Exercise set 1. Chapter I. Topological structures §1 open sets, nbhds, closed sets (E2)
(Continued)

(6) A subset A meets every dense set iff its interior is not empty.

(7) Consider the following properties of a topology X: (I) X has a countable base (II) X has a countable dense set (III) Every subset of X all of whose points are ~~countable~~ isolated is countable (IV) Every set of non-empty mutually disjoint open sets is countable.

Show that (I) $\Rightarrow$ (II) $\Rightarrow$ (IV) and that (I) $\Rightarrow$ (III) $\Rightarrow$ (IV). In a metric space, IV $\Rightarrow$ (I), so that all properties (I) – (IV) are equivalent

(8) Any isolated point of $\overline{A}$ must already belong to A.

(9) Let X be set. On subsets of X let $M \to \overline{M}$ be an operation satisfying (1) $\overline{\phi} = \phi$ (2) $M \subseteq \overline{M}$ (3) $\overline{\overline{M}} = \overline{M}$ (4) $\overline{M \cup N} = \overline{M} \cup \overline{N}$. Show that $\exists\,!$ topology on X s/t the closure of any $M \subseteq X$ is precisely $\overline{M}$.