I·§3·11 Let $\mathcal{T}$ be a topology on a set X. Let A be a dense subset of X. Show that the set of topologies $\mathcal{T}'$ s/t ① $\mathcal{T}'$ is finer than $\mathcal{T}$, ② A is dense for $\mathcal{T}'$, ③ The topology induced on A by $\mathcal{T}'$ is the same as that induced by $\mathcal{T}$ admits maximal elements. Such an element is called A-maximal.

A topology $\mathcal{T}_0$ on X is A-maximal $\Leftrightarrow$ $\mathcal{T}_0 = \{M \mid M \cap A \text{ is open in } A \text{ and dense (for } \mathcal{T}_0) \text{ in } M\}$.

Let $\mathcal{T}_0$ be A-maximal. Then ① $\mathcal{T}_0$ induces the discrete top on $\complement A$, and $\complement A$ is closed (in $\mathcal{T}_0$). and ② if the topology on A is quasi-maximal, then so is $\mathcal{T}_0$. (See I-§2-6 ⓐ below)

I-§2-6 ⓐ Quasi-maximal top: a top maximal w.r.t. not admitting isolated points

TFAE ① Quasi-maximal ② There's no isolated point & any subset without isolated points is open.

I·§7.6 Let I be an uncountable index set. Let X_i be a discrete top space, each X_i having at least two elts. On $\prod X_i$ consider the topology generated by subsets $\prod M_i$, where $M_i \subseteq X_i$ and $M_i = X_i$ except for at most countably many i. In this topology, countable intersections of opens is open; no point admits a countable FSN.