¶ (9) (a) An ultrafilter that is finer than a finite intersection of filters is finer than one of them.
(b) Give an example of an ultrafilter which is finer than the intersection of infinitely many filters but not finer than any of them.

(10) The intersection of all elements of an ultrafilter contains at most one point. If it is a singleton then the ultrafilter is the trivial one associated to that singleton.

(11) If $A \notin \mathcal{U}$ ultrafilter (on X, with $A \subseteq X$), then the trace on A of $\mathcal{U}$ is $\mathcal{P}(A)$.

(12) The elementary filter associated with a sequence all of whose elements are distinct is not ultra.

(13) $f: X \to X'$ set map is injective $\iff$ $\forall$ filter base $\mathcal{B}$ on X, $f^{-1}(f\mathcal{B})$ is equivalent to $\mathcal{B}$.

(14) Suppose $f: X \twoheadrightarrow X'$ (onto). Then $f(\mathcal{F})$ is a filter on X' for a filter $\mathcal{F}$ on X.

¶ (15) Let $f: \mathbb{N} \to \mathbb{N}$ be an <u>onto</u> mapping with <u>finite fibres</u>. For a sequence $\{x_n\}$ in a set X, let $y_n := x_{f(n)}$. Then the elementary filters corresponding to $\{x_n\}$ and $\{y_n\}$ are ~~equivalent~~ the same.

Deduce: if $\{a_n\}$ and $\{b_n\}$ are sequences s/t the elementary filter corr. to $\{b_n\}$ is finer than that corr. to $\{a_n\}$, then $\{b_n\}$ is equivalent to a subsequence of $\{a_n\}$.

¶ (16) Let Φ be a countable linearly ordered set of elementary filters on a set X. Then $\exists$ an elementary filter finer than all members of Φ.
(Hint: Show that the union of the filters in Φ has a countable base.)