approximation, a nuclear physicist. The only explanation I could think of was that there had been some kind of mistake. Perhaps the organizers wanted to invite a Srinivasan, or a Krishnan or a Gopalan, and their computer found my name in its database! A couple of months passed without any communication from the organizers, and that seemed to lend credibility to my theory. But one day I received an email asking for the title of my talk. So, after sending in the most provocative title I could think of, I recalled my interaction with Prof. Rajasekaran.

Over the last five years, I have been running at CAT a DST-funded winter school on the Physics of Beams. The purpose of this School is to attract bright students to this new, and in India unknown, field. In this context I was told that Prof. Rajasekaran was someone who had been championing the need to build new accelerators at higher energies. His point is that if new experiments, at higher energies, are not forthcoming, then high-energy physics is in the danger of becoming sterile. I had of course heard of Prof. Rajasekaran, though I had never met him, so I sent him a letter asking if he would speak at the School on this perspective, and he graciously agreed. At the School he gave an extremely lucid and enjoyable overview of particle physics for the students. At the end of his talk he strongly emphasised the importance of looking for new techniques of acceleration, that will allow us to do experiments at much higher energies than are possible today: beyond, say, 10 TeV. This endorsement of accelerator physics by a leading particle theorist had a deep impact on the students, and many of them expressed interest in the field, with a couple of them coming

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to CAT in the summer to do projects in beam physics. Prof. Rajasekaran was also kind enough to come again a few years later, and give a similar talk at the Fourth School.

The theme of my talk today will be along the lines delineated above: ideas for new acceleration techniques, and a discussion on the need of accelerators for high-energy physics.

Present methods of acceleration

DC accelerators

The simplest way to accelerate a charged particle is to 'drop' it in an electrostatic potential. This is the principle of DC accelerators, such as van de Graff accelerators. The limitation of this kind of accelerator is that it is a 'single-push' device; since electrostatics is conservative, once the accelerated particle comes out of the accelerator, you cannot take it back to the 'top of the hill' for reacceleration (without losing all the energy gained). Therefore, DC accelerators are limited by practical issues such as the maximum voltage that can be sustained before breakdown. Typically this is around 20 MV, which means that you cannot accelerate electrons or protons to more than around 20 MeV by this technique.

RF accelerators

In order to accelerate charged particles to beyond around 20 MeV, one needs to provide multiple 'pushes', and for this one needs a non-conservative or time-varying field, i.e. an electromagnetic field. The vast majority of accelerators today therefore use the principle of *RF acceleration*. Here, electromagnetic radiation, typically in the radio-frequency (RF) range, fills a cylindrical cavity, which supports a TM mode (that has an axial electric field). Since a cylindrical cavity only supports modes with a phase velocity greater than c, it is necessary to slow down the electromagnetic wave by periodically loading the structure. A charged particle beam injected along the axis of this loaded cylinder will now be accelerated. Because of the time-varying nature of the fields, it is possible to accelerate particles repeatedly, either in the same RF cavity (circular accelerator), or in successive ones (linear accelerator).

The size of an accelerator depends on the accelerating gradient produced in the RF cavity. For normal conducting cavities, that are made of copper and operate at room temperature, typical accelerating gradients that one can achieve are of the order of 10 MV/m. With this gradient, for example, a 1 TeV linear collider would need to be 100 km long!

Therefore, for very high-energy accelerators, people today use superconducting RF cavities. These are typically made of Niobium, and operate at 4.2 K or 1.8 K. Because the cavities are superconducting, there is very little power dissipation in the cavities, and therefore they are very efficient. One can also achieve higher gradients in these cavities; typically of the order of 100 MV/m (or actually many tens). For example, the TESLA collider being built in Germany is designed for a beam energy of 1 TeV, and will be 30 km long.

The above numbers make one thing very clear. RF acceleration can produce gradients of no more than 100 MV/m, and with this gradient one cannot cross the 10 TeV frontier; even a multi-TeV collider will be many tens of kilometres long. In order to cross the 10 TeV frontier, therefore, one needs to look at new methods of acceleration, that can provide higher, much higher, gradients.

Plasma-based Accelerators

The basic idea of plasma acceleration was first proposed and studied by Tajima and Dawson in 1979 (T. Tajima and J.M. Dawson, *Phys. Rev. Lett.* **43**, 267 (1979)). Since the mid-90s, there has been an explosion in the field, with a number of very exciting experimental results.

There are different schemes of plasma acceleration, the most promising of which is Laser Wakefield Acceleration (LWFA). In this scheme, a short pulse (< 1 ps), high intensity ($> 10^{18}$ W/cm^2) laser is shot through a gaseous medium. The ponderomotive force of the laser excites a longitudinal plasma wave, with a velocity close to the speed of light. If, at the same time, one injects an electron beam into the plasma, the electron beam can be accelerated.

In a number of experiments performed around the world, many milestones have been achieved. Accelerating gradients of 100 GV/m, actual acceleration to an energy of 400 MeV, and acceleration of a charge of around 1 nC, have all been demonstrated. Of course, many problems remain. The main problem is that plasma acceleration has been demonstrated only over distances of a few mm, largely because of the diffraction of the laser over larger distances. Second, the electrons typically have a large energy-spread (almost 100%), so that the actual number of electrons at the highest energy may be small. Much work is therefore on to understand and overcome these problems. So, while the principle of plasma acceleration is well established, we are still far from building a plasma accelerator.

However, the potential is enormous. An accelerating gradient of 100 GV/m has already been demonstrated, which is *three orders of magnitude* higher than the best you can achieve through RF acceleration. Which means in a plasma accelerator one can achieve 1 GeV acceleration in 1 cm, and 10 TeV in 100 m! The 1 TeV TESLA collider could be built in 10 m (rather than 30 km). Of course, it may not be possible to sustain acceleration in a plasma accelerator for more than a few cms, but in this case one could think of having multiple acceleration modules (exactly as one has many RF cavities in a linear RF accelerator).

Plasma acceleration is a totally new and exciting way of accelerating particles. While there is a long way to go before one can actually demonstrate a plasma accelerator, there is also much reason to put in that effort. If successful, plasma acceleration would make it possible to cross the 10 TeV frontier.

Are accelerators needed for high-energy physics?

In general, the answer to this question is obviously 'yes'. Even theorists must pay at least lip service to the importance of getting theories confirmed by experiments. The question is therefore being asked in the more restricted context of high-energy physics in the country.

In India, all experimental high-energy physics is done in the form of international collaborations: at CERN, Fermilab, etc. This is obviously a good thing, because it gives us access to data from the latest experiments, enables international exposure, competitive research, etc. And the quality of this work has been very high. However, it seems to me that one can achieve greater depth and a broader base in the field only if there is experimental activity within the country. Obviously, one is not talking about exploring the energy frontier, but rather about experiments at lower energies, such as B-factories at 5 GeV and tau/charm factories at 2.5 GeV. In particular, the Chinese experience shows that interesting and competitive research can be done even at lower, multi-GeV, energies.

The obvious problem with doing high-energy experiments within the country is the lack of an accelerator: a sufficiently serious problem that completely justifies the present state of affairs. However, since the last few months, there is now working at CAT a 450 MeV electron storage ring, INDUS-1, which is by far the highest energy accelerator in the country. Therefore the ability to build a low-energy storage ring has been demonstrated. Further, work has started on building INDUS-2, a 2.5 GeV storage ring, which, in my opinion, should be working in around five years from today. Once that happens, we will have shown the ability, in essence, to build a multi-GeV collider.

In the changed scenario, where multi-GeV accelerators can be built in the country in a time frame that is not unduly long, it seems that the possibility of doing high-energy physics experiments within the country warrants re-examination.

The point I am making is a limited one: that an avenue long ignored, should now be explored. It may be that, after serious thought, the conclusion is that the time is still not ripe for an indigenous experimental high-energy physics programme; perhaps because by the time say a tau/charm factory is built, all the interesting experiments will have been done. But that conclusion should be the result of mature deliberation, and not a consequence of either inertia or a casual brush-off.

In this spirit let me offer the following points for thought:

(a) After INDUS-2 is built, say five years from now, INDUS-1 (450 MeV) will be largely redundant. Can we then convert INDUS-1 into a collider?

(b) Can one do parasitic fixed-target experiments at INDUS-2 (2.5 GeV)?

(c) Can one build a new multi-GeV electron storage-ring collider, perhaps as part of an Asian collaboration?

(d) Can one think of a CEBAF-type continuous electron beam facility for nuclear physics as well as high-power free-electron lasers?

(e) Can one think of a multi-GeV linear collider, again perhaps as part of an Asian or other collaboration?

To summarize: with the commissioning of INDUS-1, one can assert that it is now becoming possible to build a multi-GeV circular accelerator in India; therefore, this is the right time to start thinking about a possible experimental programme in high-energy physics within the country.

Finally, I would like to congratulate Prof. Rajasekaran on a long and fruitful research career, and wish him all the best for an equally active research career in the years to come.

Chapter 7

Ortho and Para Supersymmetric Quantum Mechanics of Arbitrary Order

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It is a great honour to talk at the Rajaji Symposium. Rajaji is one of the stalwarts in high energy physics in this country. He is an excellent speaker and has inspired a generation of High Energy Physicists. Apart from being a good phenomenologist, he has also done some interesting work in mathematical physics. My collaboration with him is in fact in this field.

I first saw Rajaji in 1971 when he give his famous nonabelian gauge theory lectures at SINP, Calcutta where I was a graduate student. I must confess that I did not appreciate its significance at that time. Our first strong overlap was when we were together for a month at University of McMaster, Canada in 1990. We stayed together and that is when I discovered the weakness of Rajaji for Pizza and Coke !

In early 1992 both Rajaji and myself attended a workshop at ISI, Calcutta. While I talked about Chern-Simons term and charged vortices, he talked about ortho-fermions and ortho-bosons the work which he had done with Dr.A.K. Mishra from his institute. As usual his seminar was very clear and I immediately realized that something can be done.

Let me explain why I felt so. Since 1984 I have been working in the area of supersymmetric quantum mechanics (SQM) where there is a symmetry between bosons and fermions and since 1990 I have been working about anyons and fractional statistics. Further, only few months ago, I had written a paper about para-supersymmetric quantum mechanics (PSQM) [1], where there is a symmetry between bosons and para-fermions. So I felt that one could now construct ortho supersymmetric quantum mechanics (OSQM) where there will be a symmetry between bosons and ortho-fermions. I suggested this to Rajaji and we exchanged our papers and agreed to meet the next day after reading each others papers. We had detailed discussions on the next day and we agreed to pursue this problem. The entire work including paper writing was done via e-mails. We had disagreement and fights but it is to the credit of Rajaji that he never took these physics fights personally. I am glad to note

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here that when in 1995 Cooper, Sukhatme and I wrote a Physics Reports on SQM [2], we included this important work there.

To appreciate this work, let us first explain what one means by ortho-Fermi statistics [3]. This statistics is characterized by a new exclusion principle which is more “exclusive” then the Pauli exclusion principle: an orbital state shall not contain more than one particle, whatever the spin direction. Further, the wave function is antisymmetric in spatial indices alone, with the order of the spin indices frozen.

For the special case of a single ortho-fermion of order p (which is what is required for constructing OSQM), the creation and annihilation operators c_α^+ and c_α satisfy ($\alpha, \beta = 1, 2, \ldots, p$).

$$c_\alpha c_\beta^+ + \delta_{\alpha,\beta} \sum_{r=1}^{p} c_r^+ c_r = \delta_{\alpha,\beta} \; ; \; c_\alpha c_\beta = 0 \; . \tag{7.1}$$

This equation implies that

$$c_1 c_1^+ = c_2 c_2^+ = \; c_p c_p^+ \; . \tag{7.2}$$

For comparison sake, let us notice that the fermionic operators b, b^+ satisfy

$$\{b, b^+\} = 1 \; , \quad b^2 = 0 = b^{+2} \; , \tag{7.3}$$

while the para-fermions of order p satisfy

$$[[a^+, a], a] = -2a \; , \quad [[a^+, a], a^+] = 2a^+ \; , \quad a^p = (a^+)^p = 0 \; . \tag{7.4}$$

Let us now recall how, using the above algebras (10.3) and (10.4), SQM and PSQM have been constructed. In SQM one defines super charge Q, Q^+ satisfying

$$\{Q, Q^+\} = 2H \; , \quad Q^2 = 0 = (Q^+)^2 \; , \quad [H, Q] = 0 = [H, Q^+] \; , \tag{7.5}$$

while using eq. (10.4), Rubakov and Spiridomov [4] wrote down the following PSQM of order 2

$$Q^3 = 0 = (Q^+)^3 \; , \quad [H, Q] = 0 = [H, Q^+] \; , \tag{7.6}$$

$$Q^2 Q^+ + Q Q^+ Q + Q^+ Q^2 = 4QH \; . \tag{7.7}$$

These and other authors were unable to generalize relation (10.7) to any arbitrary order p. In fact, Durand et al. [5] discussed this issue in some detail and concluded that the multi-linear part of the higher order PSQM ($p \geq 3$) cannot be characterized with one universal algebraic relation. However, subsequently I showed [1] that this is not so and that the PSQM of order p is characterized by the nontrivial relation

$$Q^p Q^+ + Q^{p-1} Q^+ Q + \ldots + Q^+ Q^p = 2p Q^{p-1} H \; , \; p = 1, 2, \ldots \; , \tag{7.8}$$

in addition to the obvious generalization of eq. (10.6). Note that unlike in SQM (and also in OSQM, as we shall see shortly), in PSQM, the Hamiltonian cannot be written in terms of supercharges alone, since the inverse of Q^{p-1} does not exist. Rajaji was unhappy about this feature and subsequently we [6] gave an alternative formulation (see below) where one is able to express H in terms of PSQM charges.

Since the ortho-Fermi operators satisfy relations (10.1) and (10.2), we suggested that OSQM should be characterized by the algebra [7]

$$Q_\alpha Q_\beta = 0 \; , \quad [H, Q_\alpha] = 0 = [H, Q_\alpha^+] \; , \tag{7.9}$$

$$Q_\alpha Q_\beta^+ + \delta_{\alpha,\beta} \sum_{r=1}^{p} Q_r^+ Q_r = 2\delta_{\alpha\beta} H \ . \tag{7.10}$$

From here we deduce that $Q_1 Q_1^+ = Q_2 Q_2^+ = \ldots = Q_p Q_p^+$.

A useful representation of SQM algebra (10.5) is given by

$$Q = \begin{pmatrix} 0 & p - iW \\ 0 & 0 \end{pmatrix} \ , \quad 2H = \begin{bmatrix} p^2 + W^2 - W' & 0 \\ 0 & p^2 + W^2 + W' \end{bmatrix} \ , \tag{7.11}$$

where W' means derivative with respect to x. Similarly a useful representation of the PSQM of order p is given by

$$(Q)_{\alpha\beta} = (P - iW_\beta)\delta_{\alpha,\beta+1} \ , (Q^+)_{\alpha\beta} = (P + iW_\beta)\delta_{\alpha+1,\beta} \tag{7.12}$$

while the corresponding H is a $(p+1) \times (p+1)$ diagonal matrix: $2H = h_r \delta_{rr}$ where

$$h_r \ = \ P^2 + W_r^2 - W_r' + C_r \ , \quad r = 1, 2, \ldots, p \tag{7.13}$$

$$h_{p+1} \ = \ P^2 + W_p^2 + W_p' + C_p \ . \tag{7.14}$$

This H commutes with the supercharges provided

$$W_r^2 + W_r' + C_r = W_{r+1}^2 - W_{r+1}' + C_{r+1} \ , \ r = 1, 2, \ldots, p-1 \ , \tag{7.15}$$

with the arbitrary constants $C_1, C_2, \ldots, C_p$ satisfying the constraint

$$\sum_{r=1}^{p} C_r = 0 \ . \tag{7.16}$$

On the other hand, the p OSQM charges Q_α are again $(p+1) \times (p+1)$ matrices as given by

$$(Q_\alpha)_{rs} = (P - iW_\alpha)\delta_{r,1}\delta_{s,\alpha+1} \tag{7.17}$$

while the corresponding H is a $(p+1) \times (p+1)$diagonal matrix: $2H = H_r \delta_{rr}$ where

$$H_1 = P^2 + W_1^2 + W_1' \ , \ H_{r+1} = P^2 + W_r^2 - W_r' \ , \ r = 1, 2, \ldots, p \ . \tag{7.18}$$

On further demanding the condition $Q_1 Q_1^+ = \ldots = Q_p Q_p^+$, we get the constraint

$$W_r^2 + W_r' = W_s^2 + W_s' \ , \quad r, s = 1, 2, \ldots, p \ . \tag{7.19}$$

Some of the important salient features of SQM, PSQM and OSQM are

1. Whereas in SQM, all the excited states are always two-fold degenerate, in OSQM of order p, all the excited states are (p+1)-fold degenerate. On the other hand, in PSQM of order p, p'th and higher excited states are (p+1)-fold degenerate while nothing definite can be said about the other excited states.

2. In SQM and OSQM, all the energy eigenvalues are positive semidefinite with the ground state energy $E_0 = 0$ (> 0) corresponding to unbroken (broken) symmetry, no such conclusion can be drawn in PSQM case unless $C_1 = C_2 = \ldots = C_p = 0$ in which case symmetry is unbroken (broken) if $E_0 = 0$ $(>)$.

3. A model of conformal SQM, PSQM and OSQM has also been constructed.

One of the draw back of the PSQM as formulated here is that, H is not expressible in terms of Q and Q^+. We have given [6] a new formulation of PSQM where this defect can be cured. In particular, we showed that, in the new formulation, the constants $C_1, C_2, \ldots, C_p = 0$ and then H^2 is given in terms of Q and Q^+ by [6]

$$H^2 = \frac{1}{4}\left[(Q^+Q)^2 + (QQ^+)^2 - QQ^+Q^+Q\right] \ . \tag{7.20}$$

We also use it to construct the PSQM of infinite order whose algebra (i.e. $H = \frac{1}{2}QQ^+$) corresponds to the single mode version of the algebra describing Greenberg's infinite statistics [8].

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Chapter 8

Fock Spaces and Quantum Statistics

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Abstract.
A brief description of new forms of quantum statistics is provided. It is demonstrated that a concept of generalized Fock spaces enables one to obtain a unified picture for various statistics and associated algebras.

Many new kinds of quantum statistics were discovered during the last decade. These discoveries owe their origin to various fundamental questions concerning the physical states of matter. Thus a possible small violation of Pauli's exclusion principle has led to the formulation of quons which obey infinite statistics. A notion of impenetrability of point particles in one dimension has given rise to null statistics. A whole new family of quantum statistics *viz.* orthostatistics were obtained while analyzing the consequences of infinite repulsion between electrons occupying the same orbital state.

In dimensions equal or greater than three, particle statistics are classified according to the representations of permutation group, and in two dimension, *via* braid group representations. It has been often argued that particles in a one-dimensional system would not cross each other, and so the notion of statistics is void in 1-D [1]. It is important to note here that whatever approach one uses for constructing the statistics, the exercise ultimately leads to specifying the permissible physical states for the system.

In this article, we describe various new forms of recently discovered quantum statistics. and highlight the underlying motivation for this endeavour. A number of these statistics have been constructed at our Institute in collaboration with Professor G. Rajasekaran.

In spite of the fact that a large literature which now exists, a unified picture for various statistics and their associated algebra has not emerged. To achieve this objective, we have introduced the concept of generalized Fock spaces. Starting with this basic notion, it is possible to show that more than one statistics can be postulated in a given Fock space, and many different algebraic realizations can be constructed for any particular statistics. In the theory of generalized Fock spaces, the key element is the notion of independence of the permutation ordered states. The largest linear vector space constructed in this way is the

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super Fock space. The subsequent specification of a subset of states in this space as null states leads to many reduced Fock spaces. All these spaces are collectively called as generalized Fock spaces. We construct creation ($c^\dagger$), annihilation (c) and number (N) operators in the generalized Fock spaces. The creation and annihilation operators, even for a particular Fock space, are not unique. Consequently, many statistics and algebras can exist in a given Fock space. On the other hand, a universal representation for the number operator valid for all forms of statistics and algebra exists. A brief description of this formalism is also provided in this article.

The Bose and Fermi statistics are based on the symmetry postulates, and these are described through canonical commutation and anticommutation relations, respectively.

$$[c_j, c_k^\dagger]_{\substack{-\\+}} = \delta_{jk} \quad ; \quad [c_j, c_k]_{\substack{-\\+}} = 0 \tag{1}$$

Many new kinds of quantum statistics are obtained by appropriate generalizations of these known commutation relations. The eq.(1) can be equivalently written as

$$[c_i, [c_i, c_k^\dagger]_{\substack{-\\+}}] = 2\delta_{ij} c_k \quad ; \quad [c_i, [c_j^\dagger, c_k^\dagger]_{\substack{-\\+}}] = 2\delta_{ij} c_k^\dagger \;\substack{-\\+}\; 2\delta_{ik} c_j^\dagger \tag{2}$$

$$[c_i, [c_j, c_k]_{\substack{-\\+}}] = 0 \tag{3}$$

The general solutions for c and $c^\dagger$ satisfying eqs.(2,3) lead to parabose and parafermi statistics, of which fermi and bose statistics are specific examples [2, 3].

Next, the replacement

$$[c_j, c_k^\dagger]_{\substack{-\\+}} = \delta_{jk} \quad \rightarrow \quad [c_j, c_k^\dagger]_q = \delta_{jk} \quad \equiv \quad c_j c_k^\dagger + q c_k^\dagger c_j = \delta_{jk} \quad ; \quad -1 < q < 1 \tag{4}$$

has led to the concept of quons which obey infinite statistics [4, 5, 6, 7]. In this statistics, each permuted state is an independent state. For $q = +(-)1$, fermionic anticommutation (bosonic commutation) relation is recovered. But for $q = 1 - |\epsilon|$ (ϵ is an infinitesimal), a small violation of Pauli's exclusion principle results.

As a counterpoint to infinite statistics, null statistics can also be constructed [8]. In this statistics, no permutation is allowed and particles are frozen in their initial order. The quanta satisfying null statistics are characterized through the algebra

$$c_k c_j^\dagger \;=\; 0 \quad \text{for} \quad k \neq j \;\; ; \;\; c_j c_j^\dagger \;=\; 1 - \sum_{k<j} c_k^\dagger c_k \;\; ; \;\; c_i c_j \;=\; 0 \quad \text{for} \quad i < j \tag{5}$$

The number operator for infinite statistics when $q = 0$ ($c_i c_j^\dagger = \delta_{ij}$) is

$$N_k \;=\; \sum_{n_g..n_k..n_m} \sum_{\mu} \left(c_g^{\dagger n_g} \ldots c_k^{\dagger n_k} \ldots c_m^{\dagger n_m}; \mu \right) c_k^\dagger c_k \left(c_m^{n_m} \ldots c_k^{n_k} \ldots c_g^{n_g}; \mu \right) \tag{6}$$

where μ labels each distinct permutation of operators inside the bracket. Interestingly, if the summation over μ is removed, the number operator for the null statistics is obtained.

$$N_k = \sum_{n_1, n_2 \ldots n_k} c_1^{\dagger n_1} c_2^{\dagger n_2} \ldots c_k^{\dagger n_k} c_k^\dagger c_k c_k^{n_k} \ldots c_2^{n_2} c_1^{n_1} \tag{7}$$

The infinite statistics is obtained by modifying the canonical commutation relation between c and $c^\dagger$ to a q-commutator. Note that no cc relation exists in infinite statistics, as each permuted state is an independent state. Instead of modifying the $cc^\dagger$ relation as in eq,(4), if cc relation in eq.(1) is q deformed

$$[c_j, c_k]_{\mp} = 0 \quad \rightarrow \quad [c_j, c_k]_q = 0 \quad \equiv \quad c_j c_k + q c_k c_j = 0 \tag{8}$$

(where q is real and finite), a q-statistics which is based on quantum group algebra is obtained.

The constraint that an orbital state shall not contain more than one particle irrespective of their spin direction has led to the formulation of a new family of quantum statistics, namely, orthostatistics [9, 10, 11]. These statistics are described using the algebra

$$c_{k\alpha} c^\dagger_{p\beta} \pm \delta_{\alpha\beta} \sum_\gamma c^\dagger_{p\gamma} c_{k\gamma} \;=\; \delta_{kp}\delta_{\alpha\beta} \tag{9}$$

$$c_{k\alpha} c_{p\beta} \pm c_{p\alpha} c_{k\beta} \;=\; 0 \tag{10}$$

Here k, p refer to spatial coordinates and α, β denote spin components. The positive and negative signs respectively correspond to orthofermi and orthobose statistics. A multiparticle wave function for othofermions (orthobosons) is antisymmetric (symmetric) in k, m, whereas no symmetry constraint is required for the indices α, β. An exchange in the former set of indices lead to fermi (bose) statistics, whereas the later satisfy infinite statistics. Thus ortostatistics represent first instance wherein quanta having composite statistical character (that is indices belonging to different class exhibit uncorrelated permutation properties) is proposed.

Orthostatistics arise when particles occupying same orbital mutually interact through infinite repulsive potential. We have earlier shown that a system of electrons subject to no double occupancy in a single orbital state can be described by two distinct set of algebras, one corresponding to orthofermi statistics and other defining the Hubbard statistics [9, 10]. But recently it has been shown that such a system is described by the orthofermistatistics [12].

It has been shown earlier that infinite statistics arises as a consequence of deformation of canonical commutation or anticommutation relation. Similarly, a deformation of the orthostatistics algebra, that is,

$$c_{k\alpha} c^\dagger_{m\beta} + q\delta_{\alpha\beta} \sum_\gamma c^\dagger_{m\gamma} c_{k\gamma} \;=\; \delta_{km}\delta_{\alpha\beta} \quad , \quad -1 < q < 1 \tag{11}$$

gives rise to doubly-infinite statistics; one with respect to the pair of indices (k, m) and other for the pair (α, β) [10].

The canonical commutation relation follows by mapping $\{x, \frac{\partial}{\partial x}\}$ to $\{b, b^\dagger\}$. This shows a close connection between particle statistics and differential calculus. Surprisingly, no mapping of the differential dx to a creation operator had been reported. We have completed this task both for commutative and noncommutative spaces. In this case of noncommutative space, the mapping of dx to a creation operator has led to a new notion of statistical transmutation under particle exchange [13].

To understand the various types of statistics and algebras of creation and annihilation operators, and interconnections among them, we start with the underlying space of allowed state vectors or Fock space. Given a set of quantum numbers $g, h, i...$ with the respective occupancy being $n_g, n_h, n_i...$, all possible multiparticle state vectors are

$$|n_g, n_h \ldots n_m; \mu\rangle \quad , \quad \mu = 1, 2 \ldots s \tag{12}$$

where s is the total number of distinct permutations and μ labels each of these permuted states. We assume the existence of a unique vacuum state

$$|0\rangle \equiv |0,0,0\ldots 0\rangle \tag{13}$$

All these states are linearly independent, but need not be orthogonal or normalized

$$\langle n_g^{'}, n_h^{'} \ldots n_m^{'}; \alpha | n_g, n_h \ldots n_m; \nu\rangle \;=\; \delta_{n_g^{'} n_g}\, \delta_{n_h^{'} n_h} \ldots \delta_{n_m^{'} n_m}\, M_{\mu\nu} \tag{14}$$

with M being a $s \times s$ hermitian matrix. We choose it to be positive definite.

From the set of linearly independent state vectors, an orthonormal set of vectors $\{\| n_g \ldots n_m; \mu \gg\}$ can be obtained

$$\| \, n_g \ldots n_m; \mu \gg = \sum_\nu X_{\nu\mu} |n_g \ldots n_m; \nu\rangle \tag{15}$$

Alternatively, starting with the orthonormal vectors $\{\| \, n_g \ldots n_m; \mu \gg\}$, the vectors $\{|n_g, n_h \ldots n_m; \mu\rangle\}$ can be constructed by taking the inverse of relation (15). X is a nonsingular matrix. Although X is not unique and depends on the particular orthogonalization procedure, we have

$$M^{-1} \;=\; X X^\dagger \tag{16}$$

Choosing a nonsingular matrix X and determining the inner product matrix M as above will ensure the positivity of the matrix M.

The set of state vectors considered here constitute super Fock space. Infinite statistics resides in this Fock space. Using the projection operator

$$P(n_g \ldots n_k \ldots n_m) = \sum_{\lambda,\nu} |n_g \ldots n_m; \nu\rangle (M^{-1})_{\nu\lambda} \langle n_g \ldots n_m; \lambda| \tag{17}$$

the number operator can be written as

$$N_k \;=\; \sum_{n_g \ldots n_k \ldots n_m} n_k P(n_g \ldots n_k \ldots n_m) \tag{18}$$

which satisfies the following properties

$$N_k |n_g \ldots n_k \ldots n_m; \mu\rangle \;=\; n_k |n_g \ldots n_k \ldots n_m; \mu\rangle \quad ; \quad [N_k, N_j]_- \;=\; 0 \tag{19}$$

The creation operator is defined as

$$c_j^\dagger \;=\; \sum_{n_g \ldots n_j \ldots n_m} \sum_{\mu\nu} A_{\mu\nu}\, |1_j n_g \ldots n_j \ldots n_m; \mu\rangle \, \langle n_g \ldots n_j \ldots n_m; \nu| \tag{20}$$

and c_j as the hermitian conjugate of $c_j^\dagger$. $A_{\mu\nu}$ are a set of arbitrary (complex) numbers. Even at this stage, it is possible to verify that

$$[c_j^\dagger, N_k]_- \;=\; -c_j^\dagger \, \delta_{jk} \tag{21}$$

The ordered state vectors can be constructed using a string of $c^\dagger$ acting on the vacuum state. Consequently we also have

$$\sum_\nu A_{\mu\nu}\, M_{\nu\lambda} \;=\; \delta_{\mu\lambda} \quad ; \quad A \;=\; M^{-1} \tag{22}$$

We have provided here a unique representation of the number operator. But many different representations of creation and annihilation operators are possible through different choices of matrices A, X and M. The number operator N can be expressed in terms of $c^\dagger$ and c by solving Eqs.(17,18) and (20). Since $c^\dagger$ and c are not uniquely defined many different expressions for N in terms of $c^\dagger$, c can be obtained.

Next we consider reduced Fock spaces. All known forms of statistics other than the infinite statistics reside in reduced Fock spaces, which are obtained by postulating relations like

$$\sum_\mu B^p_\mu |n_g, n_h \ldots ; \mu >= 0 \ ; \ p = 1, 2, \ldots r \tag{23}$$

where r < s and B^p_μ are constants. The vector space dimension in the sector $\{n_g, n_h \ldots\}$ is now reduced to $d = s - r$. The formalism developed for the super Fock space is also valid for reduced Fock spaces. But μ and ν now ranges from 1...d, and X, M and A are $d \times d$ matrices.

Arbitrariness in the matrix A appearing in the creation operator expression (20) can be exploited to generate many relations involving $c^\dagger$ $c^\dagger$. Therefore, many different forms of statistics specified by different $c^\dagger$ $c^\dagger$ relations can be constructed in a given Fock space. All these statistics are interconnected. No connection exists between the statistics and the associated algebras residing in different Fock spaces. It may also be mentioned here that multiplicity of statistics are not possible in the super Fock space and in the Fock space of frozen order. Only infinite and null statistics respectively reside in these two Fock spaces. But even here, many algebras involving c and $c^\dagger$ are possible.

Thus, by decoupling the notion of the underlying Fock space from c and $c^\dagger$, we are able to define different forms of statistics in a representation independent manner. Subsequently, one can construct creation, annihilation operators and their algebra in any desired representation.

The general formalism not only unifies and classifies various forms of quantum statistics, but also enables us to construct many new kinds of statistics and algebras for single and two-indexed systems in a systematic manner. Some of these are: (i) null statistics, (ii) orthostatistics, (iii) Hubbard statistics (iv) doubly-infinite statistics, (v) complex q or fractional statistics in one dimension. Many c $c^\dagger$ algebras representing these statistics are also constructed. Besides, the notion of generalized Fock space leads to the concept of statistical transmutation in a quantum plane, and it is rich enough to handle quantum group related algebraic structures [10].

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Chapter 9

How do we know the charges of quarks?

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Abstract. Experimental tests performed in the past to determine quark charges are reviewed. It is pointed out that only experiments involving two real photons can really distinguish between the Gell-Mann-Zweig scheme of fractional charges from the Han-Nambu scheme of integer charges in the context of a theory with gauged colour interactions. While many experiments with two real photons have been performed, the only experiment with a definitive statement on quark charges was one on hard scattering of real photons at CERN. Some theoretical issues not fully resolved are pointed out.

One of the questions which interested Prof. Rajasekaran from the time he worked on gauge theories in TIFR was the question of quark charges—are they fractional (in accordance with the Gell-Mann-Zweig scheme [1]) or are they integral (in accordance with the Han-Nambu scheme [2])? The question was not easily resolved in the context of a theory with gauged colour interactions. He continued to work on it when he moved to Madras.

The colour degree of freedom was incorporated in the Gell-Mann-Zweig (GZ) scheme with a colour triplet of quarks of each flavour, with all the members having the same (fractional) charge. In the Han-Nambu (HN) scheme, the members of the colour triplet were quarks with different charges for the same flavour, the colour-averaged charge being fractional. Thus, the charge operator could be written as

$$Q = Q_0 + Q_8, \tag{9.1}$$

where Q_0 is the colour-singlet charge, viz., $\frac{2}{3}$ for up quarks, and $-\frac{1}{3}$ for down quarks, and Q_8 is the colour contribution to the total charge Q, which vanishes in the GZ scheme, and is given in the HN scheme by

$$Q_8 = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}, \tag{9.2}$$

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where the matrix Q_8 is in colour space. The presence of Q_8 gives rise to the charges (1,1,0) for the members of the colour triplet for each up quark, and (0,0,−1) for each down quark.

At first sight it appears that it would be easy to decide between the two models in experiments like deep-inelastic scattering which use electromagnetic probes. However the first requirement in these cases is that the relevant energy should be above the threshold for excitation of colour [3]. Below the colour threshold, only Q_0 would be effective, giving no difference between the predictions of the two models.

It was thus believed that deep-inelastic scattering experiments, which presumably had high enough energies to excite colour, would reveal the true quark charges. However, it was shown by Rajasekaran and Roy [4], and Pati and Salam [5], that in a theory in which colour is gauged [6], colour breaking induces mixing between the electroweak and colour gauge fields. As a result, the effective charge operator for large squared momentum-transfer takes the form

$$Q_{\text{eff}}(q^2) = Q_0 + Q_8 \left(\frac{-m_g^2}{q^2 - m_g^2} \right). \tag{9.3}$$

Thus for $|q^2| >> m_g^2$, $Q_{\text{eff}} \approx Q_0$. It follows that no difference between the two models would be seen asymptotically in deep-inelastic scattering (DIS), so far as quark charges are concerned. However, gluons in the HN scheme carry electric charge. Hence they would also contribute to the DIS structure functions, and if this effect is large, detecting it would signal integrally charged quarks.

It is curious that even though the longitudinal polarization vector of massive spin-one gluons goes for large $|q|$ as q^μ/m_g, the gluon contribution to the structure function F_2 is finite for $Q^2 \to \infty$ [4]; it is however suppressed.

With the advent of high energy e^+e^- colliding beams at PETRA there was a possibility of looking for direct production of charged-gluon jets. With the assumption that colour was excited at these energies, the additional gluon [7] and coloured scalar contributions [8] was expected to provide a test of the HN model in two-jet production experiments. In particular, the angular distribution of jets is different in the HN model as compared to the $1 + \cos^2\theta$ distribution in the GZ model [7, 8]. This, to my knowledge, was never tested by experimentalists. The three-jet experiments, in spite of a scaling violation contribution of three-gluon production in the HN model, merely provided a limit on the gluon mass [9].

The problem of nonobservation of the octet charge for large $|q^2|$ may be avoided in two-photon experiments, first carried out at PETRA. For, in such experiments, the relevant observable is Q^2_{eff}, which has a contribution Q_8^2. Hence even below colour threshold, a colour singlet projection of Q_8^2 would survive, and would help discriminating between the two charge schemes. The problem, however, was that in PETRA experiments the $|q^2|$ values were never close to zero, and the experiments could accommodate the GZ as well as HN models, for a range of values of m_g [10].

In the case of single photon production in $e^+e^- \to 2$ jets, there would be contributions from photons radiated off final-state quarks (and also gluons in the case of HN scheme), as well as from initial state e^+ or e^-. The contribution coming from final-state radiation would measure the sum of the squares of quark or gluon effective charges. However, since one of the photons involved is virtual, the resultant effect is not very sensitive to the model [11]. Some experimental papers do attempt to make a comparison, without conclusive results [12].

It seemed clear that the best test of quark charges would be in processes where two strictly real photons were involved. However, high energy processes involving real photons are plagued with uncertainties due to large errors.

To date, an inexhaustive list of various experiments dealing with two real photons is:

1. LEP (ALEPH, DELPHI, L3 and OPAL collaborations): $e^+e^- \to \gamma\gamma + X$ [13].

2. NA3 spectrometer: $\pi^\pm + C \to \gamma\gamma + X$ (200 GeV/c) [14].

3. AFS: $pp \to \gamma\gamma + X$ [15].

4. Tevatron (CDF and D0 collaborations): $p\bar{p} \to \gamma\gamma + X$ [16].

5. E-706 (Fermilab): $\pi^\pm, p + N(\mathrm{Be}, \mathrm{Cu}, \mathrm{H}) \to \gamma\gamma + X$ (515 GeV/c $\pi^\pm$ or 530 and 800 GeV/c p) [17].

6. NA14: $\gamma N(\mathrm{Li}^6) \to \gamma X$ [18, 19].

A good experimental test would be in photon-pair production in $e^+e^- \to \gamma\gamma + 2$ jets. In this case, the colour-singlet projection of the square of octet charges would contribute in the case of HN, and a quantity

$$\sum_{\text{flavour}} |\langle Q_0 Q_8 Q_8 \rangle_{\text{colour ave.}}|^2 \tag{9.4}$$

would be measurable. The effective charge factor

$$\sum_{\text{flavour}} |\langle Q^3 \rangle_{\text{colour ave.}}|^2 \tag{9.5}$$

amounts to 11/9 in HN as compared to 131/243 in GZ, for five flavours). Data does exist; however no comparison has been attempted.

Comparison of experimental results with theoretical predictions in HN and GZ schemes have been made in some hadronic experiments [20]. However, one has to have some reservations about these because of the following factors. (i) Uncertainties in quark and gluon distributions. Particularly, since gluons could also contribute to DIS, even in the zeroth order in α_{QCD}, their distributions as determined from a fit to DIS experimental data would be different in HN and GZ schemes. (ii) Higher-order corrections in HN model are not known, and a simple "K factor" calculated for GZ scheme has to be assumed.

The experiment which has led to a definitive statement, modulo the assumptions already mentioned, is the NA14 experiment on hard scattering of real photons [18, 19]. Here the prediction in HN model is larger than that in the GZ model by a factor of 2.65 for four flavours (10/3 in HN scheme as against 34/27 in GZ scheme). The HN Born contribution has been simply scaled up by the same QCD K factor as in the GZ contribution. HN model seems disfavoured by at least two standard deviations. Fig. 1, which is taken from [18], shows a comparison of the data with theoretical expectations from the two schemes. Note that the gluon contribution is not included, or else the factor of 2.65 could be replaced by something larger.

Present day results from Tevatron have the potential for making more definitive statements. However, no comparison has been made.

Other experiments where some comparison could be made is radiative decays of hadrons, particularly, two-photon decays of mesons [21].

Theoretical topics which have not been sufficiently investigated are (i) possible phenomenon of colour oscillations [22] (ii) effects that might be seen at colour threshold (iii) masses and spectrum of coloured Higgs scalars, and the possibility of colour singlet bound

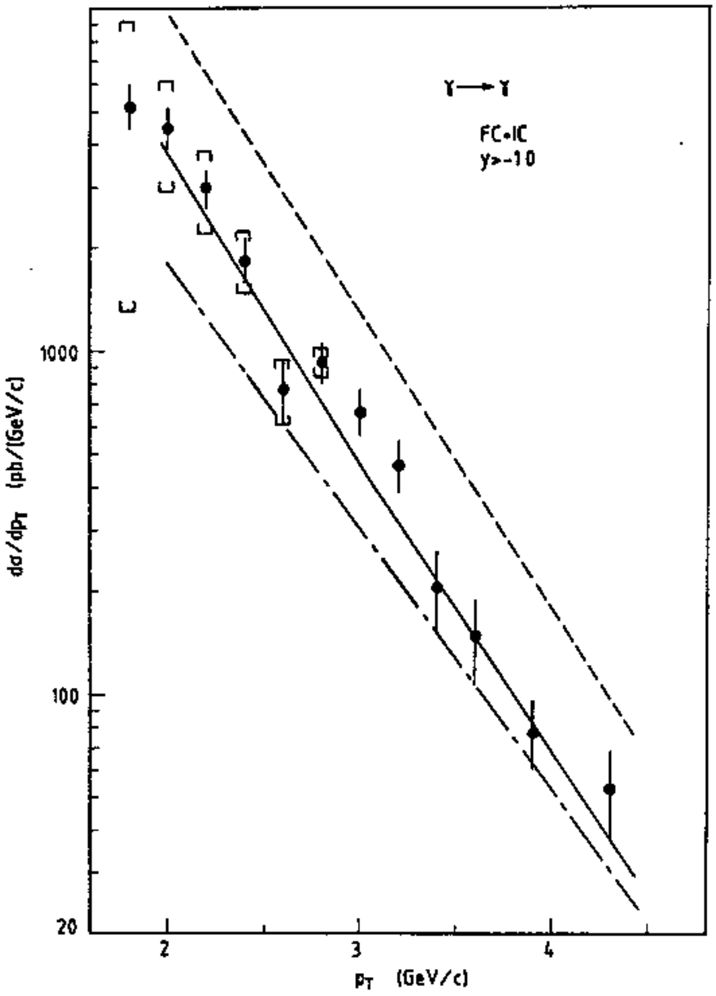

Figure 9.1: The p_T distribution of inclusive prompt photons from the NA14 experiment [18]. The dash-dotted and solid curves correspond respectively to the Born term and the Born term together with next-to-leading order QCD corrections in GZ model. The dashed curve is 2.65 times the solid curve, the expectation from the gauged HN model.

states of these among themselves or with quarks (for a discussion on a similar problem with unconfined fractionally charged quarks, see [23]).

In conclusion, data from NA14 experiment on hard scattering of real photons seems to indicate that the HN model is strongly disfavoured. However, this depends on some assumptions stated earlier. e^+e^- experimental data from LEP could be more definitive. Some theoretical issues like those of colour oscillations, however, need to be addresses in that context.

I thank Prof. Rajasekaran for a critical reading of the manuscript.

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Chapter 10

Overview of semi-inclusive reactions in deep inelastic scattering

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I thank the organisers for having invited me to participate in this symposium honouring Prof.Rajasekaran. Rajaji has had a major influence on all particle physicists in our country for four decades and it is a privilege to be here honouring him.

Abstract.
Several interesting experiments reported recently are reviewed with some elementary theoretical analysis.

Introduction

In semi-inclusive deep inelastic scattering one studies a reaction of the type

$$l(k) + \mathrm{Target(P)} \to l(k') + \mathrm{hadron}(P_h) + X \ ,$$

where a lepton l of momentum k strikes a target (momentum $= P$) and scatters to have a momentum k' , producing several hadrons, out of which a hadron (momentum $= P_h$) is detected. We use the following scaling variables to describe the scattering process:

$$x = \frac{Q^2}{2q.P}, \qquad z = \frac{P.P_h}{P.q}, \qquad Q^2 = -q^2 = -(k-k')^2 \ . \tag{10.1}$$

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The number of hadrons emitted is proportional to the quark structure functions and the fragmentation function, and is given by

$$N_T^h \approx \sum_{q,\bar{q}} e_i^2 q_i(x,Q^2) D_{q_i}^h(z,Q^2) , \tag{10.2}$$

where $q_i(x,Q^2)$ represents the structure function, and is the distribution of the ith flavour quark of momentum fraction x of the target hadron, and $D_{q_i}^h(z,Q^2)$ is the fragmentation function, which is equal to the probability that the struck quark emits a hadron h with energy fraction z of the energy of the parent quark.

Use of Semi-Inclusive Reaction to Obtain Structure Functions

One can extract information about structure functions from the semi-inclusive processes. We assume the fragmentation functions obey the following relations (from Iso-spin invariance):

$$D_u^{\pi^+} = D_d^{\pi^-}, \qquad D_u^{\pi^-} = D_d^{\pi^+}, \qquad D_s^{\pi^+} = D_s^{\pi^-} . \tag{10.3}$$

As an example, one can easily verify from Eq.(10.2), choosing the target as proton or neutron and h as a pion, that

$$\frac{N_n^{\pi^+} - N_n^{\pi^-}}{N_p^{\pi^+} - N_p^{\pi^-}} = \frac{4d(x)/u(x) - 1}{4 - d(x)/u(x)} , \tag{10.4}$$

which is independent of the fragmentation functions. The following analysis [1] has been useful in extracting the quantity

$$\frac{\overline{d}(x) - \overline{u}(x)}{u(x) - d(x)} = \frac{J(z)[1-r(x,z)] - [1+r(x,z)]}{J(z)[1-r(x,z)] + [1+r(x,z)]} , \tag{10.5}$$

where

$$r(x,z) = \frac{N_p^{\pi^-}(x,z) - N_n^{\pi^-}(x,z)}{N_p^{\pi^+}(x,z) - N_n^{\pi^-}(x,z)} , \tag{10.6}$$

and

$$J(z) = \frac{3}{5}\,\frac{1 + D_u^{\pi^-}/D_u^{\pi^+}}{1 - D_u^{\pi^-}/D_u^{\pi^+}} . \tag{10.7}$$

This information has been used to check the violation of the Gottfried sum rule from the value 1/3,

$$\int_0^1 \frac{dx}{x}[F_2^p(x) - F_2^n(x)] = \frac{1}{x} - \frac{2}{x}\int_0^1 [\overline{d}(x) - \overline{u}(x)]dx , \tag{10.8}$$

by determining the extra contribution arising from the second term.

If one assumes SU(3) model with phenomenological breaking introduced for fragmentation functions [3] further relations can be obtained, for example,

$$\frac{(N_p^{K^\pm} - N_p^{K^0}) - (N_n^{K^\pm} - N_n^{K^0})}{N_p^{K^\pm} - N_p^{K^0}} = \frac{5(u(x)+\overline{u}(x)-d(x)-\overline{d}(x))}{4u(x)+4\overline{u}(x)-d(x)-\overline{d}(x)} . \tag{10.9}$$

Here $N_i^{K^\pm}$ stands for the combination $(N_i^{K^+} + N_i^{K^-})$ and $N_i^{K^0}$ for $(N_i^{K^0} + N_i^{\overline{K}^0})$.

Polarised Semi-inclusive Processes

(a) Longitudinally polarisation : The initial lepton and the target are longitudinally polarised. The two spins can be aligned parallel to each other (denoted by $\uparrow\uparrow$) or antiparallel to each other (denoted by $\uparrow\downarrow$). The longitudinal asymmetry is defined by

$$\begin{aligned} A_l^h &= \frac{\int N_{\uparrow\uparrow}^h dz \;-\; \int N_{\uparrow\downarrow}^h dz}{\int N_{\uparrow\uparrow}^h dz \;+\; \int N_{\uparrow\downarrow}^h dz} \\ &= \frac{\sum e_i^2 \Delta q_i(x,Q^2) \int D_q^h(z,Q^2) dz}{\sum e_i^2 q_i(x,Q^2) \int D_q^h(z,Q^2) dz} \; . \end{aligned} \tag{10.10}$$

Data from SMC and HERMES [2] have been analysed under two different assumptions for the sea-quark structure functions, namely, $\delta u_s/u_s = \delta d_s/d_s$ and $\delta u_s = \delta d_s$. This has been used in determining $\Delta\Sigma$, leading to $\Delta\Sigma \;=\; 0.3 \pm .04 \pm .09$ at $Q^2 \;=\; 2.5$GeV. One again has the sum rule

$$\frac{\left(N_{\uparrow\uparrow}^{\pi^+} \;-\; N_{\uparrow\uparrow}^{\pi^-}\right) \;-\; \left(N_{\uparrow\downarrow}^{\pi^+} \;-\; N_{\uparrow\downarrow}^{\pi^-}\right)}{\left(N_{\uparrow\uparrow}^{\pi^+} \;-\; N_{\uparrow\uparrow}^{\pi^-}\right) \;+\; \left(N_{\uparrow\downarrow}^{\pi^+} \;-\; N_{\uparrow\downarrow}^{\pi^-}\right)} \;=\; \frac{4\Delta u_v - \Delta d_v}{4u_v - d_v} \; . \tag{10.11}$$

(b) Transverse Spin Asymmetry : One of the outstanding problems in hadron-hadron scattering is the understanding of transverse polarisation Λ^0 in the reaction (initial state unpolarised)

$$p \;+\; p \;\;\rightarrow\;\; \Lambda^0 \;+\; X \; , \tag{10.12}$$

at large transverse momentum (p_T), where one expects perturbative QCD to be valid. However perturbative QCD predicts polarisation to be proportional to $\alpha m/\sqrt{s}$, as it needs a flip in helicity. This comes for the T-odd term, $\mathbf{S}_\Lambda.(\mathbf{p}_\Lambda \times \mathbf{p}_p)$ where $\mathbf{p}_p$ is the incoming proton momentum, $\mathbf{S}_\Lambda, \mathbf{p}_\Lambda$ the spin and momentum of Λ^0 respectively. A similar effect is observed in the left- right asymmetry of the π^+ produced when a proton scatters off a transversely polarised target proton in the reaction

$$p \;+\; \vec{p} \rightarrow \pi^+(\text{large } p_T) + X \; . \tag{10.13}$$

Recently the same effect has been observed in semi-inclusive deep inelastic scattering at SMC and HERMES [4]. This is understood by assuming that the fragmentation function of a transversely polarised quark depends on the transverse momentum of the produced hadron (Collins effect) [5]. One can parametrise the fragmentation function as

$$D_q^h(\mathbf{p}_q, \mathbf{s}_q, z, \mathbf{p}_{hT}) \;\;=\;\; D_q^h(z, \mathbf{p}_{hT}) \;+\; \frac{1}{2}(\Delta D_q^h(z, p_{p_T}))\frac{\mathbf{s}_q.(\mathbf{p}_q \times \mathbf{p}_{hT})}{|\mathbf{p}_q \times \mathbf{p}_{hT}|)} \; , \tag{10.14}$$

where $\mathbf{p}_q$ is the parent quark momentum before fragmentation, $\mathbf{s}_q$ is its spin and $\mathbf{p}_{hT}$ is the transverse momentum of the hadron. The analysing power defined by

$$A_q^h \;\;=\;\; \frac{\Delta D_q^h(z, p_{p_T})}{2 D_q^h(z, p_{p_T})} \; , \tag{10.15}$$

is the experimental quantity which was measured by the experimental groups at SMC and HERMES.

It remains a challenge to understand these fragmentation fractions.

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Chapter 11

$\Lambda_{Phys} = 0$ in Kaluza-Klein Formalism and Relation to Randall-Sundrum Background

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Abstract.

The recently proposed Randall-Sundrum 5-dimensional background metric, which solves the hierarchy problem, is obtained from modified Kaluza-Klein formalism by imposing vanishing of 4-dimensional cosmological constant Λ_{Phys}. Quantum fluctuations are studied and the condition for massless graviton is found to be the same as that for vanishing Λ_{Phys}. The orbifold picture is obtained by coupling scalars to higher dimensional gravity and insisting correct dynamics in 4-dimensions. Extension to higher dimensions (> 5) are considered with the extra dimensions taken as $S^1 \times S^1$ and S^2.

It is a pleasure to contribute the following at the one-day symposium to honour Prof. G. Rajasekaran. He knows me since 1983 and I have much benefitted by his insight and clarity of thought. He evinced interest in Kaluza-Klein theory in the mid eighties and this resulted in our attempts to provide a unified description of various phases of the quantum mechanical wavefunction.

Randall and Sundrum [1] (hereinafter referred to as RS) have proposed a 5-dimensional theory based on a non-factorizable geometry, which solves the hierarchy problem, by an exponential suppression factor. The exponential arises from the background metric of the 5-dimensional theory. The fifth dimension is in an orbifold S^1/Z_2 and the gravity can propagate in the bulk, while the standard model fields are confined in one boundary. The background metric (Eqn.2 of RS)

$$(ds)^2 \;\; = \;\; e^{-2kr_c\theta}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2\ d\theta^2, \qquad (11.1)$$

where k is a scale of the order of the Planck scale, x^μ are the coordinates for the familiar four dimensional space-time, $-\pi \leq \theta \leq \pi$ (with (x, θ) and $(x, -\theta)$ identified and the 3-brane

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located at $\theta = 0, \pi$) and r_c sets the size of the fifth dimension. Goldberger and Wise [2] have shown that scalar fields with a 5-dimensional mass term have, after integrating over the fifth dimension, modes with 4-dimensional masses that are *exponentially* suppressed, thereby bringing Planck scale to TeV scale. Recently, Flachi and Toms [3] have considered a bulk scalar field with a positive mass term and showed a way of obtaining a scalar field with imaginary mass term on the brane, by a non-minimal coupling to the space-time curvature. This gives the possibility to generate spontaneous symmetry breaking using geometry. Cosmological studies [4] and inflationary solutions [5] with RS background metric are promising. Thus it is convincing that the RS metric is able to address a wide variety of important issues and opens up new thrust in high energy physics. So it will be worthwhile to understand the possible origin of it and its relation to the Kaluza-Klein theory which is incorporated in string theory. *It is the purpose of this paper to relate the RS background metric to Kaluza-Klein formalism by insisting on the vanishing of the 4-dimensional cosmological constant Λ_{Phys} to provide a reason for RS backgorund choice. Fluctuations of the metric will also be considered for consistency with the masslessness of graviton and extension to higher dimensions* (> 5) *will be indicated.* Although the results obtained here are not entirely new, this study provides a origin of the RS background (the background RS metric in [1] is ad hoc and no reasons were given for the choice) and relates $\Lambda_{Phys} = 0$ condition to the warp factor of RS metric and masslessness condition for graviton.

In Kaluza-Klein formalism, the higher dimensional gravity is taken on $M^4 \times B^n$, where M^4 is the usual 4-dimensional space-time continuum and B^n is the manifold (compact or non-compact) of extra dimensions. For pure gravity, the metric is taken to be

$$(ds)^2 = g_{\mu\nu} dx^\mu dx^\nu + g_{ij} dy^i dy^j. \tag{11.2}$$

In order to realize non-Abelian gauge fields of metric in origin, the manifold B^n has to be Ricci non-flat (i-e., $R_{ij} \neq 0$)[6]. Without introducing matter fields in B^n (such as monopoles [7] or scalars [8]), Nagarajan, the present author and Lakshmibala [9] have shown that the metric choice (2) is incompatible with $\Lambda_{Phys} = 0$. In order to obtain $\Lambda_{Phys} = 0$ and $R_{ij} \neq 0$, we[9] used non-factorizable ansatz for the higher dimensional metric. Similar ansatze have been studied earlier [10]. Among the various cases considered, it was shown [9] that the choice

$$(ds)^2 = f(y) g_{\mu\nu} dx^\mu dx^\nu + g_{ij} dy^i dy^j, \tag{11.3}$$

has been able to realize both $\Lambda_{Phys} = 0$, $R_{ij} \neq 0$ and consistent with Poincare symmetry in M^4. The function $f(y)$ (y standing for y^i, $i = 1$ to n) has been shown to be determinable from the equations of motion of the action

$$S = \int d^4x d^n y \sqrt{-\mathcal{G}} \Big\{ \hat{R} + 2\Lambda \Big\}, \tag{11.4}$$

where $\mathcal{G}_{AB}$ is the metric (3) ($A = \mu, i$), $\hat{R}$ is the Ricci scalar $\mathcal{G}^{AB}\hat{R}_{AB}$ for the $(4+n)$-dimensional space and Λ is the cosmological constant in the higher dimensional gravity.

The equations of motion from the action (4) are

$$\begin{aligned} R_{\mu\nu} &= g_{\mu\nu} \Big\{ \frac{2\Lambda f(y)}{(2+n)} - \frac{1}{2} \nabla_i \nabla^i f(y) \\ &- \frac{1}{2f(y)} \nabla_i f(y)\ \nabla^i f(y) \Big\}, \end{aligned} \tag{11.5}$$

on M^4 and

$$
\begin{aligned}
R_{ij} &= \frac{2\Lambda}{(2+n)} g_{ij}(y) + \frac{2}{f(y)} \nabla_i \nabla_j f(y) \\
&- \frac{1}{f^2(y)} \nabla_i f(y)\ \nabla_j f(y),
\end{aligned}
\tag{11.6}
$$

on B^n, where ∇_i is the covariant derivative on B^n. Taking the trace of (6) and using it in (5), allows us to define Λ_{Phys} in 4-dimensional space-time, as

$$
\begin{aligned}
R_{\mu\nu} &\equiv \Lambda_{Phys}\ g_{\mu\nu}, \\
\Lambda_{Phys} &= \frac{3}{2}\Big\{\nabla_i \nabla^i - \frac{R^{(n)}}{3} - \frac{2}{3}\Lambda\Big\} f(y).
\end{aligned}
\tag{11.7}
$$

Imposing $\Lambda_{Phys} = 0$, we[9] obtained an eigenvalue equation for $f(y)$ as

$$
\nabla_i \nabla^i f(y) = \frac{1}{3}\Big\{2\Lambda + R^{(n)}\Big\} f(y),
\tag{11.8}
$$

which can be solved for $f(y)$ once B^n is specified. This is *the main result in Ref.9.* Further, we [9] showed that the choice (3), for a scalar field coupled to higher dimensional gravity $\mathcal{L} = \mathcal{G}^{AB}\partial_A \Phi \partial_B \Phi$, after dimensional reduction using (3) (integrating over y^i coordinates), correctly reproduces the dynamics of massive scalar fields in M^4. Nevertheless, the hierarchy problem has not been addressed to in Ref.9.

Now, we will relate (3) and (8) to the RS background metric. For this, we restrict to 5-dimensional space-time and take B^n to be S^1, to begin with. Then the y-coordinate is bounded $0 \leq y \leq 2\pi r_c$, where r_c is the radius of S^1. Since the manifold S^1 is flat, (8) assumes the simple form

$$
\frac{d^2 f(y)}{dy^2} = \frac{2\Lambda}{3} f(y),
\tag{11.9}
$$

whose solution is

$$
f(y) = e^{\pm\sqrt{\frac{2\Lambda}{3}}y},
\tag{11.10}
$$

and the metric (3) becomes

$$
(ds)^2 = e^{\pm\sqrt{\frac{2\Lambda}{3}}y} g_{\mu\nu} dx^\mu dx^\nu + (dy)^2,
\tag{11.11}
$$

which is the same as RS background metric (1), if we identify $\sqrt{\frac{2\Lambda}{3}}$ with $2k$ and use $\theta = y/r_c$. $e^{-\sqrt{\frac{2\Lambda}{3}}y}$ corresponds to the RS background and $e^{\sqrt{\frac{2\Lambda}{3}}y}$ to an alternate background in which the brane attracts ordinary matter where as it is repulsive in RS background, as studied by Mück, Viswanathan and Volovich [11]. In this way, the RS background metric is related to the Kaluza-Klein metric with y-dependent factor in front of $g_{\mu\nu}$ on M^4. This provides a geometric meaning to the constant k as related to the cosmological constant in higher dimensional gravity.

Now, we examine, before obtaining the orbifold picture, whether the solution (10) is consistent with the quantum fluctuations of the metric (11). First, we consider a Minkowski background for M^4, (the results obtained remain unchanged otherwise as well) and fluctuate $\eta_{\mu\nu}$ in M^4. This is described by considering

$$(ds)^2 = e^{\pm\sqrt{\frac{2\Lambda}{3}}y}\{\eta_{\mu\nu} + h_{\mu\nu}(x)\}dx^\mu dx^\nu + (dy)^2, \tag{11.12}$$

where $h_{\mu\nu}(x)$ is the fluctuation or graviton field. The non-vanishing connections (upto linear in $h_{\mu\nu}$) are

$$\begin{aligned}
\tilde{\Gamma}^\mu_{\nu\rho} &= \frac{1}{2}\{\partial_\nu h^\mu_\rho + \partial_\rho h^\mu_\nu - \eta^{\mu\lambda}\partial_\lambda h_{\nu\rho}\}, \\
\tilde{\Gamma}^\mu_{\nu 5} &= \mp\frac{1}{2}\sqrt{\frac{2\Lambda}{3}}\;\delta^\mu_\nu, \\
\tilde{\Gamma}^5_{\mu\nu} &= \pm\frac{1}{2}\sqrt{\frac{2\Lambda}{3}}\;e^{\pm\sqrt{\frac{2\Lambda}{3}}y}\{\eta_{\mu\nu} + h_{\mu\nu}\},
\end{aligned} \tag{11.13}$$

where $\tilde{\Gamma}$ stands for 5-dimensional connection. Then it follows,

$$\begin{aligned}
\tilde{R}_{\mu\nu} &= \frac{1}{2}\{\partial_{nu}\partial_\rho h^\rho_\mu - \partial_\mu\partial_\nu h^\lambda_\lambda + \partial_\rho\partial_\mu h^\rho_\nu \\
&- \partial_\lambda\partial^\lambda h_{\mu\nu}\} + \frac{2\Lambda}{3}e^{\pm\sqrt{\frac{2\Lambda}{3}}y}(\eta_{\mu\nu} + h_{\mu\nu}), \\
\tilde{R}_{\mu 5} &= 0, \\
\tilde{R}_{55} &= -\frac{2\Lambda}{3}, \\
\tilde{R} &\equiv \mathcal{G}^{AB}\tilde{R}_{AB} \\
&= e^{\mp\sqrt{\frac{2\Lambda}{3}}y}\{\partial^\mu\partial_\rho h^\rho_\mu - \partial_\lambda\partial^\lambda h^\mu_\mu\} - 5\frac{2\Lambda}{3}.
\end{aligned} \tag{11.14}$$

Substituting (14) in Einstein's equation in 5-dimensional gravitational action with Λ term,

$$\tilde{R}_{AB} - \frac{1}{2}\mathcal{G}_{AB}\tilde{R} = \Lambda\mathcal{G}_{AB}, \tag{11.15}$$

the $A = B = 5$ component gives

$$\Lambda + \frac{1}{2}e^{\mp\sqrt{\frac{2\Lambda}{3}}y}(\partial_\mu\partial^\mu h^\lambda_\lambda - \partial_\rho\partial^\mu h^\rho_\mu) = \Lambda, \tag{11.16}$$

which is satisfied if

$$h^\lambda_\lambda = 0 \quad ; \quad \partial_\rho h^\rho_\mu = 0. \tag{11.17}$$

So, the usual gauge condition on the fluctuations $h_{\mu\nu}$ is algebraically derived. With these conditions on $h_{\mu\nu}$, the $A = \mu,\ B = \nu$ part of (15) readily gives

$$\partial_\lambda\partial^\lambda h_{\mu\nu} = 0, \tag{11.18}$$

showing the *masslessness of the graviton fields $h_{\mu\nu}$.* Thus, the solution (10) is consistent with massless graviton.

We have considered the fluctuations in M^4 in (12). It will be necessary to check whether fluctuations in S^1 are consistent with (10) and (11). Fluctuations in S^1 can be taken into account by replacing y in the exponent in (10) and (11), by $s(y) \simeq y + \epsilon(y)$ and replacing $(dy)^2$ by $(1+\varepsilon(y))(dy)^2$. Since $(1+\varepsilon(y))$ can be absorbed by redefining the coordinate y as $y' = \int \sqrt{(1+\varepsilon(y))}dy$, the fluctuated metric is taken as

$$(ds)^2 = e^{\pm\sqrt{\frac{2\Lambda}{3}}\, s(y)}(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + (dy)^2. \tag{11.19}$$

The 5-dimensional connections are evaluated using (19) and the non-vanishing ones are: $\tilde{\Gamma}^\mu_{\nu\rho}$ is same as in the first expression in (15) and,

$$\begin{aligned}
\tilde{\Gamma}^\mu_{\nu 5} &= \mp\frac{1}{2}\sqrt{\frac{2\Lambda}{3}}\, s'(y)\ \delta^\mu_\nu, \\
\tilde{\Gamma}^5_{\mu\nu} &= \pm\frac{1}{2}\sqrt{\frac{2\Lambda}{3}}\, s'(y)\ e^{\pm\sqrt{\frac{2\Lambda}{3}}\, s(y)}(\eta_{\mu\nu} + h_{\mu\nu}),
\end{aligned} \tag{11.20}$$

where $s'(y) = \frac{ds(y)}{dy}$. Then it follows that

$$\begin{aligned}
\tilde{R}_{\mu\nu} &= \frac{1}{2}\big\{\partial_\lambda\partial_\mu h^\lambda_\nu - \partial_\mu\partial_\nu h^\lambda_\lambda + \eta^{\lambda\rho}\partial_\nu\partial_\rho h_{\mu\lambda} \\
&- \eta^{\lambda\rho}\partial_\lambda\partial_\rho h_{\mu\nu}\big\} - \frac{1}{2}\sqrt{\frac{2\Lambda}{3}}\, s''(y)e^{\pm\sqrt{\frac{2\Lambda}{3}}\, s(y)}(\eta_{\mu\nu} + h_{\mu\nu}) \\
&+ \frac{2\Lambda}{3}(s'(y))^2\ e^{\pm\sqrt{\frac{2\Lambda}{3}}\, s(y)}(\eta_{\mu\nu} + h_{\mu\nu}), \\
\tilde{R}_{\mu 5} &= 0, \\
\tilde{R}_{55} &= 2\sqrt{\frac{2\Lambda}{3}}\, s''(y) - \frac{2\Lambda}{3}\ (s'(y))^2, \\
\tilde{R} &= e^{\mp\sqrt{\frac{2\Lambda}{3}}\, s(y)}\{\partial^\mu\partial_\rho h^\rho_\mu - \partial_\lambda\partial^\lambda h^\mu_\mu\} \\
&- 5\frac{2\Lambda}{3}\ (s'(y))^2 + 4\frac{2\Lambda}{3}s''(y).
\end{aligned} \tag{11.21}$$

Now $A = B = 5$ part of (15) becomes

$$\Lambda(s'(y))^2 + \frac{1}{2}e^{\mp\sqrt{\frac{2\Lambda}{3}}\, s(y)}(\partial_\nu\partial^\nu h^\mu_\mu - \partial_\rho\partial^\mu h^\rho_\mu) = \Lambda,$$

which will be satisfied *if we take*

$$h^\mu_\mu = 0 \quad ; \quad \partial_\rho h^\rho_\mu = 0, \tag{11.22}$$

the familiar gauge choice on $h_{\mu\nu}$ *and if*

$$(s'(y))^2 = 1, \tag{11.23}$$

and so

$$s(y) = \pm y, \tag{11.24}$$

thereby recovering (12). The $A = \mu$, $B = \nu$ part of (15) then leads to (18). Thus the fluctuations in S^1 as given by (19) are consistent with (12), $\Lambda_{Phys} = 0$ and massless graviton.

So far the fifth dimension is taken to be S^1. It remains to relate this to RS [1] with the choice of $\mid y \mid$ for the eponent in (11). For this we consider the action for a scalar field in the 5-dimensional background (11). It is

$$S = \frac{1}{2}\int d^4x \int dy\sqrt{\mathcal{G}}\Big\{\mathcal{G}^{AB}\partial_A\Phi\partial_B\Phi - M^2\Phi^2\Big\}, \tag{11.25}$$

and expand $\Phi(x,y)$ as

$$\Phi(x,y) = \sum_n \phi_n(x)\chi_n(y). \tag{11.26}$$

Substituting (26) in (25), taking the negative sign in the exponent for example and assuming

$$\int dy\; e^{-\sqrt{\frac{2\Lambda}{3}}\;y}\chi_n(y)\chi_m(y) = \delta_{mn}, \tag{11.27}$$

the 4-dimensional derivative term becomes

$$\int \textstyle\sum_n g^{\mu\nu}\partial_\mu\phi_n(x)\;\partial_\nu\phi_n(x)\;d^4x, \tag{11.28}$$

the standard kinetic energy term for n-scalar fields. The assumption (27) for $\chi_n(y)$ on S^1 can be justified by considering orthogonal polynomials on a circle with measure [12] $e^{-\sqrt{\frac{2\Lambda}{3}}\;y}$. Then the action (25) becomes,

$$\begin{aligned} S &= \frac{1}{2}\int d^4x\sqrt{-g}\Big\{\sum_n g^{\mu\nu}\partial_\mu\phi_n(x)\;\partial_\nu\phi_n(x) \\ &+ \int dy e^{-2\sqrt{\frac{2\Lambda}{3}}\;y}\sum_{m,n}\phi_n(x)\phi_m(x)\partial_y\chi_n(y)\partial_y\chi_m(y) \\ &- M^2\int dy e^{-2\sqrt{\frac{2\Lambda}{3}}\;y}\sum_{m,n}\phi_n(x)\phi_m(x)\chi_n(y)\chi_m(y)\Big\}. \end{aligned} \tag{11.29}$$

It will be natural to realize the second and third terms in (28) along with (27) as $-\sum_n M_n^2\phi_n(x)\phi_n(x)$ so that M_n can be interpretted as the mass of the nth scalar field in M^4. To realize this, we require an eigenvalue equation for $\chi_n(y)$ with M_n^2 as eigenvalue. The first order derivatives in the second term in (28) can be converted to second order derivative on $\chi_n(y)$ by a partial integration over dy. This produces the surface term

$$e^{-2\sqrt{\frac{2\Lambda}{3}}\;y}\chi_n(y)\;\partial_y\chi_m(y)|_{-r_c}^{r_c}.$$

The requirement $\chi_n(r_c) = \chi_n(-r_c)$ of identifying $y = r_c$ and $-r_c$ which is so for S^1 is not sufficient to remove the surface term, due to the exponent. *In addition, if we choose the exponent in (11) as* $e^{\pm\sqrt{\frac{2\Lambda}{3}}\;|y|}$, *then the surface term vanishes.* The resulting eigenvalue equation for $\chi_n(y)$ with (27) then can be solved for the eigenvalue M_n. This eigenvalue equation is

$$\begin{aligned} \frac{d}{dy}\Big\{e^{-2\sqrt{\frac{2\Lambda}{3}}\;|y|}\frac{d}{dy}\chi_m(y)\Big\} &+ M^2 e^{-2\sqrt{\frac{2\Lambda}{3}}\;|y|}\chi_m(y) \\ &= M_m^2 e^{-\sqrt{\frac{2\Lambda}{3}}\;|y|}\;\chi_m(y), \end{aligned} \tag{11.30}$$

agreeing with Eqn.7 of Goldberger and Wise [2], which becomes Bessel equation and solved numerically for M_n in Ref.2. Thus the correct dynamics of the scalar fields with exponentially suppressed mass is obtained. The same reasoning holds good for $e^{\sqrt{\frac{2\Lambda}{3}}\,|y|}$. The choice of $|\, y \,|$ for the exponent, necessitated here by the vanishing of the surface term, thus gives the orbifold structure to the fifth dimension, with $y = 0$ and r_c as fixed points.

This study allows us to extend RS background to higher dimensions (> 5), by considering (3) and (8) for which $\Lambda_{Phys} = 0$ and $R_{ij} \neq 0$. We illustrate this first by considering a torus $S^1 \times S^1$ and then S^2, for the extra dimensions. The torus is Ricci flat and the equation (8) becomes

$$\frac{d^2 f(y_1, y_2)}{dy_1^2} + \frac{d^2 f(y_1, y_2)}{dy_2^2} \;=\; \frac{2\Lambda}{3} f(y_1, y_2). \tag{11.31}$$

The separable solution is

$$f(y_1, y_2) \;=\; e^{\pm\sqrt{\frac{\Lambda}{3}}\, y_1}\, e^{\pm\sqrt{\frac{\Lambda}{3}}\, y_2}, \tag{11.32}$$

where $y_{1,2}$ are the coordinates on $S^1 \times S^1$. The same analysis goes through with 'doubling' and with replacing $y_{1,2}$ by $|\, y_{1,2} \,|$ in the exponents. If the sizes of the two circles are taken as r_1 and r_2, then the 4-dimensional Planck scale will become (analogue of Eqn.15 of RS)

$$\tfrac{3}{\Lambda}\{1 - e^{-\sqrt{\frac{\Lambda}{3}}\, r_1 \pi}\}\; \{1 - e^{-\sqrt{\frac{\Lambda}{3}}\, r_2 \pi}\}, \tag{11.33}$$

so that by choosing the parameters the hierarchy problem can be addressed to. The coupling of scalars with the extended RS background here can be studied by expanding $\Phi(x, y_1, y_2)$ as $\sum_{\ell_1 \ell_2} \phi_{\ell_1 \ell_2}(x) \chi_{\ell_1}(y_1) \chi_{\ell_2}(y_2)$.

We will next consider the case of extension with the extra dimensions taken to be on S^2. This has the property $R_{ij} \neq 0$. Using the standard metric on S^2, to begin with, with the size of S^2 as a, we have $R^{(2)} = -\frac{2}{a^2}$ and (8) becomes

$$\begin{aligned} &\Big\{ sin^2\theta \partial_\theta^2 + sin\theta\; cos\theta\; \partial_\theta \\ &-\tfrac{a^2}{3} sin^2\theta\; (\Lambda + R^{(2)}) \Big\} f(\theta, \phi) + \partial_\phi^2 f(\theta, \phi) = 0, \end{aligned} \tag{11.34}$$

where $0 \leq \theta \leq \pi,\; -\pi \leq \phi \leq \pi$. The differential operator above is suggestive of separation of θ and ϕ as $f(\theta, \phi) \;=\; F(\theta)\; D(\phi)$ and then we find,

$$\begin{aligned} &\tfrac{d^2 D(\phi)}{d\phi^2} = \alpha D(\phi), \\ &\tfrac{d^2 F(\theta)}{d\theta^2} + cot\theta\; \tfrac{dF(\theta)}{d\theta} \\ &-\{\tfrac{a^2}{3}(\Lambda + R^{(2)}) - \tfrac{\alpha}{sin^2\theta}\} F(\theta) = 0, \end{aligned} \tag{11.35}$$

where α is the separation constant. The first equation is solved for $D(\phi)$ as $D(\phi) \;=\; e^{\pm\sqrt{\alpha}\phi}$ and the second one, after using $\mu \;=\; cos\theta$, becomes

$$\frac{d}{d\mu}\{(1 - \mu^2)\frac{dF}{d\mu}\} - \Big\{\frac{a^2}{3}(\Lambda + R^{(2)}) - \frac{\alpha}{1 - \mu^2}\Big\} = 0, \tag{11.36}$$

which is the associated Legendre differential equation. Writing $\nu(\nu + 1) \;=\; -\frac{a^2}{3}(\Lambda + R^{(2)})$ and $\beta \;=\; i\sqrt{\alpha}$, the solution is $F(\theta) \;=\; P_\nu^\beta(cos\theta)$ and so,

$$f(\theta, \phi) \;=\; P_\nu^\beta(cos\theta)\; e^{\pm\sqrt{\alpha}\phi}. \tag{11.37}$$

Thus the generalized RS background for $M^4 \times S^2$ is

$$(ds)^2 = P_\nu^\beta(cos\theta)e^{\pm\sqrt{\alpha}\phi}\eta_{\mu\nu}dx^\mu dx^\nu + g_{ij}(\theta,\phi)dy^i dy^j, \tag{11.38}$$

where i,j stand for $\theta,\ \phi$. Restricting the coordinate ϕ to $\mid \phi \mid$ and $P_\nu^\beta(cos\theta)$ to $P_\nu^\beta(\mid cos\theta \mid)$, the required suppression of 4-dimensional Planck scale can be obtained. With this choice, the coupling of a 6-dimensional scalar field with higher dimensional gravity can be described by

$$S \;=\; \frac{1}{2}\int d^4x \int d^2y\ \sqrt{\mathcal{G}}\Big\{\mathcal{G}^{AB}\partial_A\Phi\partial_B\Phi - M^2\Phi^2\Big\}, \tag{11.39}$$

where $\mathcal{G}_{AB}$ corresponds to (38). Expanding $\Phi(x,\theta,\phi)$ as

$$\Phi(x,\theta,\phi) \;=\; \sum_n \phi_n(x)\chi_n(\theta,\phi), \tag{11.40}$$

where n *collectively* stands for the modes on S^2, imposing

$$\begin{aligned}&\int d\theta d\phi\sqrt{g}f(\theta,\phi)\chi_n(\theta,\phi)\chi_m(\theta,\phi)\\ &= a^2\int sin\theta P_\nu^\beta(\mid cos\theta \mid)e^{-\sqrt{\alpha}\mid\phi\mid}\chi_n(\theta,\phi)\chi_m(\theta,\phi)d\theta d\phi\\ &= \delta_{nm},\end{aligned}$$

and after a partial integration, the action becomes

$$S \;=\; \frac{1}{2}\int \sum_n \Big\{\eta^{\mu\nu}\partial_\mu\phi_n(x)\partial_\nu\phi_n(x) - M_n^2\phi_n^2(x)\Big\}\ d^4x, \tag{11.41}$$

the standard action for free scalar fields in M^4. The masses M_n are determined by the eigenvalue equation,

$$\begin{aligned}&\partial_i\Big\{f^2(\theta,\phi)\sqrt{g}g^{ij}\partial_j\chi_m(\theta,\phi)\Big\} + M^2\sqrt{g}f^2(\theta,\phi)\chi_m(\theta,\phi)\\ &= \sqrt{g}f(\theta,\phi)M_m^2\chi_m(\theta,\phi).\end{aligned} \tag{11.42}$$

This eigenvalue equation for 4-dimensional masses M_m, is a generalization of Eqn.7 of Goldberger and Wise [2] for RS background on $M^4 \times B^2$, where B^2 is S^2 with the use of $\mid cos\theta \mid$ and $\mid \phi \mid$.

This extension to higher dimensions (> 5) encourages us to view the background (3) *directly* and consider the quantum fluctuations in M^4 to obtain conditions on $f(y)$ for massless gravitons. This condition *precisely* turns out to be (8), obtained by setting $\Lambda_{Phys} = 0$. We briefly give this derivation now. The quantum fluctustions on M^4 on the background (3) can be described by the metric

$$ds^2 \;=\; f(y)(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + g_{ij}(y)dy^i dy^j. \tag{11.43}$$

The non-vanishing connection coefficients are obtained as

$$\begin{aligned}
\tilde{\Gamma}^{\mu}_{\lambda\rho} &= \frac{1}{2}(\partial_\lambda h^{\mu}_{\rho} + \partial_\rho h^{\mu}_{\lambda} - \partial^{\mu} h_{\lambda\rho}), \\
\tilde{\Gamma}^{\mu}_{\lambda i} &= \frac{1}{2f(y)}(\partial_i f(y))\delta^{\mu}_{\lambda}, \\
\tilde{\Gamma}^{i}_{\mu\nu} &= -\frac{1}{2}g^{ik}(\partial_k f(y))(\eta_{\mu\nu} + h_{\mu\nu}), \\
\tilde{\Gamma}^{i}_{jm} &= \Gamma^{i}_{jm}.
\end{aligned} \tag{11.44}$$

Using these, it follows,

$$\begin{aligned}
\tilde{R}_{\mu\nu} &= -\frac{1}{2}\{\partial_\nu\partial_\mu h^{\lambda}_{\lambda} - \partial_\rho\partial_\mu h^{\rho}_{\nu} \\
&- \partial_\rho\partial_{nu} h^{\rho}_{\mu} + \partial_\rho\partial^\rho h_{\mu\nu}\} \\
&- \frac{1}{2}\{(\nabla_i\nabla^i f(y)) + \frac{1}{f}(\nabla_i f(y))(\nabla^i f(y))\}(\eta_{\mu\nu} + h_{\mu\nu}), \\
\tilde{R}_{\mu i} &= 0, \\
\tilde{R}_{ij} &= R_{ij} + \frac{1}{f^2}(\nabla_i f(y))(\nabla_j f(y)) \\
&- \frac{2}{f(y)}(\nabla_i\nabla_j f(y)), \\
\tilde{R} &= R^{(n)} - \frac{1}{f(y)}(\partial_\rho\partial^\rho h^{\lambda}_{\lambda} - \partial_\rho\partial^\nu h^{\rho}_{\nu}) \\
&- \frac{4}{f(y)}(\nabla^i\nabla_i f(y)) - \frac{1}{f^2(y)}(\nabla_i f(y))(\nabla^i f(y)).
\end{aligned} \tag{11.45}$$

Using these results, the $A = \mu,\ B = \nu$ part of the Einstein equation gives the equation of motion for the graviton field $h_{\mu\nu}$. In this the mass term will be proportional to $h_{\mu\nu}$. Equating this to zero, we obtain (1) $h^{\lambda}_{\lambda} = 0$; $\partial_\rho h^{\rho}_{\mu} = 0$, which are the familiar gauge choices and (2) $\nabla_i\nabla^i f(y) = (2\Lambda + R^{(n)})\frac{1}{3}f(y)$, which is *exactly* the same equation (8), obtained by imposing $\Lambda_{Phys} = 0$. Thus the massless graviton requirement and vanishing of the 4-dimensional cosmological constant are intimately (and mysteriously) connected in the background metric (3).

To summarize, by insisting on $\Lambda_{Phys} = 0$ and the extra dimensional space B^n to have $R_{ij} \neq 0$, the Kaluza-Klein ansatz (3) with the warp factor $f(y)$ satisfying (8) has been obtained. By restricting to 5-dimensional theory, the metric (11) is derived and it is the RS background to begin with. Quantum fluctuations in M^4 and S^1 (through the warp factor) do not alter the background and consistent with massless graviton. By considering scalar fields in the background (11), the correct 4-dimensional scalar field dynamics is obtained by choosing $\mid y \mid$ in the exponent, which then is the RS^1 background. This study provides a geometric meaning to the constant k of RS as related to the cosmological constant in the higher dimensions. The consistency of the metric with $\Lambda_{Phys} = 0$ and massless graviton is very important. Although the results obtained so far are not entirely new, this study provides the important connection between the warp factor and vanshing of the physical cosmological constant. Further Eqn.8 can be used to construct generalized RS background for higher dimensional (> 5) theory. This is illustrated by taking two examples, $M^4 \times S^1 \times S^1$

and $M^4 \times S^2$. The connection between the vanishing of the physical cosmological constant and the masslessness of gravitons is obtained and this connection is very significant, although the true physical origin remains mysterious within the background (3).

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Chapter 12

Quantum Groups, q-Dynamics and Rajaji

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Abstract.
We sketch briefly the essentials of the quantum groups and their application to the dynamics of a q-deformed simple harmonic oscillator moving on a quantum line, defined in the q-deformed cotangent (momentum phase) space. In this endeavour, the quantum group $GL_{qp}(2)$- and the conventional rotational invariances are respected together. During the course of this discussion, we touch upon Raja*ji*'s personality as a critical physicist and a bold and adventurous man of mathematical physics.

The basic idea behind the concept of "deformation" in theoretical physics is quite old one. In fact, the two most successful and well-tested theories of 20th century, namely; the quantum mechanics and the special theory of relativity, can be thought of as the "deformed" versions of their "undeformed" counterparts: the classical mechanics and the Galilean relativity. The deformation parameters in these theories are supposed to be the Planck constant ($\hbar$) and the speed of light (c) [1, 2] respectively (which turn out to be the two fundamental constants of nature). In the limit when $\hbar \to 0$ and $c \to \infty$, we get back the corresponding "undeformed" physical theories. Long ago, it was proposed that space-time might become noncommutative [3, 4, 5] if we probe the deeper structure of matter with energies much higher than the typical scale of energy for quantum mechanics. Nearly a couple of decades ago, this idea got a shot in its arms in the context of inverse scattering method (and Yang-Baxter equations) applied to the integrable systems [6] and it was conjectured that the deformation of groups based on the quasi-triangular Hopf algebras [7] together with the ideas of noncommutative geometry [8] might provide a "fundamental length" (l_p) in the context of space-time quantization. This will complete the *trio* (i.e. $\hbar, c, l_p$) of fundamental constants of nature and will, thereby, enable us to express physical quantities in terms of these natural units. Recently, there has been an upsurge of interest in the noncommutative spaces [8, 9] in the context of branes in string theory and matrix model of M-theory. However, we shall discuss here some aspects of noncommutativity associated with the space-time structure in the framework of quantum

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groups *alone* and will not touch upon the noncommutativity associated with the string/M-theory.

Let us begin with a q-deformation (where q is a dimensionless quantity), introduced as a noncommutativity parameter for the spacetime coordinates in the D- dimensional flat Minkowski (configuration) manifold, as

$$x_\mu x_\nu = q\, x_\nu x_\mu, \qquad (\mu, \nu = 0, 1, 2, \dots D-1). \tag{1}$$

It can be checked that (1) is *invariant under the Lorentz boost transformations* iff $\mu < \nu$. Moreover, if we reduce (1) to a two dimensional "quantum plane" in space (i.e. $\mu = 1, \nu = 2$)

$$x\, y = q\, y\, x, \tag{2}$$

we see that the conventional rotational invariance for a two dimensional "undeformed" plane is violated. In some sense, the homogeneity and isotropy of space-time becomes questionable because of the loss of these two conventional invariances. In the limit $q \to 1$, the "quantum plane" reduces to an ordinary plane with its rotational symmetry intact.

It was a challenging problem to develop a consistent q-deformed dynamics where conventional invariances were respected. In this context, the Lagrangian and Hamiltonian formulation of a q-deformed dynamics was considered in the tangent and cotangent spaces, defined over 2D q-deformed configuration space (corresponding to the definition (2)) [10]. In this approach, however, the conventional rotational invariance was lost and the status of a one-dimensional physical system was not clear. On the positive side of this approach, a rigorous $GL_{qp}(2)$ invariant differential calculus was developed and then it was applied to the construction of a consistent q-dynamics. In another interesting attempt, a q-deformation was introduced in the Heisenberg algebra [11]. As a result, it was impossible to maintain the Hermiticity property of the phase variables *together*. This led to the introduction of a new coordinate variable in the algebra. Consequently, a single point particle was forced to move on *two trajectories* at a given value of the evolution parameter for $q \neq 1$ (which was not found to be a physically interesting feature). In an altogether different approach, a q-deformation was introduced in the cotangent (momentum phase) space defined over a one-dimensional configuration manifold [12]. In this endeavour, a "quantum-line" was defined in the 2D cotangent manifold as

$$x(t)\; \pi(t) = q\; \pi(t)\; x(t), \tag{3}$$

where t is a commuting real evolution parameter and $x(t)$ and $\pi(t)$ are the phase space variables. In relation (3), the conventional rotational invariance is maintained because a rotation *does not mix a coordinate with its momentum.* However, a rigorous differential calculus was not developed in this approach and *dynamics was discussed by exploiting the on-shell conditions alone.* It was also required that the solutions to equations of motion should be such that the quantum-line (3) is satisfied for all values of the evolution parameter. This way of deformation was generalized to the multi-dimensional systems [13, 14, 15]

$$x_\mu x_\nu = x_\nu x_\mu, \quad \pi_\mu \pi_\nu = \pi_\nu \pi_\mu, \quad x_\mu \pi_\nu = q\, \pi_\nu x_\mu, \tag{4}$$

and the dynamics of (non)relativistic systems was discussed by exploiting the on-shell conditions only. *Prof. G. Rajasekaran (popularly known as "Rajaji" in the physics community of India) is blessed with a very critical mind. Not only he is critical about others' work,* *he is self-critical too. In fact, he was very much critical about these relations in Eq. (4) and argued that there must be some quantum group symmetry behind this choice of relations. His criticism spurred our interest in this problem a great deal.* As a result, we were able to find that, under the following transformations for the pair(s) of phase variable(s): $(x_0, \pi_0), (x_1, \pi_1) \ldots (x_{D-1}, \pi_{D-1})$:

$$\begin{array}{lll} x_\mu & \rightarrow & A\, x_\mu + B\, \pi_\mu, \\ \pi_\mu & \rightarrow & C\, x_\mu + D\, \pi_\mu, \end{array} \tag{5}$$

where A, B, C, D are the elements of a 2×2 matrix belonging to the quantum group $GL_{qp}(2)$ and obeying the braiding relations in rows and columns (with $q, p \in \mathcal{C}/\{0\}$) as:

$$\begin{array}{ll} AB = p\, BA, \quad AC = q\, CA, \quad BC = (q/p)\, CB, \quad BD = q\, DB, \\ CD = p\, DC, \quad AD - DA = (p - q^{-1})\, BC = (q - p^{-1})\, CB, \end{array} \tag{6a}$$

the relations (4) *remain invariant for any arbitrary ordering of μ and ν if the parameters of the group are restricted to obey $pq = 1$.* In fact, relations (4) respect the conventional Lorentz invariance as well as the quantum group $GL_{q,q^{-1}}(2)$

$$\begin{array}{l} AB = q^{-1}\, BA, \quad AC = q\, CA, \quad BC = q^2\, CB, \\ BD = q\, DB, \quad CD = q^{-1}\, DC, \quad AD = DA, \end{array} \tag{6b}$$

invariance *together* if we assume the commutativity of elements A, B, C, D of the above quantum group with the phase variables x_μ and π_μ. It will be noticed that the relationship (6b) has been derived from (6a) for $pq = 1$ (and $GL_{q,q^{-1}}(2) \neq GL_q(2)$). In fact, relations (4) and symmetry transformations (6a,6b) were exploited for the discussion of a consistent q-dynamics for some physical systems in the multi-dimensional phase space [16].

To develop a consistent q-dynamics in the 2D phase space for a one dimensional simple harmonic oscillator (1D-SHO), we shall exploit the definition of a quantum-line (3) in the phase space. This relationship remains invariant under the conventional rotations as well as the following quantum group symmetry transformations

$$\begin{pmatrix} x \\ \pi \end{pmatrix} \rightarrow \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x \\ \pi \end{pmatrix}, \tag{7}$$

where A, B, C, D are the elements of the quantum group $GL_{qp}(2)$ that obey relations (6a). It will be noticed that the quantum-line (3) is also invariant under transformations corresponding to the quantum group $GL_q(2)$ which is endowed with elements (A, B, C, D) *that obey relations (6a) for $q = p$* [13]. In fact, both these quantum groups possess identity, inverse, closure property and associativity under the binary operation $(\cdot)$ as the *matrix multiplication.* However, these quantum groups form what are known as *pseudo-groups.* To elaborate this point, let us examine the group properties of the simpler group $GL_q(2)$. The identity element (I) of this group can be defined from its typical general element (T) as:

$$T_{ij} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in GL_q(2), \quad \text{and} \quad T_{ij} \rightarrow \delta_{ij} \equiv I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{8}$$

for $A = D = 1$ and $B = C = 0$. The determinant of T ($det\ T$) turns out to be the central for this group in the sense that it commutes with all the elements (A, B, C, D)

$$\begin{array}{l} det\ T = AD - qBC = DA - q^{-1}CB, \\ (det\ T)\ (A, B, C, D) = (A, B, C, D)\ (det\ T), \end{array} \tag{9}$$

as can be seen from the q-commutation relations of elements A, B, C, D belonging to the quantum group $GL_q(2)$. Now the inverse of T can be defined as

$$T^{-1} = \frac{1}{AD - qBC} \begin{pmatrix} D & -q^{-1}B \\ -qC & A \end{pmatrix} \equiv \frac{1}{DA - q^{-1}CB} \begin{pmatrix} D & -q^{-1}B \\ -qC & A \end{pmatrix}, \tag{10}$$

because $T \cdot T^{-1} = T^{-1} \cdot T = I$. The closure property ($T \cdot T' = T''$), under matrix multiplication, can be seen by taking two matrices T and $T' \in GL_q(2)$ and demonstrating

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = \begin{pmatrix} A'' & B'' \\ C'' & D'' \end{pmatrix} \in GL_q(2), \tag{11}$$

where it is assumed that elements of the above two matrices, belonging to the quantum group $GL_q(2)$, commute among themselves (i.e., $[T_{ij}, T'_{kl}] = 0$). Exploiting the closure property, it can be checked that the associativity property is also satisfied, i.e.,

$$T \cdot (T' \cdot T'') = (T \cdot T') \cdot T''. \tag{12}$$

The requirement that $[T_{ij}, T'_{kl}] = 0$, entails upon quantum group (e.g. $GL_q(2)$) to be a *pseudo-group*. This is because of the fact that: (i) the elements of the product $T^2 = T \cdot T$ do not belong to the quantum group (e.g. $GL_q(2)$), and (ii) the condition $[T_{ij}, T'_{kl}] = 0$ is not taken into account while defining the inverse matrix (T^{-1}) where the elements of T^{-1} and T are taken to be non-commuting in the proof: $T \cdot T^{-1} = T^{-1} \cdot T = I$.

In our work [16], a consistent q-dynamics was developed where conventional rotational (and/or Lorentz) symmetry invariance, together with a quantum group symmetry invariance, was maintained. A $GL_{qp}(2)$ invariant differential calculus, consistent with the Yang-Baxter equations, was developed in 2D (momentum) phase space and then it was applied for the discussion of dynamics of some physical systems. As an example here, we begin with the following Lagrangian for the 1D-SHO [16]

$$L = P\, \dot{x}^2 - Q\, x^2, \tag{13}$$

where $\dot{x}, x$ are the velocity and position variables (with $x\dot{x} = pq\ \dot{x}x$) and P and Q are the parameters which are, in general, non-commutative. A general discussion for the least action principle leads to the definition and derivation of the canonical momentum (π) and the Euler-Lagrange equation of motion (EOM) as

$$\pi = \frac{1}{p}\, \frac{\partial L}{\partial \dot{x}}, \quad \dot{\pi} = \frac{1}{p}\, \frac{\partial L}{\partial x}. \tag{14}$$

The differential calculus (with $dx = \dot{x}\ dt, d\pi = \dot{\pi}\ dt$ where t is a real commuting evolution parameter) leads to the derivation of the following basic relations [16]

$$\begin{aligned} &x\, \pi = q\, \pi\, x, \quad \dot{x}\, \dot{\pi} = q\, \dot{\pi}\, \dot{x}, \quad \pi\, \dot{x} = p\, \dot{x}\, \pi, \\ &\pi\, \dot{\pi} = pq\, \dot{\pi}\, \pi, \qquad x\, \dot{\pi} = q\, \dot{\pi}\, x + (pq - 1)\, \dot{x}\, \pi, \end{aligned} \tag{15}$$

which restrict any arbitrary general Lagrangian ($L(x, \dot{x})$) to satisfy

$$\begin{aligned} &x\, \frac{\partial L}{\partial \dot{x}} = q\, \frac{\partial L}{\partial \dot{x}}\, x, \quad \dot{x}\, \frac{\partial L}{\partial x} = q\, \frac{\partial L}{\partial x}\, \dot{x}, \quad \frac{\partial L}{\partial \dot{x}}\, \dot{x} = p\, \frac{\partial L}{\partial \dot{x}}\, \dot{x}, \\ &\frac{\partial L}{\partial \dot{x}}\, \frac{\partial L}{\partial x} = pq\, \frac{\partial L}{\partial x}\, \frac{\partial L}{\partial \dot{x}}, \quad x\, \frac{\partial L}{\partial x} = q\, \frac{\partial L}{\partial x}\, x + (pq - 1)\, \dot{x}\, \frac{\partial L}{\partial \dot{x}}. \end{aligned} \tag{16}$$

We demand that the Lagrangian (13) should satisfy all the basic conditions listed in (16). As it turns out, there are two interesting sectors of dynamics for 1D-SHO, described by (13).

These are: (i) when the parameters of deformations are restricted to satisfy $pq = 1$, and (ii) when $pq \neq 1$ for the discussion of solutions to EOM. For the former case, consistent with the differential calculus [16], we obtain the following q-commutation relations

$$x\,\dot{x} = \dot{x}\,x, \quad P\,Q = Q\,P, \quad \xi\,P = q\,P\,\xi, \quad \xi\,Q = q\,Q\,\xi, \tag{17}$$

where ξ stands for $x, \dot{x}$ (i.e. $\xi = x, \dot{x}$). Exploiting these relations, we obtain the following EOM for the system under consideration

$$\ddot{x} = -P^{-1}Q\,x \equiv -\omega^2\,x, \qquad (\omega^2 = P^{-1}Q), \tag{18}$$

which has its solution, at any arbitrary value of the evolution parameter t, as

$$x(t) = e^{i\omega t}\,A + e^{-i\omega t}\,B, \tag{19}$$

where A and B are the non-commuting constants which can be fixed in terms of the initial conditions of the dynamics, as given below

$$A = \tfrac{1}{2}\,[\,x(0) + \omega^{-1}\dot{x}(0)\,], \quad B = \tfrac{1}{2}\,[\,x(0) - \omega^{-1}\dot{x}(0)\,]. \tag{20}$$

The consistency requirements of $GL_{qp}(2)$ invariant differential calculus [16] vis-a-vis relations (17) and (20), lead to

$$AB = BA, \quad B\omega = \omega B, \quad A\omega = \omega A, \quad x\omega = \omega x, \quad \dot{x}\omega = \omega\dot{x}. \tag{21}$$

Furthermore, it can be checked that all the q-commutation relations [16] among $x, \pi, \dot{x}, \dot{\pi}$ are satisfied at any arbitrary value of t. *Thus, we have a completely consistent dynamics in a noncommutative phase space for* $pq = 1$. We have paid a price, however. As it has turned out, all the variables (i.e. $A, B, \omega, x(0), \dot{x}(0)$), present in the solution $x(t)$, are commutative in nature *like we have in conventional classical mechanics.* Thus, in some sense, the nature of this dynamics is *trivial.* In other words, as far as the evolution of the system is concerned, we do not see the effect of q-deformation (or noncommutativity) on the dynamics.

This is the point where *boldness and adventure of "Rajaji" (as a mathematical physicist) came to the fore. He argued that we must find out a non-trivial solution to the equation of motion where parameters of the deformation are not restricted to unity (i.e, $pq \neq 1$).* Thus, let us consider this non-trivial sector of dynamics for 1D-SHO. The Lagrangian (13) *satisfies all but one relations in (16)* with the following q-commutation relations

$$PQ = QP, \quad \xi\,Q = pq^2\,Q\,\xi, \quad P\,\xi = p\,\xi\,P, \quad (\xi = x, \dot{x}). \tag{22}$$

The problematic relation of (16) (the last one!) forces us to require:

$$(pq - 1)(Qx^2 - pq\,P\dot{x}^2) = 0, \tag{23}$$

which emerges from the basic condition: $x\dot{\pi} = q\,\dot{\pi}x + (pq-1)\,\dot{x}\pi$ of Eq. (15). The EOM for the system in this sector (where $pq \neq 1$)

$$\ddot{x} = -\tfrac{1}{p^2q^2}\,P^{-1}Q\,x \equiv -\omega^2\,x, \quad (\omega^2 = \tfrac{1}{p^2q^2}\,P^{-1}Q), \tag{24}$$

has the following general solution in terms of constants A, B and ω

$$x(t) = e^{i\omega t}\,A + e^{-i\omega t}\,B. \tag{25}$$

However, the *restriction (23) is satisfied if and only if: either* $A = 0$ *or* $B = 0$. With the expression for A and B, given in (20), this leads to the restriction: $\dot{x}(0) = \pm\, \omega\, x(0)$. Now the general form of the *exponential* evolution for the 1D-SHO is either

$$x(t) = e^{i\omega t}\, A, \quad \text{with} \quad A\omega = pq\, \omega A, \tag{26}$$

or,

$$x(t) = e^{-i\omega t}\, B, \quad \text{with} \quad B\omega = pq\, \omega B. \tag{27}$$

As a consequence, the dynamics for $pq \neq 1$ does not evolve in the 2D (velocity) phase space but degenerates into a restricted 1D region. In this region, both the restrictions: $\dot{x}(t) = \pm\, \omega\, x(t)$ and $Q\, x^2 = pq\, P\, \dot{x}^2$ are satisfied for all values of the evolution parameter t. In fact, it can be checked that the evolution of the system, described by Eq. (26) and/or Eq.(27), is such that these conditions are very precisely satisfied.

It will be noticed that, unlike the dynamical sector for $pq = 1$, here the constants ω and A (or B) do not commute with each-other and still there exists a consistent "time" evolution for the system, *albeit a restricted one*. In fact, as a result of the q-deformation, the evolution of the 1D-SHO is *strictly* on a 1D "quantum-line" *even-though the whole (velocity) phase space* is allowed for its evolution †. *This new feature is completely different from the discussion of a 1D-SHO in the framework of classical mechanics*. It will be a nice idea to discuss the supersymmetric version of this system in the framework of q-deformed dynamics. It will be very interesting to explore the possibility of $\hbar$-deformation over q-deformation and look for the new aspects of dynamics when q and $\hbar$ both are present.

† The discussion of dynamics in the cotangent (momentum phase) space has been carried out in the Hamiltonian formulation as well [16].

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Chapter 13

Geometric description of Hamiltonian Chaos

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Introduction

Recently a geometric approach has been developed to explain the origin of chaos in Hamiltonian systems [1, 2, 3, 4, 5]. This approach exploits the fact that the trajectories of a Hamiltonian system can be viewed as geodesics on a Riemannian manifold endowed with a suitable metric.

Consider a dynamical system described by the Lagrangian

$$L(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2} a_{ik}(\mathbf{q}) \dot{q}^i \dot{q}^k - V(\mathbf{q}), \tag{13.1}$$

or equivalently the Hamiltonian,

$$H(\mathbf{p}, \mathbf{q}) = \frac{1}{2} a^{ik}(\mathbf{q}) p_i p_k + V(\mathbf{q}). \tag{13.2}$$

For a fixed energy E, define the metric tensor

$$g_{ij} = (E - V) a_{ij}, \tag{13.3}$$

and the 'interval' or the 'proper time' ds by

$$ds^2 = g_{ij} dq^i dq^j = 2(E - V)^2 dt^2. \tag{13.4}$$

It can then be shown that the equations of motion can be written as geodesic equations in a Riemannian manifold:

$$\frac{d^2 q^i}{ds^2} + \Gamma^i_{jk} \frac{dq^i}{ds} \frac{dq^k}{ds} = 0, \tag{13.5}$$

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where Γ^i_{jk} are the Christoffel symbols associated with g_{ij}.

Further, the Jacobi-Levi-Civita equation for the geodesic spread [6] is essentially the tangent flow equations in the geometric form [1]:

$$\frac{\nabla}{ds}\left(\frac{\nabla J^i}{ds}\right) + \left(R^i_{jkl} J^k \frac{dq^l}{ds}\right)\frac{dq^j}{ds} = 0, \tag{13.6}$$

where $J^i = \delta q^i$ denotes the tangent vector, $\frac{\nabla}{ds}$ is the covariant derivative along a geodesic and R^i_{jkl} is the Riemann curvature tensor.

By simple algebraic manipulations of Eq.(6), it can be shown that

$$\frac{1}{2}\frac{d^2}{ds^2}||\mathbf{J}||^2 + K^{(2)}(\mathbf{J},\mathbf{v})||\mathbf{J}||^2 - \left\|\frac{\nabla}{ds}\mathbf{J}\right\|^2 = 0, \tag{13.7}$$

where

$$||\mathbf{J}||^2 = g_{ij}J^iJ^j, \tag{13.8}$$

$$v^i = \frac{dq^i}{ds}, with < \mathbf{J},\mathbf{v} >= g_{ij}J^iv^j = 0, \tag{13.9}$$

and $K^{(2)}(\mathbf{J},\mathbf{v})$ is the sectional curvature given by

$$K^{(2)}(\mathbf{J},\mathbf{v}) = R_{\mu\nu\lambda\eta}\frac{J^\mu}{\|J\|}\frac{dq^\nu}{ds}\frac{J^\eta}{\|J\|}\frac{dq^\lambda}{ds}. \tag{13.10}$$

It is obvious from Eq.(7) that any point with $K^{(2)} < 0$ is an unstable point. Hence it is expected that the average of $K^{(2)}$ over those points in the manifold where it is negative, will give information about the degree of chaos for that manifold. It would be a great advantage if the average of $K^{(2)}$ is a good quantifier of chaos as its computation involves only integration over a constant energy surface. This is in contrast to the computation of Lyapunov exponent which involves integrating out the equations of motion and the tangent flow. In this article, we explore the relation between this geometric quantity and the Lyapunov exponent for Hamiltonian systems with two degrees of freedom.

Sectional curvature as global measure of chaos

It has been argued that it is necessary to go to a higher dimensional manifold by augmenting the configuration space, to get reliable information about the degree of chaos in the system [1, 4]. Then, with a suitable of choice of $\mathbf{J}$ it turns out that the sectional curvature for a system with two degrees of freedom is given by

$$K^{(2)}(\dot{\mathbf{q}},\mathbf{q}) = \frac{1}{2(E-V)}\left[\frac{\partial^2 V}{\partial q_1^2}\dot{q}_2^2 + \frac{\partial^2 V}{\partial q_2^2}\dot{q}_1^2 - 2\frac{\partial^2 V}{\partial q_1 \partial q_2}\dot{q}_1\dot{q}_2\right]. \tag{13.11}$$

$< K^{(2)}_- >$ is the average of the negative values assumed by the sectional curvature over a constant energy surface Σ_E, and is given by

$$\begin{aligned} < K^{(2)}_- > &= \frac{1}{A(\Sigma_E)}\int_{\Sigma_E} d\sigma_E K^{(2)}_- \\ &= \frac{1}{A(\Sigma_E)}\int d\mathbf{p}d\mathbf{q}\delta[H(\mathbf{p},\mathbf{q})-E]\Theta(-K^{(2)})K^{(2)}(\mathbf{p},\mathbf{q}), \end{aligned} \tag{13.12}$$

where the area $A(\Sigma_E)$ is given by

$$A(\Sigma_E) = \int_{\Sigma_E} d\sigma_E = \int d\mathbf{p}d\mathbf{q}\delta[H(\mathbf{p},\mathbf{q}) - E], \tag{13.13}$$

and Θ is the step function. This quantity $< K_-^{(2)} >$ is the object of central interest in this article.

Stability analysis for Hamiltonians with two degrees of freedom indicates that $\det(\partial q^2 V) < 0$ is a sufficient condition for chaos. This is the Toda-Brumer criterion for chaos [7]. When $det(\partial_q^2 V) > 0$ also, chaos is possible provided there are regions in which $\frac{\partial^2 V}{\partial q_1^2} < 0$ and $\frac{\partial^2 V}{\partial q_2^2} < 0$. From the expression for $K^{(2)}$ it can be shown that (i) $det(\partial_q^2 V) < 0$ is a sufficient condition for negativity of $K^{(2)}$ in some regions of the constant energy manifold, and (ii) $K^{(2)}$ is negative in regions with $\frac{\partial^2 V}{\partial q_1^2} < 0$, $\frac{\partial^2 V}{\partial q_2^2} < 0$ even when $det(\partial_q^2 V) > 0$. Hence $K^{(2)}$ is negative precisely when the stability analysis indicates chaos, including the Toda-Brumer Criterion.

We make a detailed comparison between $< K_-^{(2)} >$ and λ for the system of Coupled quartic oscillators (CQO) whose Hamiltonian is

$$H = \frac{p_1^2}{2} + \frac{p_1^2}{2} + q_1^4 + q_2^4 + \alpha q_1^2 q_2^2, \tag{13.14}$$

where α is a parameter [8]. We compute $< K_-^{(2)} >$ and the maximal Lyapunov exponent λ for $E = 1024$ and several values of α. λ is computed for 2000 random initial conditions for each value of α corresponding to this energy. ¿From this, we find $< \lambda >$, the average value of the Lyapunov exponent, and the fraction of phase space corresponding to chaotic trajectories, f over the 2000 initial conditions for a particular value of α. To facilitate comparison among $< K_-^{(2)} >$, f and $< \lambda >^2$ we plot them in the same graph against α in Fig.1. $< K_-^{(2)} >$ and $< \lambda >^2$ are normalized such that at $\alpha = 15$, the scaled values of $< K_-^{(2)} >$ and $< \lambda >^2$ are equal to $f(= 0.904)$ at $\alpha = 15$.

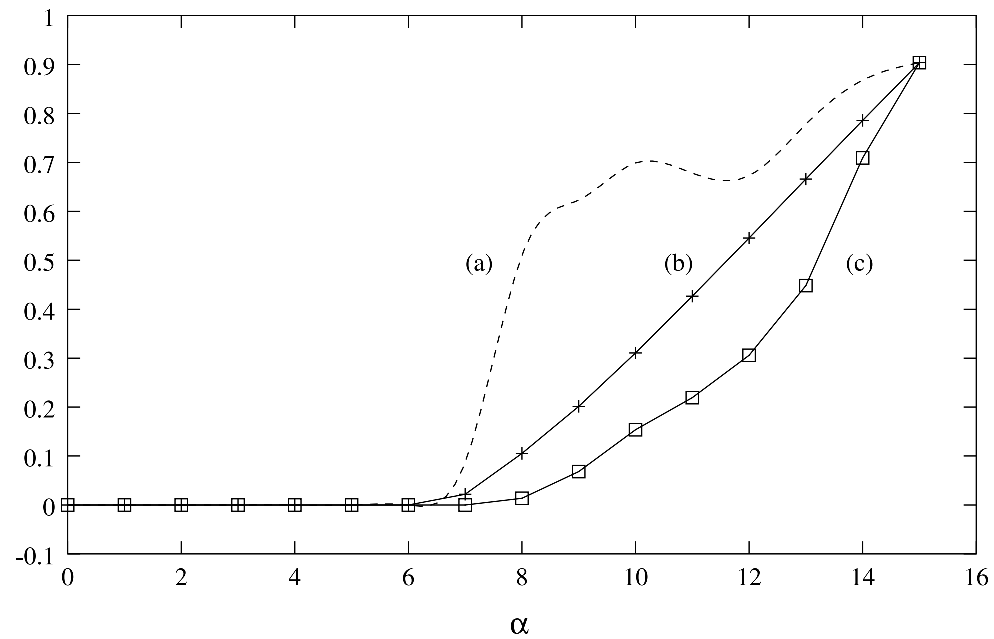

Figure 13.1: Plots of (a) f, (b)-.1533 $\langle K_-^{(2)} \rangle$, and (c) 0.1012 $\langle \lambda \rangle^2$ against α.

It is remarkable that all the quantities are zero for $\alpha < 6$. This follows from an analysis of the negativity of $K^{(2)}$ and the Toda-Brumer criterion. It is seen that $< K_-^{(2)} >$ is closely related to $< \lambda >^2$ rather than f. This can be understood from the scaling properties of $K^{(2)}$ and λ. Under the scaling transformation, $p_i \to \beta^{\frac{1}{2}} p_i$, $q_i \to \beta^{\frac{1}{4}} q_i$, the energy scales as $E \to \beta E$ and $K^{(2)}$ scales as $K^{(2)} \to \beta^{\frac{1}{2}} K^{(2)}$. Hence $< K_-^{(2)} >$ scales as $< K_-^{(2)} > \sim E^{\frac{1}{2}}$ when the energy is varied. It can be shown that $\lambda \to \beta^{\frac{1}{4}} \lambda$ under the scaling transformation. Hence $\lambda \sim E^{\frac{1}{4}}$ when the energy is varied. Obviously, the scaling property is valid for any value of energy. Our numerical data confirm these scaling relations, as well as the fact that the fraction of phase space f corresponding to chaotic trajectories does not vary with E for a given α. This is the reason why $< K_-^{(2)} >$ has close correspondence with $< \lambda >^2$, rather than with f.

We have considered another system known as "Yang-Mills-Higgs Hamiltonian" [9] with

$$H = \frac{p_1^2}{2} + \frac{p_2^2}{2} + 3q_1^2 q_2^2 + \frac{3}{4} q_1^4 + \frac{3}{2} \kappa q_2^4 - \frac{1}{2} \kappa q_2^2. \tag{13.15}$$

Here also, it is found that $< K_-^{(2)} >$ and $< \lambda >^2$ are reasonably close to each other(apart from a scale factor) [8]. Moreover, it is found that both $< K_-^2 >$ and $< \lambda >^2$ scale as $E^{\frac{1}{2}}$, even though the potential is not a homogeneous quartic.

Conclusions

A recently proposed geometric description of Hamiltonian chaos has been very successful, both in qualitative and quantitative aspects. For systems with a small number of degrees of freedom, it has been claimed that $< K_-^{(2)} >$ (a suitably defined sectional curvature averaged over its negative values over a constant energy manifold) is an indicator of the global degree of chaoticity at that energy. If true, this is a great advantage, as $K^{(2)}$ is an integral over the constant energy surface, and one can dispense with the tedious numerical evolution of the trajectories and the 'difference' vector to estimate chaos in the system.

We have investigated in detail, the relations between $< K_-^{(2)} >$, and the average maximal Lyapunov exponent $< \lambda >$, for some Hamiltonian systems with two degrees of freedom.

We find that $< K_-^{(2)} >$ is closely related to $< \lambda >^2$. Both $< K_-^{(2)} >$ and $< \lambda >^2$ scale as $E^{\frac{1}{2}}$, for the quartic potentials we have considered. In fact, for potentials which are of n^{th} degree in the coordinates, it is easy to see that both $< K_-^{(2)} >$ and $< \lambda >^2$ scale as $E^{\frac{(n-2)}{n}}$. Thus the linear relation between $< K_-^{(2)} >$ and $< \lambda >^2$ is expected to be a general feature of Hamiltonian systems.

What about systems with large number of degrees of freedom (N)? For such systems, it has been shown that the 'global degree of chaoticity' can be obtained by computing the mean Ricci curvature, averaged over a constant energy manifold, independent of the dynamics of the system. It is possible to make an analytic estimate of the largest Lyapunov exponent λ, by making a Gaussian hypothesis about the statistics of curvature fluctuations along a geodesic. This exponent is in very good agreement with the corresponding exponent obtained from tangent dynamics[3]. Moreover it has been demonstrated that the geometric approach can even provide a frame work of understanding phase transition which are related to the singular behaviour of curvature fluctuations [5]. In fact, when a system undergoes a phase transition, the fluctuations of the configurational space curvature, when plotted as a function of either the temperature or energy of the system exhibit a singular behaviour at the phase transition point, which can be qualitatively reproduced using geometric methods.This has been explicitly verified for some model large -N Hamiltonians. The success of this approach has led Casetti, Pettini and Cohen [5] to put forth the 'Topological Hypothesis', whose essential content is that phase transitions would be related at a deeper level to a change in the topology of the configuration space of the system. Hence, the geometric approach to Hamiltonian dynamics has been successful in its many applications and is a promising and interesting field of enquiry.

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Chapter 14

Quantum dynamics of superposed states and classical nonlinear maps

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Introduction

The phase space dynamics of classical systems whose evolution is governed by nonlinear equations has been studied extensively for over several years now. The long-time behaviour of such systems and the degree of randomness they exhibit varying from mere ergodicity to chaos have been analysed extensively using both analytical and numerical tools. While much investigation has been undertaken on low-dimensional maps and flows, and the classical dynamics that arises in the phase space of these systems examined, their quantum counterparts have not been investigated in great detail. While in broad terms, chaos in classical systems can be traced back to the nonlinear equations that govern the system's temporal evolution, the quantum counterparts obey the Schrodinger equation, (or the Liouville equation, for mixed states). These are linear equations, and information loss in these systems must arise from repeated measurements and the effects of the uncertainty principle. But the precise manner in which such information loss arises is poorly understood, and needs further study.

One aspect of such a study is the identification of suitable quantum systems where the discrete time evolution of an appropriate parameter is guided by a classical nonlinear map equation. Such an example of a quantum phenomenon arises in the context of wave packet revivals, where the dynamics of an appropriate phase is identical to the classical rotation map on the circle [1]. This map exhibits ergodicity, and not chaos. An obvious extension then would be the identification of a quantum system where the underlying dynamics of an appropriate phase exhibits full-fledged chaos. The temporal dynamics of such a phase would be given by a chaotic one-dimensional map, like the Bernoulli shift map. The shift map not merely serves as a paradigm of chaos, but also displays the ultimate stage in the hierarchy of randomness. In fact, in a certain sense, the Bernoulli shift is as random as a coin toss. It is therefore a matter of considerable interest to identify quantum mechanical analogs of this map. We have examined such a case, namely, the quantum cloning of the photon, an example which gets more impetus due to the fact that it is the optical analog of the quantum network given for relative phase estimation [2]. We outline the essential features of both the wave packet revival phenomenon and the cloning problem in the subsequent sections. These two examples have been selected as they display both the extreme situations in the hierarchy of randomness that arises in nonlinear dynamics - namely, mere ergodicity and fully developed chaos. In the next section we review the problem of wave packet revivals, and in Section 3 we examine the evolution of a suitable phase, in repeated cloning of the photon through stimulated emission from two-level atoms.

Wave Packet Revivals

Revivals of a non-stationary state of a Hamiltonian may be regarded as the quantum analogs of recurrences in a classical dynamical system. In general, when a quantum mechanical system is in a state $|\psi\rangle$, the autocorrelation function $|\langle\psi(0)|\psi(t)\rangle|^2$ decays from its initial value of unity as the time t increases. Under very specific conditions however, it may return to its initial value at certain instants of time, signalling a full revival. In the generic case though, any return would be to a value in a small neighborhood of unity [3], which we shall term a "near revival". This would be the counterpart of a Poincaré recurrence [4] in a classical system, although, of course, there is no direct connection between revivals and classical periodic orbits in the phase space of the same physical system. Due to the fact that measuring devices have a non-zero least count, in practice near revivals rather than exact revivals are the readily identifiable phenomena of interest.

In a system whose evolution is governed by a nonlinear Hamiltonian, both revivals and fractional revivals can occur. Fractional revivals correspond to a situation where the wave packet evolves to a superposition of similar wave packets. Both revivals and fractional revivals of wave packets are of considerable current interest in many contexts, for example, in atom optics [5], propagation of coherent light through a Kerr medium [6], the dynamics of population inversion of atoms interacting with radiation [3, 7, 8] etc.

Our interest here is in near revivals. It is possible to suitably cycle the parameters in the governing Hamiltonian to suppress fractional revivals [9]. This is because as a by-product of the cyclic variation of the parameters in the Hamiltonian, the state vector may pick up an extra geometric phase . This phase can be exploited to get rid of fractional revivals. Once fractional revivals are suppressed the system can be effectively modelled by a linear Hamiltonian which allows for full or near revivals alone to occur. Further, the introduction of a time period T over which the parameters are cycled, also discretizes time in a natural way, and it is only at instants separated by an interval T that the Hamiltonian returns to its original self. We therefore look for revivals of a state only at the instants $T, 2T, \cdots$. Correspondingly, we analyze the problem in terms of returns in a discrete-time map. Due to the presence of the geometric phase the revival times get shifted by the time required to cover the angular excess (or decrement) representing the anholonomy in the semiclassical limit, namely, the corresponding Hannay angle [10].

For definiteness, we demonstrate this in the case of a unitarily deformed oscillator Hamiltonian whose eigenstates are generalized coherent states, while the spectrum remains linear (equi-spaced). These states provide a setting in which anholonomies can be measured experimentally [11, 12] and analyzed theoretically [13, 14], in a physical range extending from the extreme quantum regime to the semiclassical limit. The latter can thus serve as a clear signature of the dynamics of wave packet revivals. In what follows therefore, we discuss the revival problem with special emphasis on the oscillator coherent states.

Consider a system with a time-independent hermitian Hamiltonian H, with spectrum $\{E_n\}$ and eigenstates $\{|\phi_n\rangle\}$. Let the system be prepared in an initial state $|\psi(0)\rangle$ that is a superposition of the $\{|\phi_n\rangle\}$, sharply peaked about some n_0. We expand E_n as

$$E_n \; = \; E_{n_0} + (n - n_0)\, E'_{n_0} + (1/2)(n - n_0)^2\, E''_{n_0} + \cdots \tag{14.1}$$

As we wish to analyze only revivals and fractional revivals, we retain only terms up to the second order in Eq. (14.1) and by suitably shifting n we have the quadratic form

$$E_n \; = \; C_0 \, + \, C_1\, n \, + \, C_2\, n^2\,. \tag{14.2}$$

The coefficients C_i evidently depend on the parameters that occur in H. We shall assume, without loss of generality that $C_1,\, C_2 \,>\, 0$. In the $|\phi_n\rangle$-basis the time evolution operator $U(t) = \exp\left[-i\, H\, t/\hbar\right]$ has the representation

$$U(t) = \sum_n^\infty \exp[-i\, (C_0 + C_1\, n + C_2\, n^2) t/\hbar] |\phi_n\rangle\langle\phi_n\,|. \tag{14.3}$$

For a full revival (the auto-correlation function $\mathcal{C}(t) \;=\; 1$) to occur at time t, $U(t)$ must reduce to the unit operator (apart from a possible overall phase factor), i.e., $(C_1 n + C_2 n^2)t$ must be an integer multiple of $2\pi\hbar$ for every n in the summation. The following cases arise:

(i) $C_1 \neq 0$, $C_2 = 0$ (equi-spaced or linear spectrum): Revivals of an initial state occur with a period $T_{rev} = 2\pi\hbar/C_1$.

(ii) $C_1 = 0$, $C_2 \neq 0$: Revivals occur with a period $T_{rev} = 2\pi\hbar/C_2$.

(iii) $C_1, C_2 \neq 0$, $C_1/C_2 \;=$ a rational number r/s: Once again, full revivals occur with a fundamental revival time $T_{rev} = 2\pi\hbar s/C_2$. A specific example is provided by the Hamiltonian ${a^\dagger}^2 a^2 \;=\; a^\dagger a(a^\dagger a - 1)$ that is relevant to wave packets propagating in a Kerr medium. It is evident that, in this case, E_n is proportional to $n(n-1)$ which is an even integer for every n.

(iv) $C_1, C_2 \neq 0$, C_1/C_2 irrational (the generic case): As the condition $(C_2 n^2 + C_1 n)t \;=\; 2\pi\hbar m$ ($m =$ integer) cannot be satisfied for *all* n at any value of t, full revivals are no longer possible. However, at certain instants of time the quantity $\mathcal{C}(t)$ could come *arbitrarily* close to unity, producing a near revival.

Between occurrences of full (or near) revivals, the wave packet breaks up into a finite sum of subsidiary packets at specific instants of time, *provided* the spectrum is nonlinear, i.e., $C_2 \neq 0$ [15]. These fractional revivals occur at times t given by $t \;=\; \pi\hbar r/C_2\, s$ where r and s are mutually prime integers. It can be shown that at these instants the evolution operator U can be expressed as a finite sum of operators U_p, in each of which the phase factor multiplying the projection operator $|\phi_n\rangle\,\langle\phi_n|$ is linear in n: That is,

$$U\left(\pi\hbar r/C_2\, s\right) \;\; = \;\; \sum_{p=0}^{l-1} a_p^{(r,s)} \;\; U_p\,, \tag{14.4}$$

with

$$a_p^{(r,s)} \;\; = \;\; (1/l) \sum_{k=0}^{l-1} \exp\left[-(i\,\pi\, k^2 r/s) \; + \; (2\, i\, \pi\, kp/l)\right]\,, \tag{14.5}$$

and

$$U_p \;\; = \;\; \sum_{n=0}^{\infty} \exp\left(-i\, n\theta_p\right)\, |\phi_n\rangle\, \langle\phi_n|\,, \;\; \theta_p \;\; = \;\; \pi\; \left[(C_1 r/C_2\, s) \; + \; (2p/l)\right]\,. \tag{14.6}$$

It is this decomposition of U which is responsible for fractional revivals, as can be seen by examining the action of the operator U_p on the initial state. For instance, if the initial state $|\psi(0)\rangle$ is the "coherent state" $|z\rangle$ given by

$$|z\rangle \;=\; \exp\left(-|z|^2/2\right)\;\sum_{n=0}^{\infty}\frac{z^n}{\sqrt{n!}}\,|\phi_n\rangle\;,\; z\in\mathbb{C}\;, \tag{14.7}$$

Eqs. (14.4)–(14.6) yield

$$|\psi(\pi\hbar r/C_2 s)\rangle \;\;=\;\; \exp\left(-i\pi C_0 r/C_2\, s\right)\sum_{p=0}^{l-1} a_p^{(r,s)}\;\; |z\exp(-i\,\theta_p)\rangle\,. \tag{14.8}$$

The state at time $\pi\hbar r/C_2 s$ is therefore a weighted sum of wave packets, each of which has the same form as the initial state $|z\rangle$.

It is clear from the foregoing that fractional revivals arise from the quadratic dependence of E_n on n, and further, a whole host of such revivals of varying intensities can appear in a given case, depending on the precise values of C_2 and the integers r and s for which these revivals are detectable. However, by inducing suitable geometric phases in the basis states $|\phi_n\rangle$, we can eliminate these partial revivals and restore near revivals at specific instants of time.

To demonstrate this, we now consider the situation in which the Hamiltonian contains a set of "slow" parameters $\mathbf{R}$ that can be varied adiabatically and cyclically with period T, as in Berry's original setting for the geometric phase: i.e., $\mathbf{R}_T = \mathbf{R}_0$. For a general nonlinear spectrum E_n, one may expect that the dependence of the geometric phases on n is of the form

$$\gamma_n = \Theta_0 \,+\, \Theta_1\, n \,+\, \Theta_2\, n^2 \,+\, \cdots \tag{14.9}$$

This is generic, as it only requires that γ_n be a regular function of n. As in the expansion of E_n, only terms up to $O(n^2)$ in the above equation are relevant for our present purposes. The coefficients Θ_i will clearly depend on the manner in which the parameters $\mathbf{R}$ are varied. The unitary operator responsible for the time evolution of the wave packet will now get modified to

$$U(kT) = \sum_n \exp\left[i\,k(\nu_0 \,+\, \nu_1 n \,+\, \nu_2 n^2)\right]\;|\phi_n\,,\mathbf{R}_0\rangle\,\langle\phi_n,\mathbf{R}_0| \tag{14.10}$$

where

$$\nu_i = \Theta_i - (1/\hbar)\int_0^T C_i(t)\,dt\;,(i \;=\; 0,1,2)\;. \tag{14.11}$$

From the earlier discussion, it is clear that all fractional revivals of a wave packet will be eliminated if the coefficient ν_2 of n^2 in the foregoing expression for $U(kT)$ vanishes: this happens if we arrange the variation of the parameters such that $\Theta_2 \;=\; (1/\hbar)\int_0^T C_2(t)\,dt$. Once this is done, the exponent in $U(kT)$ has only terms that are linear in n. Hence, at the relevant times kT $(k = 1, 2, \cdots)$, the wave packet will no longer exhibit fractional revivals.

After the cancellation of the n^2 term in the phase factors in Eq. (14.10), the effective time development operator at time kT $(k = 1, 2, \cdots)$ is of the form

$$U(kT) \;\;=\;\; \exp(ik\nu_0)\sum_n \exp(ikn\nu_1)\;|\phi_n\,,\mathbf{R}_0\rangle\,\langle\phi_n,\mathbf{R}_0|\,. \tag{14.12}$$

Therefore an initial coherent state $|\psi(0)\rangle \;=\; |z\rangle$ given by Eq. (14.7) evolves to

$$|\psi(kT)\rangle \;\;=\;\; |z\,\exp(ik\nu_1)\rangle\;. \tag{14.13}$$

Correspondingly, the autocorrelation function becomes

$$\mathcal{C}(kT) \;=\; \exp\left[2|z|^2\left(\cos k\nu_1 - 1\right)\right] . \qquad (14.14)$$

Thus, if $k\nu_1$ happens to be an integer multiple of 2π (recall that ν_1 depends on T), then full revivals occur with $T_{rev} = kT$ as the basic revival time. In general, however, ν_1 is an irrational number (modulo 2π). Therefore, if θ_0 is the phase of the complex number z labelling the initial state $|\psi(0)\rangle$, and θ_k that of the corresponding number at time kT, Eq. (14.13) shows that the discrete time evolution of the state is entirely equivalent to the circle map

$$\theta_k \;=\; \theta_{k-1} + \nu_1 \pmod{2\pi} \qquad (14.15)$$

corresponding to a rotation.

Equivalently, we can write

$$\theta_{k+1} \;=\; \theta_k - 2\pi\Delta \pmod{2\pi}. \qquad (14.16)$$

If Δ is a rational number p/q, every orbit of the map is periodic with a period q. Correspondingly, every initial state $|z(0)\rangle$ has revivals at times $qT,\ 2qT,\ \cdots$.

The generic case however, corresponds to an irrational value of Δ. As is well known, the map no longer has any periodic orbits, but the iterates of *any* θ_0 cover S^1 densely as $k \to \infty$. The dynamics is ergodic but not mixing, with a uniform invariant density $\rho(\theta) = 1/(2\pi)$. Although we no longer have *exact* revivals in principle, the system exhibits *near* revivals – precisely the analog of Poincaré recurrences. Hence, the following complete analysis of the statistics of near revivals can be done.

Consider a prescribed small angular interval I_ϵ of size $2\pi\epsilon$, located symetrically about the initial phase θ_0 on S^1. Let $\Delta \equiv [\Delta] + \delta$, where $[\Delta]$ stands for the integer defined by $\Delta - 1 < [\Delta] < \Delta$ (for *either* sign of Δ), so that δ is an irrational number satisfying $0 < \delta < 1$. It is then easy to see that $\theta_k \in I_\epsilon \Rightarrow \{k\delta\} < \epsilon$, where $\{x\}$ denotes the fractional part of x and the corresponding correlation function then merely differs from unity by a term of order ϵ^2. To order ϵ, therefore, we may regard a return of θ_k to I_ϵ as a (near) revival. The statistics of the occurrence times of such revivals is then identical to that of the recurrences to an angular interval of size $2\pi\epsilon$ in the rotation map. The solution to the latter problem is given by certain gap theorems for interval exchange transformations [16, 17]. Applying these to the case at hand, we obtain the following results in the long-time limit, after the transients due to specific initial conditions have died out and the invariant measure is attained. The mean time between successive near revivals is, in units of T, the reciprocal of the invariant measure of I_ϵ and is just T/ϵ, as one may expect. The *distribution* of recurrence times is, however, quite remarkable [18]. In general (i.e., for arbitrary ϵ and δ), it is concentrated at no more than *three* points $T_1 = k_1 T$, $T_2 = k_2 T$ and $T_3 = T_1 + T_2$, where k_1 and k_2 are the least positive integers such that

$$\{k_1\delta\} \;<\; \epsilon \ \text{and}\ 1 - \{k_2\delta\} \;<\; \epsilon \qquad (14.17)$$

respectively. (Recall that $\epsilon \ll 1$; as long as $\epsilon < 1/2$, $k_1 \neq k_2$.) It is easy to show that $\epsilon \leq \{k_1\delta\} + 1 - \{k_2\delta\}$: when the equality sign applies, only *two* recurrence times (T_1 and T_2) occur.

We have considered the example of wave packet revivals where the phase in question evolves like the map variable of the rotation map, which is not chaotic. and demonstrated how results from ergodicity theory can be used to make predictions on revival times. In the next section, we discuss another example from quantum mechanics, namely photon cloning through light-atom interactions. We identify a phase in this problem which evolves in discrete time like the map variable corresponding to the Bernoulli shift map.

Quantum Cloning

Photon cloning has had a rich history. When a two-level atom in the excited state releases a photon through stimulated emission due to interaction with an incident photon, the emitted photon is guaranteed to have the same polarization as the incident photon. In this sense, a perfect copy of the incident photon has been produced. In their seminal paper [19] Wootters and Zurek argue that given such a quantum cloning machine that ideally copies some polarization states, arbitrary quantum superpositions of these states cannot be exactly cloned. However,the extent to which these superposed states can be cloned have been examined and quantifiers of the efficiency of cloning have been given both in terms of the Hilbert-Schmidt norm of the difference between appropriate density matrices [20], and a suitably defined fidelity factor [21]. In turn, both input state-dependent and universal quantum copying machines have been proposed, and their designs have been suggested [22] using both logic gates and optical devices.

However, one aspect which is not examined here is the nature of information loss in the relstive phase between superpositions of basis states when the bases are ideally cloned *repeatedly* in a copying machine. The standard prescription to determine the relative phase between two beams of light into which an initial beam is split, is to conduct an interference experiment. This has a parallel in a very different area - namely, quantum networks and information theory, where a set-up has been given to estimate the relative phase between two qubits [2]. Inspiration for this set-up can be traced back to the two fundamental properties of quantum states- superposition and interference. These two principles have also played a major role in other related areas of quantum information theory like the development of rapid search algorithms in a given database, algorithms for drastically reducing the number of computational steps in a given problem, and so on. Dividing information which was originally contained in a single bit into a superposition of qubits, is achieved in network theory through an Hadamard transformation. This is the analog of what a beam-splitter does in conventional optics. The relative phase between two qubits itself is determined in the context of networks by using appropriate X-OR gates. A suitable combination of Hadamard transformations and the X-OR operation is the analog of an interference experiment in optics conducted to determine the relative phase between two states.

It is a matter of interest that a similar combination has been successfully suggested to solve the problem posed by Deutch [23], concerning Boolean functions. Here, one is required to determine in a *single* measurement whether a given function is a constant or a balanced function, where, in general there are four options, two corresponding to constant functions and two to balanced functions. The procedure has been successfully extended to solve a more generalized version of the Deutsch problem subsequently proposed by Deutsh and Jozsa [24].

Returning to the problem at hand, namely, relative phase estimation, we first briefly review the essential features of the network proposed to estimate the relative phase between two qubits. This is based on the theory of quantum Fourier transforms [25], and we outline the pertinent features below: Let $a \ \epsilon \ \{0.1\}^m$. a is a thus a binary string with m entries, each being 0 or 1. In decimal notation, $a \ \epsilon \ \{0, 1, 2, ...2^m - 1\}$. The quantum Fourier transform on the additive group of integers modulo 2^m is a mapping done on a and is defined as [2]

$$|a\rangle \rightarrow \sum_{y=0}^{2^m-1} exp(2\pi i a y/2^m)|y> . \tag{14.18}$$

Here, $y \ \epsilon \ \{0, 1, 2, ...2^m - 1\}$ and $|a\rangle$ has m entries, each being 0 or 1. In binary notation, the above equation reads as

$$|a\rangle \rightarrow \sum_{y=0}^{2^m-1} exp(2\pi i(0.a_1a_2a_3....a_m)y)|y> . \tag{14.19}$$

This follows from the fact that if $a = (a_1a_2a_3...a_m)$, where each of the a_ks takes on the value 0 or 1, and $k = (1, 2, ...m)$, $a/2^m = (0.a_1a_2a_3...a_m)$. It can then be checked that for a given value of $|y\rangle$ say $|y_1y_2y_3...y_m\rangle$, $exp(2\pi i(0.a_1a_2a_3...a_m)(y_1y_2y_3...y_m))|y_1y_2y_3...y_m\rangle$ can again be written as the product

$$exp(2\pi i(0.a_m)y_1)|y_1\rangle \ \ exp(2\pi i(0.a_{m-1}a_m)y_2)|y_2\rangle...exp(2\pi i(0.a_1a_2a_3...a_m)y_m)|y_m\rangle \ .$$

In the above we have omitted the integral part of $(a)(y)$. It can also be independently verified that the RHS of Eq.(14.18) can be factorized in the form

$$(|0\rangle + exp(2\pi i(0.a_m))|1\rangle) \ \ (|0\rangle + exp(2\pi i(0.a_{m-1}a_m))|1\rangle) \ \ ...(|0\rangle + exp(2\pi i(0.a_1a_2a_3...a_m))|1\rangle) \ .$$

and that each term tnat arises here can be identified with a corresponding term in the product form given earlier. In this sense the quantum Fourier transform of $|a\rangle$ is factorizable.

Good estimators for arbitrary phase differences between states can be obtained using the principle of the quantum Fourier transform outlined above. We briefly discuss a procedure for doing this using networks, and subsequently draw parallels between this procedure and a photon cloning experiment that we will outline. Further, in both cases we will indicate how an appropriate relative phase exhibits chaos. The network for estimatimg relative phases relies on the implementation of a set of n unitary transformations U_{2^n} on qubits. Let $|\psi\rangle$ be an eigenvector of U_{2^n} such that

$$U_{2^n}|\psi\rangle \ \ = exp(2\pi i 2^n \phi)|\psi\rangle \ . \tag{14.20}$$

Here, $0 \ \leq \ \phi \ \leq \ 2\pi$. Apart from the state $|\psi\rangle$ a 'control' qubit is also provided. U_{2^n} is called a 'controlled' unitary operator as it's action is such that if the control qubit is in the state $|0\rangle$, it performs the identity operation on $|\psi\rangle$, and it pulls out a factor $exp(2\pi i 2^n \phi)$, if the control qubit is in the state $|1\rangle$. These are the only two states allowed for the control qubit. Thus we have,

$$U_{2^n} \ \ |0\rangle|\psi\rangle \ \rightarrow \ |0\rangle|\psi\rangle, \tag{14.21}$$

and

$$U_{2^n} \ |1\rangle|\psi\rangle \ \rightarrow \ exp(2\pi i 2^n \phi)|1\rangle|\psi\rangle \ . \tag{14.22}$$

Clearly, the controlled unitary operation mimics the X-OR operation in logic gates as it introduces a relative phase between the two states $|0\rangle$ and $|1\rangle$.

We now discuss the actual network [2]. It initially consists of an auxiliary qubit $|\psi\rangle$ and n qubits, each prepared in the state $|0\rangle$. We shall label these n qubits $|0\rangle_i$, with $i = 0, 1, 2, ...n - 1$. An Hadamard transformation is now done on the $|0\rangle_i$s, so that each splits up to a superposition of $(|0\rangle \;+\; |1\rangle)$, apart from the normalization. The operation U_{2^0} is now performed on $|0\rangle_0|\psi\rangle$. This leaves $|\psi\rangle$ unchanged and converts $|0\rangle - 0$ to $|\phi\rangle_0$ which is $(|0\rangle + exp(2\pi i2^0\phi)|1\rangle)$. This is followed by the operation U_{2^1} on $|\psi\rangle$ and $|0\rangle_1$. The operation clearly converts them to the output state $|\psi\rangle$ and $|\phi\rangle_1$ which is simply $(|0\rangle + exp(2\pi i2^1\phi)|1\rangle)$. In the next stage, the operation U_{2^2} is carried out on the pair $|\psi\rangle$ and $|0\rangle_3$, and so on. There are totally n stages in this operation and the net output clearly consists of the auxiliary qubit $|\psi\rangle$ tigether with the set $|\phi\rangle_i$ with $i \;=\; 0, 1, 2, .n-1$. This set can be written as $\sum_{y=0}^{2^m-1}\; exp(2\pi i\phi y)|y\rangle$. From our earlier discussion it is evident that the network has implemented a quantum Fourier transform on $|\phi\rangle$. However, ϕ is not in general a rational number, and only an m-bit estimate of ϕ can be obtained by doing the inverse quantum Fouries transform of the above output.

Now consider a realistic situation, where the network above is slightly modified, as follows: We begin the discussion at that stage when the first unitary transformation has been already implemented and $|0\rangle_0$ has been converted to $|\phi\rangle_0$. Let us suppose that the phase ϕ is measured with a suitable device, and the measured value is used as an input in the first stage of the program- namely, in implementing the unitary transformation U_{2^1} on the appropriate states. The generic ϕ, being irrational, an initial error has already been introduced in implementing U_{2^1}. If the new phase is measured and used to implement U_{2^2} in the second stage this error magnifies. At the end of n steps of the program, a small error ϵ in the initial phase measurement has increased to $2^n\epsilon$. In fact the phase ϕ is governed by the map equation

$$\phi_{n+1} \;=\; 2\phi_n, \tag{14.23}$$

where the suffix denotes the stage in the operation, and ϕ is considered modulo 2π. The initial error ϵ in the measurement of ϕ is also governed by a similar equation. This is simply the map equation for the Bernoulli shift map, which displays chaos and has a Lyapunov exponent equal to ln 2.

We now return to the photon cloning experiment we mentioned earlier and examine the possibility of finding the equivalent of the network problem in optics, replacing the Hadamard operation and the X-OR gates with coherent states, beam splitters, light-atom interactions through stimulated emission and an interference experiment. We denotee the initial state of the radiation field in the experiment that we outline below, by the standard coherent state $|z\rangle$ given in Eq.(14.7).Other states of the radiation field that arise during the cloning procedure will be described by appropriate superpositions of $|z\rangle$ and the genaralized coherent states $\{|n,z\rangle\}$, with $n\;=1,2,...$The generalized coherent states are got by repeated application of the operator$(a^\dagger - z^*)$ on $|z\rangle$. The set $\{|z\rangle, |n,z\rangle\}$ form an orthonormal basis. In contrast to these states the photon-added coherent states $\{|z,n\rangle\}$, with $n \;=\; 1,2,...$ [26], do not form an orthonormal set, and can in fact be expanded in the basis given above. In general, normalizing them to unity, we have

$$|z,n\rangle \;=\; c_0|z\rangle \;+\; \sum_{p=1}^{n} c_p|p,z\rangle \;, \tag{14.24}$$

with the c_is $i = 0,1,..$ being the expansion coefficients whose modulus^2 sums up to unity.

To begin with, we consider the interaction of the radiation field $|z\rangle$ with two-level atoms in the excited state. The set-up requires the suppression of spontaneous emission of photons, so that ideal cloning can occur. A single photon in this radiation field interacts with one two-level atom and produces a clone of itself. Consequently the state of the radiation field changes to the one-photon added coherent state $|z,1\rangle$ [26]. The relative phase χ_0 between $|z\rangle$ and $|z,1\rangle$ is given by

$$\chi_0 \;=\; (1/i)\; ln\; (\frac{< z|z.1 >}{|< z|z,1 >|}) \tag{14.25}$$

with $0 \;\leq\; \chi_0 \;\leq\; 2\pi$. In the first stage of the operation that follows this, the cloned photon produces another copy of itself through stimulated emission of another excited two-level atom. Consequently, the radiation field transforms to $|z,2\rangle$. The relative phase between $|z\rangle$ and $|z,2\rangle$ is now χ_1 given by

$$\chi_1 \;=\; (1/i)\; ln\; (\frac{< z|z.2 >}{|< z|z,2 >|}) \tag{14.26}$$

This can be simplified using the fact that $|z,2\rangle$ is obtained by applying $a^\dagger$ twice on $|z\rangle$, and we get

$$\chi_1 \;=\; 2^1\chi_0. \tag{14.27}$$

In the second stage of the cloning program, each of the two photons produces a copy of itself by inteaction with two excited atoms, and therefore the radiation field is described by $|z,4\rangle$. The relative phase between this and $|z\rangle$ can be seen to be $2^2\chi_0$. At the nth stage of this cloning progra, the corresponding relative phase χ_n satisfies

$$\chi_n \;=\; 2^n\chi_0 \tag{14.28}$$

modulo 2π. This is identical to the manner in which the phase ϕ evolved in the network problem discussed earlier, and again the equation can be mapped on to the equation governing the map variable in the shift map. A small error in the estimation of the initial relative phase, magnifies exponentially, and the system displays extreme sensitivity to the initial value of the phase. To that extent the prediction of the relative phase, after many stages of the cloning program becomes meaningless.

We have outlined above the optical analog of the network that determines relative phases. An interesting problem that arises in this context is the generalization of this cloning program to include other classes of coherent states, and obtaining a quantum analog of the shift map in terms of operators. The results of this study will be reported elsewhere.

Acknowledgment : We take great pleasure in submitting this article to the Proceedings brought out in honour of Prof.G. Rajasekaran.

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Chapter 15

Felicitations

Several colleagues and friends of Rajaji who could not participate in the Rajaji Symposium sent messages wishing him well on his 65th birthday. We have reproduced some of the more detailed ones below.–Eds.

From R.H. Dalitz

I am sending this e-mail message* to congratulate you on reaching your 65th birthday and on your excellent record of academic work, whether we consider your research or your administrative work in the Institute of Mathematical Sciences at Chennai. I believe that you are also reaching your retirement age now, although I do not know whether it takes place on the date of this birthday or whether it will be delayed until the end of this Academic Year. Either way, I wish you a happy retirement!

I met you first at the Summer School which the Tata Institute for Fundamental Research at Bombay (now Mumbai) had organised to take place at Bangalore in 1961. I don't remember when it was that you came to Chicago University to work in the Enrico Fermi Institute of Nuclear Studies as a graduate student there. We must have got off with a quick start, since our first joint paper, on spins and lifetimes of light hypernuclei, was published in the first issue of Phys. Lett., dated 1962. Most likely, you arrived in Chicago in the Fall of 1961. The work you did in the years 1961-63 was a mixture of hypernuclear physics and strange resonance physics. I recall that your Ph.D. dissertation was a rebuttal of the Yang/Oakes paper which said "that the large mass differences between particles in different channels for the resonance meant that SU(3) symmetry could not hold. It must be wrong". What you proved in your Ph.D. thesis was that this conclusion was not necessarily so.

I visited Chicago for the KAON99 conference. One of the gentlemen just mentioned above, came over to me at this Conference and happened to say that he had not understood what mistake they had made until he had read your Ph.D. thesis. They haven't forgotten you there, Raja.!

I remember also the case of a multichannel system, with energy thresholds at $\ldots i, i+1, i+2, \ldots$, so that there are many thresholds. We realised that there were many ways to leave the Energy-plane, e.g., crossing over the real axis between threshold i and $(i+1)$ or that between $(i+1)$ and $(i+2)$, each getting onto a different unphysical sheet. However

*Prof. Dalitz sent this e-mail message with a request that it "be read at the celebration of Professor Rajasekaran's 65th birthday".

there was a resonance pole on both unphysical sheets, indeed one for each of these unphysical sheets. They are now known as "shadow poles". We didn't invent that name, but it is a very evocative name.

Well, we had quite a lot of fun with other questions, calculations about the 1405 MeV resonance in the hyperon-meson system—its nature is still an open question—and we made the first determination of the strength of the Lambda-Nucleon force, much weaker than the Nucleon-Nucleon force.

I think that you came with me to Oxford in 1963/4, where you worked on your thesis, before you went back to Chicago for your Ph.D. examination. We had a good time in those days. Life doesn't stop when you become "retired". I wish you a happy retirement and that you will find enjoyable and creative things to do.

Dick Dalitz,
Oxford University,
U.K.

From R.K. Bhaduri

The stage must be all set, after all the preparations and planning, for this meeting honouring Rajasekaran's contributions to physics research and development. It is too bad that I cannot be present Before conveying my regards to him, I cannot resist here to reminesence, perhaps not very accurately, some incidents during our long friendship.

In 1959, when I joined TIFR, Rajasekaran was already there. But at that time I did not know him too well. I remember him vividly in the Christmas Party of 1963 that Professor and Mrs. Peierls used to throw every year at 12 Parks road, Oxford. Rajasekaran was completing his Ph.D. with Dalitz. At the party, he was standing by the staircase near the entrance with a glass full of wine. My wife and I were already good friends of his by that time. As a good friend, I had to help him empty the wine glass before Mrs. Peierls came with more ! Next summer, in 1964, he finished writing his thesis (I recall it had to do with $Omega^-$ and SU(3)), and decided to come with us to Europe on a short vacation. He insisted on bringing the manuscript of his thesis with him, so that it does not get lost in his absence. While boating on Lake Como, we threatened to drop it in the lake unless he treated us in a restaurant in Paris. Rajaji good-naturedly agreed, and so we had a grand celebration for his Ph.D. much before it was awarded. During this time, Divakaran and Rajasekaran used to visit our home frequently, and we have nice memories of many evenings spent together.

In 1967, when I went back to TIFR, Rajasekaran was a prominent member of the theory group. I recall he was still interested in ΛHe^3 life-time, where there was confusion that could be blamed on some wrong experiments. Rajasekaran was always a very clear-thinking person, with a quick grasp. I think I would have collaborated with him, but for the fact that I left soon. I remember that at this time he was already married, with a very young and beautiful wife, Swatantra, who cooked fabulous dinners. Much later, I wanted to spend my sabbatical in Madras (in 1981), with the intention of learning QCD (I still want to). Rajasekaran had done some very good work on weak interactions too in the seventies at TIFR. And he was one of the few who actually was working on QCD in its early days. In any case, in 1981-2, he was in Japan, so I did not get to work with him. Rajasekaran had originally gone to the University there, but later joined I.M.Sc. I used to come frequently to visit Murthy, and over the years, had many discussions with Rajaji. He has always shown patience in explaining difficult topics to me.

Rajasekaran is remarkable because of his intellectual honesty. He has set an example to senior physicists everywhere with his active interest in research, and his willingness to learn from his younger colleagues. My wife and I wish him years of healthy and active life, and I hope that he continues to contribute to the high-standard work that goes on at I.M.Sc.

Rajat Bhaduri
McMaster University,
Canada

From U. Sarkar

I take this opportunity to thank you[†] for inviting me to this meeting. It would have been a great honour for me to be present in this meeting because I consider Professor Rajasekaran to be one of the best Particle Physicists in India. Most of us know how much he contributed directly to the community, but very few of us know how high his indirect contributions are. He was an idol for students like us, who did their Ph.D. in Indian Universities. His works inspired us to think that it is possible to continue high standard research from any place with any minimal facilities.

Many years I did not dared to talk with him face to face, because he was too big a figure to me. But when I could finally come closer to him, I realised how simple he is. He never let me feel that he is anything more than my fellow research scientist. At these times he clarified so many complicated physics to me in such simple language, it was really a previlage for me to be associated with him and finally write one paper together with him last year. I hope I can now continue our research collaboration for many more years. As we all see him becoming younger day by day, I hope he would continue to inspire us and give me more opportunity to collaborate with him. I wish him a very active life ahead for many more years to come.

Utpal Sarkar,
Physical Research Laboratory,
Ahmedabad, India

From Sandip Pakvasa

I assume you[‡] are in the middle of the neutrino 2001 meeting and all is going well.

What I regret even more than missing the neutrino meeting is being unable to attend the Rajaji-Fest for his 65th birthday cum retirement. I would appreciate it if you could read this message to him or during the festivities.

I have known Rajaji for over 30 years as a good friend, mentor and teacher(regardless of the fact that we are almost the same age!). Every time I have spent some time in his company, I have learnt some entirely new physics and have come back refreshed. One of the first times I ran into him in Mumbai (in 1970) I got more than physics; he gave up his

[†]Letter addressed to Prof. M.V.N. Murthy in response to the invitation to attend the Rajaji Symposium.
[‡]Letter addressed to Prof. M.V.N. Murthy in response to the invitation to attend the Rajaji Symposium.

apartment (at TIFR) so I could move in and have a place to stay! He ended up going on a pilgrimage . . . but that is another story, you will have to ask him about it!

I have enjoyed all our times together; they have been stimulating, provocative and never dull. I look forward to his active presence for a long time to come.

Here is best wishes and felicitations to Rajaji.

Sandip Pakvasa,
University of Hawaii,
U.S.A.

From J. Maharana

Greetings. First of all please accept my best wishes and felicitations for the forth coming **RAJAJI-FEST** to be held at Chennai next week. I would have been very very happy to be present on that occasion. I I go back in my memory lane, you have been one of the persons who have provided me immense inspirations over two decades while I have been struggling at the Institute of Physics to keep abreast with moving frontiers of high energy physics. To come to specifics— your work on loop space formulation along the lines of the approach of Sakita gave me and L.P. Singh impetus to carry out a Hamiltonian formulation for loops and strings- you might not remember that. This was a good opportunity for me to learn Makenko-Migdal equations and Polyakov's loop space formulations.

The second important contributions to my career came from you when I met you at Bhubaneswar on 16th December 1984 (I remember the date for another reason) where in a brief (should I say in French way) encounter you told me about Green-Schwarz anomaly cancellation result and stressed the importance of string theory at that juncture of time.

Those were certainly most valuable "few minutes" of my life. I took your words into my heart and burried for six months to learn sigma—models, string theory and studied Witten's nonabelian bosonization paper as much as I could sitting under hot roofs in Bhubaneswar with occasional load shedding of power. I went on to write my paper on supersymmetric Grassmannian model soon (nine months later); but most importantly, Veneziano realised the importance of that in relation to string theory. Consequently, my long collaborations with Veneziano started. Time and again (you might not remember your words to me in Calcutta) you encouraged me to stick to that adventurous path, especially when I was persuing alone in directions of string theory at Bhubaneswar for five years before youngsters came in. I wish you a very happy and long life and may your experience and enthusiasm continue to inspire younger generations to persue the most fundamental problems in physics drawing inspirations from your life as an example.

Jnan Maharana,
Institute of Physics,
Bhubaneswar, India

From B. Sriram Shastry

I am writing this on the occasion of the Rajaji Symposium when Professor Rajasekaran's contributions to the Indian science will be celebrated. I would like to take the opportunity to extend my warmest greetings to Professor Rajasekaran. Indeed, I as well as many other former students at the TIFR graduate school have had the honour of learning enormously from him. We learned not only the specifics of the subject, but his whole approach was inspiring. He made things magically straight forward, and emphasised so many "good habits", like calculations instead of speculations, orders of magnitude estimates, and the study of various limits of solutions to problems to check things out. I feel a sense of gratitude towards him, and know that it is true of a whole group of people, then young, in the TIFR days. He was always enthusiastic and non judgemental, and encouraged many who were not even in High Energy Physics, like myself. It was fun and it was wonderful... I hope Rajaji has many more productive and happy years in Physics.

B. Sriram Shastry,
Indian Institute of Science,
Bangalore, India

Chapter 16

Rajaji's Research Interests and Publications

His major scientific contributions

During the early period, Rajaji with Prof. R.H. Dalitz, showed that a pole of the S matrix is in general followed by a retinue of poles. This discovery of "shadow poles" not only removed a serious obstacle to the application of broken symmetry to particle physics but also leads to a reformulation of a basic tenet of the S matrix theory.

The early days of the quark model, saw him look at the possibility of "molecular hadrons" and formulated an empirical test for their identification. This has now become an important topic of current research.

Rajaji pointed out that the current $\times$ current theory of weak interactions violated CPT invariance, unless it was properly symmetrised. (The erroneous unsymmetrised form is used by some authors even now!). He was one of the early proponent of gauge theories and was actively involved in unravelling its ramifications much before it was accepted as the new paradigm of High Energy Physics. His lectures were the very first connected account of a number of topics containing the ingredients that make up the present-day Standard Model of High Energy Physics. The confinement of massless Yang-Mills quanta was conjectured by him even before the advent of QCD.

In a significant contribution the first model-independent analysis of the neutral current weak interaction was pioneered by Rajaji and K.V.L. Sarma. The equations derived by them (subsequently called "Master Equations" by J.J. Sakurai) played a crucial role in pinning down the coupling constants of this new interaction as a part and parcel of the Weinberg-Salam Model.

The remarkable properties of broken-colour QCD with intergrally-charged quarks were elucidated by him and Probir Roy. With his collaborators T. Jayaraman, S. Lakshmi Bala and S.D. Rindani, he tested the viability of the above non-standard QCD in a variety of "jet" experiments. The studies helped in uncovering many loop-holes in the experimental tests of the standard QCD. Further his work with T. Jayaraman and S.D. Rindani revealed new effects invalidating the time-honoured Equivalent Photon Method, for the production of charged particles of spin $> \frac{1}{2}$.

With A.K. Mishra, he has uncovered many new forms of quantum statistics (such as orthostatistics, null statistics, etc.). Their theory of generalized Fock spaces has enlarged the framework within which familiar quantum field theory and statistical mechanics reside.

This work is continuing. With the recent resurgence of interest in noncommutative spaces in string theory, their earlier work on new algebras is likely to have added significance.

With his collaborators at IMSc and other institutions, Rajaji initiated a neutrino-physics program involving a comprehensive analysis of the solar and "cosmic ray" neutrino problems, within a realistic three -flavour oscillation framework, superceding most of the earlier literature based on two-flavour toy models. They have also performed a global phenomenological analysis of a wide-range of observations in neutrino physics including inputs from neutrinoless double beta decay and cosmological models of dark matter. This has resulted in a broad synthesis and they show that Majorana neutrinos can be ruled out, if the small angle solution is borne out for the solar neutrinos.

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Rajaji Symposium

PROGRAMME SCHEDULE

22 February 2001

Each talk is of 20 minute duration with 5 minutes time for Chairman's remarks.

09:15 -9:30	R.Ramachandran	Inaugural address
09:30–10:45	**Session 1**	*T. Pradhan- Chair*
09:35–09:55	Lalit Sehgal	1968 - 1973: Some Reminiscences
10:00–10:20	P.P.Divakaran	Composite Fields and Fermion Masses
10:25–10:45	C.S.Unnikrishnan	Proof of absence of spooky action at a distance in Quantum Correlations
10:45–11:15	T E A B R E A K	
11:15–13:00	**Session 2**	*P M Mathews - Chair*
11:20–11:40	S. Krishnagopal	Are Accelerators needed for High Energy Physics?
11:45–12:05	Avinash Khare	Para and Ortho Supersymmetric Quantum Mechanics
12:10–12:30	A.K.Mishra	Fock Spaces and Quantum Statistics
12:35–12:55	H.S.Mani	Overview of Semi-inclusive DIS Processes
13:00–14:00	L U N C H B R E A K	
14:00–15:40	**Session 3**	*J. Pasupathy - Chair*
14:05–14:25	Rohini Godbole	Physics at future colliders
14:30–14:50	R. Cowsik	Direct Neutrino Mass Measurements
14:55–15:15	R.Parthasarathy	Kaluza-Klein Idea: Ups, Downs and Strange
15:20–15:40	R.P.Malik	Quantum Groups, q-dynamics and Rajaji
15:45–16:10	T E A B R E A K	
16:10–17:30	**Session 4**	*R. K. Kaul - Chair*
16:15–16:35	T.Jayaraman	Solitons in String Theory Compactifications
16:40–17:00	V.Srinivasan	Ramblings
17:05–17:25	M.S.Sriram	Geometric Description of Hamiltonian Chaos
18:30–20:00	**Evening Session** Open Session	*R. Balasubramanian - Chair*
20:00	**Symposium Dinner hosted by the Director**	