A Quantum Theory of Dark Matter

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Dedicated to Barry Simon on his 70th Birthday

Abstract

In this paper we present a possible quantum theory of dark matter.

1 Introduction

In this paper we discuss a possible quantum theory of dark matter. Dark matter gets its name from the fact that it does not interact well with ordinary matter or antimatter and its existence is concluded from its gravitational influence, such as bending of light from far away galaxies.

We therefore start with the assumption that dark matter lives in a Hilbert space orthogonal to the one in which ordinary matter/antimatter resides. One possibility is to consider L^2 space of a measure singular with respect to the Lebesgue measure, which is what we do in this paper.

The problem then is to identify the notion of a derivative associated with singular measures to represent the kinetic energy operator on such a Hilbert

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space of states. We could take fractional derivatives coming from a metric. The difficulty with such a choice is that for functions in L^2, of the measure in question, such fractional derivatives may not even exist almost everywhere with respect to the measure.

We therefore consider a measure theoretic derivative here which is more intrinsic to the measure.

We further consider the Hamiltonian with repulsive potentials, we choose repulsive potential to explain the uniformly present dark matter in the vacuum region of the universe, and identify the spectrum of N dark particles.

The particles are assumed to have a mass and are assumed to interact via a coulomb pair potential among themselves, effective in an ambient space.

As far as we know the formulation we have given here is new and does not appear in the literature.

2 Some Mathematical Preliminaries

We present here some measure theory needed and define the Ayyer-Krishnapur [1] Laplacian for the space we consider and for particular cases identify its spectrum.

Consider a finite positive measure μ on $\mathbb{R}$ and the Hilbert space $L^2(\mathbb{R}, \mu)$. Since μ is finite, $L^1(\mathbb{R}, \mu) \subset L^2(\mathbb{R}, \mu)$ by an application of Cauchy-Schwarz inequality. When f is non-negative we can define the distribution functions

$$\Phi_f(x) = \int_{-\infty}^{x} f(x) d\mu(x), \quad f \in L^2_+(\mathbb{R}, \mu),$$

where $\Phi_f(x)$ is now a non-negative bounded measurable function and therefore is also in $L^1(\mathbb{R}, \mu) \cap L^2(\mathbb{R}, \mu)$. The association $f \to \Phi_f$ is unique, since a finite positive measure and its distribution function are uniquely associated to each other. One can then extend this definition to real valued f presev-ing the uniqueness of $f \to \Phi_f$ (by taking the positive and negative parts respectively) and then to complex valued f by linearity. Thus, associated with every $f \in L^2(\mathbb{R}, \mu)$, there is a finite complex measure $d\nu_f = f(x)d\mu(x)$, such that the Radon-Nikodym derivative of ν_f with respect to μ is f. This association of $f \to \Phi_f$ is also unique, since the real and imaginary parts are uniquely associated to those of f.

Given $f \in L^2(\mathbb{R}, \mu)$ consider the function Φ_f defined above, this function is the "distribution function" of ν_f in the sense that

$$\Phi_f(x) = \nu_f((-\infty, x]), \quad x \in \mathbb{R}.$$

In [1] Ayyer-Krishnapur use this property to define a measure theoretic Laplacian of functions in $L^2(\mathbb{R}, \mu)$ as follows.

This existence of $\Phi_f \in L^2(\mathbb{R}, \mu)$ associated to any $f \in L^2(\mathbb{R}, \mu)$ allows us to define a quadratic form

$$\mathcal{E}_\mu(\Phi_f, \Phi_g) = \int \overline{f}(x) g(x) d\mu(x), \quad f, g \in L^2(\mathbb{R}, \mu). \tag{1}$$

Note that Φ_f is linear in f and its definition it is sesquilinear and we see that

$$\mathcal{E}(\Phi_f, \Phi_g) = \int \overline{f} g \ d\mu = \overline{\int \overline{g} f \ d\mu} = \overline{\mathcal{E}_\mu(\Phi_g, \Phi_f)},$$

so it is symmetric and positive definite, since $\mathcal{E}_\mu(\Phi_f, \Phi_f) \geq 0$.

Suppose Φ_n is a sequence in $L^2(\mathbb{R}, \mu)$ with the property that there is a $f_n \in L^2(\mathbb{R}, \mu)$ with $\Phi_n = \Phi_{f_n}$ and Φ_n converges to Φ in $L^2(\mathbb{R}, \mu)$. To show that $\mathcal{E}_\mu$ is closed, we need to show that if such f_n converges, say to f, then $\Phi = \Phi_f$.

We first note that

$$\begin{aligned} \int |\Phi_n(x) - \Phi_f(x)|^2 \ d\mu(x) \quad & \leq \int | \int^x (f_n(y) - f(y)) \ d\mu(y)|^2 \ d\mu(x) \\ & \leq \int \left(\int^x |f_n(y) - f(y)| \ d\mu(y) \right)^2 \ d\mu(x) \\ & \leq \mu(\mathbb{R})^2 \|f_n - f\|_2^2. \end{aligned}$$

Therefore if $f_n \to f$, then $\Phi_n \to \Phi_f$, but Φ_n converges to Φ by assumption, so by uniqueness of the limit $\Phi = \Phi_f$.

This shows that $\mathcal{E}_\mu$ is also closed. Now combining these facts with Theorem VIII.15, [5], we have the following

Proposition 2.1. *Let μ be a positive finite measure on $\mathbb{R}$ and let $\mathcal{E}_\mu$ be the quadratic form defined in equation (1). Then $\mathcal{E}_\mu$ is a closed, symmetric positive definite form. There is a unique self-adjoint operator Δ_μ associated with $\mathcal{E}_\mu$ such that*

$$\mathcal{E}_\mu(\Phi_f, \Phi_g) = \langle \Phi_f, \Delta_\mu \Phi_g \rangle.$$

Remark 2.2. *Since functions of bounded variation give finite signed measures, the form domain of $\mathcal{E}_\mu$ is expected to be the collection of $L^2(\mathbb{R},\mu)$ functions such that the real and imaginary parts are of bonded variation with the further property that the finite complex measure associated with them (not necessarily uniquely) are absolutely continuous with respect to μ and have a their Radon-Nikodym derivative in $L^2(\mathbb{R},\mu)$.*

We shall now extend the definition of $\mathcal{E}_\mu$ to σ-finite positive measures. For doing this we need to modify the definition of the distribution function of a measure slightly from the standard definition. The reason we can do this will be explained shortly.

Recall that the definition of $\mathcal{E}_\mu$ above came from identifying complex measures which are absolutely continuous with the given μ such that they have $L^2(\mu)$ density with respect to μ. We will preserve this property in the definition of the distribution function.

If μ is σ-finite and $f \in L^1(\mathbb{R},\mu)$, in general the 'standard' distribution function, namely,

$$\Phi_f(x) = \int_{-\infty}^{x} f(y)\ d\mu(y)$$

is well defined, but will no longer be in $L^2(\mathbb{R},\mu)$, even for compactly supported f since Φ_f will be a constant in $[a,\infty)$, for any a with $supp(f) \subset (-\infty,a]$. Therefore we give a new definition:

Let $\mathcal{P}$ be a partition of $\mathbb{R}$ as a union of intervals $\cup_{i\in\mathbb{Z}}^{\infty}(a_i,b_i]$ such that $\mu([a_i,b_i]) < \infty,\ \forall i$.

Definition 2.3. *Suppose $f \in L^2_+(\mathbb{R},\mu)$ Then we define the function $\Phi_{f,\mathcal{P}}$ associated to f by*

$$\Phi_{f,\mathcal{P}}(x) = \int_{a_i}^{x} f(y)\ d\mu(y), \quad x \in (a_i,b_i],\ i \in \mathbb{Z}. \tag{2}$$

When $f \in L^2(\mathbb{R},\mu)$, as before, then using the Jordon decomposition $f = Re(f)_+ - Re(f)_- + i(Im(f)_+ - Im(f)_-)$, in terms of the positie components of the real and imaginary parts of f, we extend the definition Φ_f as:

$$\Phi_f(x) = \Phi_{Re(f)_+,\mathcal{P}}(x) - \Phi_{Re(f)_-,\mathcal{P}}(x) + i\Phi_{Im(f)_+,\mathcal{P}}(x) - i\Phi_{Im(f)_-,\mathcal{P}}(x). \tag{3}$$

Then $\Phi_{f,\mathcal{P}}$ is well defined for any function $f \in L^2(\mathbb{R},\mu)$ and it is in $L^2_{loc}(\mathbb{R},\mu)$. It is also clear from the arguments for the finite μ case, that if

f has compact support then so does $\Phi_{f,\mathcal{P}}$ and in that case it is uniquely associated to f.

We define

$$\mathcal{E}_{\mu,\mathcal{P}}(\Phi_{f,\mathcal{P}}, \Phi_{g,\mathcal{P}}) = \int \overline{f}(x)g(x)d\mu(x), \quad f, g \in L^2(\mathbb{R}, \mu). \tag{4}$$

This quadratic form is well defined on the set of functions $\mathcal{D}_\mu$ of bounded variation having compact support such that the densities of the associated complex measures are in $L^2(\mathbb{R}, \mu)$.

Lemma 2.4. *The quadratic form defined on $\mathcal{D}_\mu \times \mathcal{D}_\mu$ is closed.*

Proof: We will show that if $\phi_n = \Phi_{f_n,\mathcal{P}}, \quad f_n \in L^2(\mathbb{R}, \mu)$, $\mathcal{D}_\mu \ni \phi_n \to \phi$ in $L^2(\mathbb{R}, \mu)$, and $\mathcal{E}_{\mu,\mathcal{P}}(\phi_n - \phi_m, \phi_n - \phi_m) \to 0$, then there is a $f \in L^2(\mathbb{R}, \mu)$ with $f_n \to f$ in $L^2(\mathbb{R}, \mu)$ and $\phi(x) = \phi_{f,\mathcal{P}}(x)$ almost every x w.r.t. μ.

First we note that $\mathcal{E}_{\mu,\mathcal{P}}(\phi_n - \phi_m, \phi_n - \phi_m) \to 0, m.n \to \infty$ implies that f_n is Cauchy in $L^2(\mathbb{R}, \mu)$ so it converges to some f there. Consider, $\Phi_{f,\mathcal{P}}$ which is well defined for almost every $x \in \mathbb{R}$. Then it is clear that since, $f_n \to f$ in $L^2(\mathbb{R}, \mu)$,

$$\sup_{x \in (a_i, b_i]} |\Phi_{f_n,\mathcal{P}}(x) - \Phi_{f,\mathcal{P}}(x)| \leq \mu((a_i, b_i])^2 \int |f_n - f|^2(x) \, d\mu(x), \quad \forall i \in \mathbb{Z}.$$

Therefore, using subsequences that converge almost everywhere, we see that ϕ and $\Phi_{f,\mathcal{P}}$ agree almost everywhere on each $(a_i, b_i]$ for each i showing that they are the same. $\square$

In the case when we consider $\mathbb{R}^n$ and the product measure $\mu = \times_1^n \nu$, we can similarly define, $\mathcal{E}_\mu$, first taking product vectors $f = \times_{i=1}^n f_i$, then their finite linear combinations and then extend the quadratic form $\mathcal{E}_\mu$ to its natural domain.

Definition 2.5. *The Ayyer-Krishnapur Laplacian Δ_μ on $L^2(\mathbb{R}^n, \mu)$ associated with a finite positive measure μ is defined as the unique self-adjoint operator associated with $\mathcal{E}_\mu$. In the case when the measure is not finite but σ-finite, then the operator associated with $(\mu, \mathcal{P})$ is the unique self-adjoint operator $\Delta_{\mu,\mathcal{P}}$ associated to $\mathcal{E}_{\mu,\mathcal{P}}$.*

Remark 2.6. *In general the complex measure associated to $\Phi_{f,\mathcal{P}}$ is the same as $f\mu$, independent of $\mathcal{P}$, however the operator $\Delta_{\mu,\mathcal{P}}$ will in general depend on $\mathcal{P}$. This forces us to make a choice below for our application.*

In the case when μ is an atomic measures on $\mathbb{R}$, we consider the partition $\mathcal{P}_1 : S_\mu = \cup_{x \in S_\mu} \{x\}$ consisting of singletons in the support $S_\mu = \{x : \mu(\{x\}) \neq 0\}$ of μ. In the theorem and the following corollary we drop the subscript $\mathcal{P}_1$ for Δ_μ.

Theorem 2.7. *Suppose μ is an atomic measure, then the self-adjoint operator associated with $\mathcal{E}_{\mu,\mathcal{P}_1}$ is given by*

$$\Delta_\mu = \sum_{x \in S_\mu} \frac{1}{\mu(\{x\})^2} |e_x\rangle\langle e_x|, \quad \text{on} \quad L^2(\mathbb{R}, \mu),$$

where $\{e_x\}$ is the orthonormal basis for $\ell^2(\mathbb{Z}, \mu)$, with $e_x(y) = 0, y \neq x$.

Proof: First note that the orthonormal basis e_x is given by the sequence $e_x(y) = 0, y \neq x, \quad e_x(x) = \frac{1}{\mu(\{x\})^{\frac{1}{2}}}$. Therefore if we find the f_x such that $e_x = \Phi_{f_x,\mathcal{P}_1}$, then we can compute the quantities

$$\mathcal{E}_{\mu,\mathcal{P}_1}(e_x, e_y), \quad x, y \in \mathbb{Z},$$

thus getting the matrix $\Delta_{\mu,\mathcal{P}_1}$ explicitly in this basis. A simple computation shows that the required f_x is just $\frac{1}{\mu(\{x\})} e_x$. Therefore computing the matrix elements we get

$$\langle e_x, \Delta_\mu e_y \rangle = \mathcal{E}_{\mu,\mathcal{P}_1}(e_x, e_y) = \begin{cases} \frac{1}{\mu(\{x\})^2}, & x = y, \\ 0, & x \neq y. \end{cases}$$

□

Clearly when μ is the counting measure on $\mathbb{Z}$, then the above theorem trivially gives:

Corollary 2.8. *In the case μ is the counting measure on $\mathbb{Z}$, the Ayyer-Krishnapur Laplacian on $\ell^2(\mathbb{Z})$ is the I operator.*

3 Hilbert space and Hamiltonian of Dark particles

In this section we discuss the quantum states of dark particles and their dynamics. We begin with a set of postulates:

Postulate 3.1. *The configuration space of dark particle is the support of an atomic measure in* $\mathbb{R}^3$ *and the support is discrete.*

Postulate 3.2. *The Hilbert space of quantum states of a dark particle is* $L^2(\mathbb{R}^3, \mu)$*,* μ *as in Postulate 3.1. The particle has mass* m *and its kinetic energy is given by the Ayyer-Krishnapur Laplacian* $\frac{h_\mu}{2m}\Delta_\mu$ *associated with* μ*, where* h_μ *is an appropriate Planck constant relevant for* μ*.*

In the following we will make the assumption:

Hypothesis 3.3. *We will take the measure* μ *in postulate 3.1 to be* $\mu = \times_{i=1}^{3}\nu$ *where* ν *is the counting measure on* $\mathbb{Z}$*. Thus the configuration space of a single dark particle is assumed to be* $\mathbb{Z}^3$*. In this case* $L^2(\mathbb{R}^3, \mu) = \ell^2(\mathbb{Z}^3)$*. For this case we will set* $h_\mu = \tilde{h}$*.*

4 N body Spectrum

We first compute the spectrum of two particle Hamiltonian and then discuss the N particle case. The Hamiltonian is assumed to consist of the kinetic energy part $\frac{1}{2m}\Delta_\mu$ and the potential is assumed to be repulsive coulomb, of strength D between the pair. Other than this we do not assume any forces affecting the pair of particles. Since the space is discrete we need to rule out the possibility of the two particles occupying the same site since that would involve infinite pair interaction, which will instantaneously move the particles apart.

In view of the Hypothesis 3.3 , the two particle Hilbert space is $\ell^2(\mathbb{Z}^3) \otimes \ell^2(\mathbb{Z}^3)$. We denote the standard orthonormal basis here by $\{\delta_x \otimes \delta_y, x, y \in \mathbb{Z}^3\}$, where $\{\delta_x, x \in \mathbb{Z}^3\}$ is the standard basis for $\ell^2(\mathbb{Z}^3)$. We denote the projection onto the subspace generated by $\delta_x \otimes \delta_y$ by $P_{(x,y)}$. We set $P_{diag} = \sum_{x\in\mathbb{Z}^3} P_{x,x}$. We will continue to write $I \otimes I$ on $\ell^2(\mathbb{Z}^3) \otimes \ell^2(\mathbb{Z}^3)$ as I. We take $|x| = \sqrt{x_1^2 + x_2^2 + x_3^2},\ x \in \mathbb{Z}^2$.

Then the Hamiltonian H for this case is

$$H = (I - P_{diag})(\frac{\tilde{h}}{2m} I \times I + I \times \frac{\tilde{h}}{2m} I) + \sum_{x,y \in \mathbb{Z}^3} \frac{D}{|x-y|} P_{(x,y)} (I - P_{diag}) \quad (5)$$

$$= \frac{\tilde{h}}{m}(I - P_{diag}) + \sum_{x \neq y \in \mathbb{Z}^3} \frac{D}{|x-y|} P_{(x,y)}. \quad (6)$$

$$= \sum_{x \neq y \in \mathbb{Z}^3} \left(\frac{1}{m} + \frac{D}{|x-y|} \right) P_{(x,y)}. \quad (7)$$

Then we immediately see that :

Theorem 4.1. *The spectrum of the two particle Hamiltonian is given by*

$$\sigma(H) = \{ \frac{\tilde{h}}{m} + \frac{D}{\sqrt{L}} : L \neq 4^a(8b+7), L, a, b \in \mathbb{N} \}.$$

Proof: . It is clear that the eigenvalues of H are the numbers $H(x,y) = \frac{\tilde{h}}{m} + \frac{D}{|x-y|}$ with the corresponding eigenvectors $\delta_x \otimes \delta_y, x \neq y$. Since $x, y \in \mathbb{Z}^3$ and a theorem of Legendre ([3], 20.10), specifies that a sum of squares of three integers can take all positive integer values except $4^k(8l+7), k, l \in \mathbb{N}$, the theorem follows. □

In the case of N particles we recognize that the Hilbert space cannot have states where any pair of particles occupy the same position, so we use the following notation for ease of writing the Hamiltonian in this case. Let

$$\mathbb{Z}^{3N}_{\#} = \{(x_1, \ldots, x_N) \in \mathbb{Z}^{3N} : x_i \neq x_j, i \neq j\}.$$

Then the N particle Hilbert space is $\ell^2(\mathbb{Z}^{3N}_{\#})$ and the N particle Hamiltonian is :

$$H_N = \frac{\tilde{h}}{2m} \sum_{x \in \mathbb{Z}^{3N}_{\#}} P_x + \sum_{x \in \mathbb{Z}^{3N}_{\#}} \sum_{1 \leq i < j \leq N} \frac{D}{|x_i - x_j|} P_x. \quad (8)$$

We can say something about the essential spectrum of H_N, these are threshold points given by the spectrum of lower particle Hamiltonians.

Theorem 4.2. *Consider the N-particle Hamiltonian H_N given by equation (8). Then the spectrum is pure point and $\sigma(H_N) \subset \sigma_{ess}(H_{N+1})$, N=1,2,....*

Proof: Since H_N is a diagonal operator in the basis for $\ell^2(\mathbb{Z}^{3N}_{\#})$, the spectrum is pure point. It is clear that when $N = 1$, the particle is free and has Hamiltonian $H_1 = \frac{1}{2m}I$ and so $\sigma(H_1) = \sigma_{ess}(H_1) = \{\frac{1}{2m}\}$.

We have

$$\begin{aligned} H_{N+1} &= \sum_{x_{N+1}\in\mathbb{Z}^3} \sum_{\substack{x\in\mathbb{Z}^{3N}_{\#} \\ x_j\neq x_{N+1},\forall j}} \left(\frac{N\tilde{h}}{2m} + \sum_{1\leq i<j\leq N} \frac{D}{|x_i - x_j|}\right) P_x \otimes P_{x_{N+1}} \\ &\quad + \sum_{y\in\mathbb{Z}^3} \sum_{\substack{x\in\mathbb{Z}^{3N}_{\#} \\ x_j\neq y,\forall j}} \left(\frac{\tilde{h}}{2m} + \sum_{j<N+1} \frac{D}{|y - x_j|}\right) P_x \otimes P_y \\ &= \sum_{x_{N+1}\in\mathbb{Z}^3} H_N(x_{N+1}) \otimes P_{x_{N+1}} + \sum_{y\in\mathbb{Z}^3} V(y) \otimes P_y \\ &= \sum_{y\in\mathbb{Z}^3} (H_N(y) \otimes P_y + V(y)). \end{aligned}$$

The operators $H_N(y) \otimes P_y$ and $V(y)$ commute, for each $y \in \mathbb{Z}^3$ and given any $x \in \mathbb{Z}^{3N}_{\#}$, we take $y : y \neq x_j, \forall j$, then

$$\lambda(x) = \sum_{1\leq i<j\leq N} \frac{D}{|x_i - x_j|}$$

is an eigenvalue of H_N and

$$\lambda(x) + \sum_{1\leq j\leq N} \frac{D}{|y - x_j|}$$

is an eigenvalue of H_{N+1}. Now taking $|y| \to \infty$, we see that these eigenvalues converge to $\lambda(x)$. Thus all the eigenvalues of H_N are in the essential spectrum of H_{N+1}. $\square$

The N body spectrum for $N \geq 3$ is unclear, so we leave it as a problem.

Problem 4.3. *Determine the N body spectrum for $N \geq 3$.*

There are several question that come up we leave with one such:

Problem 4.4. *What is the thermodynamics of the particles subjected to the theory here?*

Once we have the above N particle Hamiltonian, the dynamics is assumed to be governed by the Schrödinger equation, which in this case, will result in only change of phases of eigenvectors as time varies, since the spectrum is pure point.

5 Discussion

While in the earlier sections we presented a mathematically precise theory, in this section we present an informal discussion and some questions.

The dark energy is about 68.3% and dark matter about 26.8% of the composition in the Universe. These numbers indicate that the total energy of the dark particles as per this model given by

$$\langle \psi, H\psi \rangle = \frac{N\tilde{h}}{2m} + D \sum_{1 \le i < j \le N} \frac{1}{|x_i - x_j|},$$

should give the ratio of the dark energy to dark matter content.

If we assume that the universe is bounded then assuming further that the dark particles collectively assume a position of minimum total energy, which requires them to be as far away from each other as possible, in the absence of compelling external gravitational forces, so they will be uniformly spread out in the universe. This appears to be the case in observations. See Deb et al [2].

There is a body of work by Laurent Nottale, specifically the 'scale relativity' based work to explain dark matter see Nottale-Celerier [4]. This body of work however is unconnected with our work presented here.

If we assume that they are at equal distance from each other, L, then the total energy becomes.

$$\langle \psi, H\psi \rangle = \frac{N\tilde{h}}{2m} + D\frac{(N-1)!}{L}.$$

We assumed here that ψ is the state where the N particles assume such a configuration.

If we measure the above energy in units of the mass of the dark particle, then We need some parameters to compute the constant mD.

We note a few features about the quantum theory presented here for dark matter. Firstly there is no uncertainty for the states since the eigenstates correspond to constant momenta. The computation of energy is classical, once we know the potential function the energy in a given configuration of the particles is computed.

The second feature is the fact that in view of the repulsive nature of the pair interactions, the particles as a whole tend to stay away from each other and in a finite sized universe (such as ours), they should be uniformly diffused in space (in the absence of gravity). On the other hand in large galaxies where there is a compensatory gravitational pull, (which is not included in this paper) the density of the dark matter is expected to be larger in the centre of the galaxy than at the outskirts.

The particles subjected to this quantum mechanics change energy by physically moving to another 'location in their configuration space'. We can think of the 'space of dark particles' being in the ambient space $\mathbb{R}^3$ occupied by Baryons and the dark particles appear to 'teleport' in the ambient space when their energy is changed, because they live in a discrete space and from the structure of the Hamiltonian any change in energy happens only by a shift in the relative position of a pair of particles.

This fact raises a serious question about the consistency with the constraints imposed by relativity (our theory here is however non-relativistic).

If relativity were to apply, then we may have to demand that there is an upper bound on the amount of energy the particles can gain or lose in a given period of time. If the change position of a pair of dark particles is Δx in a period of time ΔT, and involves a shift in position from x to y, then we must have

$$\frac{|x-y|}{\Delta T} \leq c,$$

where c is the speed of light. This constraint would be natural if the physics of dark particles is consistent with relativity. This then manifests as a constraint on the change in energy possible in a given period of time, since such a change can come only by a shift in position. This feature may be exploited to suggest an experiment to test the validity of this theory.

The mechanism by which the particles subjected to the theory presented here lose energy, what form it takes, is unclear as they are not assumed to couple to Baryons, Leptons and other 'normal' matter in any way. This is a question to think about.

Another feature with this theory is the lack of specific 'statistics' though there is a natural 'exclusion principle' built in. As of now there does not seem to be a need for a 'spin-statistics' type theorem.

Coupling gravity on a collection of particles subjected to the above theory should also be interesting to study, since the configuration space is discrete. There should be quantum jumps of the particles even in the case of classical gravity.

The above discussion, makes us believe that the time of big-bang perhaps initially dark matter was created causing the inflation which expanded the universe rapidly and there was no distinction between 'quantum' and 'classical' at that stage, subsequently the production of baryons started and more complex physics emerged with their creation.

Overall the theory presented here should give new phenomena, however all this is only if experimental observations confirm that indeed the dark particles behave as per this theory and this is big 'IF'.

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References

[1] Arvind Ayyer, Manjunath Krishnapur, *Private Communication*

[2] Sanghamitra Deb, David M Goldberg, Catherine Haymens and Andrea Morendi, *Measuring dark matter ellipticity of Abell 901/902 using Particle Based Lensing*, The Astrophysical Journal, 721(1), 124 (2010).

[3] G. Hardy, E. M. Wright, *Introduction to the Theory of Numbers*, Oxford University Press.

[4] Laurent Nottale and Marie-Neëlle Cĺérier, *Derivation of the postulates of quantum mechanics from the first principles of scale relativity*, preprint, arXiv:0711.2418v1 (2007).

[5] Michael Reed, Barry Simon, *Methods of Modern Mathematical Physics*, Vol 2,