Rmk: Covering space is a local homeo: i.e., every point of $\tilde{B}$ has a nbhd that is homeomorphically mapped onto its image in B by p.
The converse is not true: e.g. $\mathbb{R}^2 \setminus \{0\} \to S^1 \times S^1$.

Lemma 2 $p: \tilde{B} \to B$ continuous. $E \subseteq B$ evenly covered. Then any connected subset F of E is also evenly covered. Components of $p^{-1}F$ are the intersections with $p^{-1}F$ of the components of $p^{-1}E$.

Lemma 3 $p: \tilde{B} \to B$ covering space. ~~If~~ For every $b \in B$ ~~∃~~ every nbhd of b contains a connected open nbhd of b that is evenly covered.

Lemma 4 $p_1: \tilde{B}_1 \to B$ continuous. Assume $\tilde{B}_1$ is locally connected and B is connected. Suppose that every point of B has a nbhd that is evenly covered by p_1. Let $\tilde{B}$ be a component of $\tilde{B}_1$ & let $p: \tilde{B} \to B$ be the restriction of p_1 to p.
Then $p: \tilde{B} \to B$ is a covering ~~∃~~ and $\tilde{B}$ is open in $\tilde{B}_1$.
Proof: Claim $p\tilde{B} = B$. ~~$b \in B$ & V nbhd of b that is evenly covered by p_1.~~
ETS $p\tilde{B}$ is open & closed in B. ~~Any~~ If an evenly covered subset of B (under p_1) intersects $p\tilde{B}$, then that whole subset belongs to $p\tilde{B}$ (as is easily seen). Thus the claim is proved. Easily seen that p is a covering space.
That $\tilde{B}$ is open follows from the first remark on page 401. QED.

Lemma 5 $p: \tilde{B} \to B$ covering. X connected, loc. connected $\subseteq B$.
$\tilde{X}$ be a component of $p^{-1}X$. Then $\tilde{X}$ is relatively open in $p^{-1}X$
And $p: \tilde{X} \to X$ is a covering.
Proof: ~~Let~~ Let $x \in X$. Let $V \overset{\text{open}}{\subseteq} B$ be s/t $x \in V$ & V evenly covered.
Choose Y connected nbhd of x in X s/t $x \in Y \subseteq V \cap X$.
The components $\tilde{Y}_i$ of $p^{-1}Y$ are the intersections with $p^{-1}Y$ of the components of $p^{-1}V$ (Lemma 2). If $\tilde{x}_i$ is the point of $\tilde{Y}_i$ s/t $p\tilde{x}_i = x$, then $\tilde{x}_i$ is interior to $\tilde{V}_i$ (the component of $p^{-1}V$) and $\tilde{Y}_i$ is therefore a nbhd of $\tilde{x}_i$ w.r.t. $p^{-1}X$. It follows easily that $p^{-1}X$ is locally connected & that every point of X has a nbhd which is evenly covered by $p^{-1}X$. Now invoke Lemma 4. QED.