I §7 Limits. Exercises.

1. Let $\mathcal{T}_1, \mathcal{T}_2$ be topologies on a set X. Then $\mathcal{T}_1$ is finer than $\mathcal{T}_2$ $\iff$ every filter convergent to $x \in X$ w.r.t $\mathcal{T}_1$ converges to x in $\mathcal{T}_2$.

2. Let $\mathcal{U}$ be any ultra filter on $\mathbb{N}$. Let $X = \mathbb{N} \cup \{\omega\}$ be the corr. top. space. No sequence in X that has infinitely many distinct terms can converge.

3. Let $f : X \to X'$ map between top spaces be continuous at $x_0 \in X$. If $\mathcal{B}$ is a filter base in X with cluster point x_0, then $f(\mathcal{B})$ has $f(x_0)$ has a cluster point.

4. (a) Let x_0 be a cluster point of a filter $\mathcal{F}$ on X top space. Similarly, let y_0 be a cluster point of a filter $\mathcal{G}$ on a top space Y. Then (x_0, y_0) is a cluster point of $\mathcal{F} \times \mathcal{G}$.

   (b) In $\mathbb{Q}^2$, give an example of a sequence $\{(x_n, y_n)\}$ with no cluster point although both $\{x_n\}$ and $\{y_n\}$ admit cluster points in $\mathbb{Q}$.

5. $X \xrightarrow{f} Y$ map between top spaces. For $x \in X$, the set of cluster points of f at x is the section $\overline{G}(x)$ of G at x (where G denotes the graph of f; $\overline{G}$ = closure of G in $X \times Y$).

7. A subset P of a topological space X is <u>primitive</u> if it is the set of limit points of an ultrafilter on X. Let γ_P denote the set of ultrafilters each of which has P for its set of limit points.

   (a) Any open set of X that meets P belongs to every ultrafilter in γ_P

   (b) If P & P' are distinct primitive sets, then there is $\mathcal{U} \in \gamma_P$ and $\mathcal{U}' \in \gamma_{P'}$ and a subset M of X s/t $M \in \mathcal{U}$ & $CM \in \mathcal{U}'$.

   (c) The image under a continuous mapping of a primitive set is contained in a primitive set.

   (d) If $x \in X$ is s/t $\{x\}$ is closed, then $\{x\}$ is primitive.