8-5 The Fundamental group of the punctured plane.

Strong Deformation Retract $A \overset{\text{subspace}}{\subseteq} X$ is a **SDR** if $\exists$
$H^{\text{cont}}: X \times I \to X$ s/t $H(x,0)=x$, $H(x,1)\in A$, $H(a,t)=a$ $\forall \begin{smallmatrix} x\in X \\ a\in A \\ t\in I \end{smallmatrix}$

Retract: $A \overset{\text{subspace}}{\subseteq} X$ is a retract if $\exists\, f^{\text{cont}}: X \to A$ s/t $f|_A = \mathrm{id}$.

Theorem: Suppose A is a SDR of X. Then, for $a\in A$, $\pi_1(A,a) \to \pi_1(X,a)$ induced by the inclusion is an isomorphism.

Example: S^1 in $\mathbb{R}^2 \setminus \{0\}$; $\mathbb{R}^2\setminus\{0\}$ included in $\mathbb{R}^3 \setminus$ z-axis;
"figure 8" or "Θ-space" in $\mathbb{R}^2 \setminus$ two points.

8-6 The Fundamental group of S^n.

Suppose $X = U \cup V$, U & V open in X, $U\cap V$ path connected. Let x_0 be a point of $U \cap V$. If both inclusions $i:(U,x_0) \to (X,x_0)$ & $j:(V,x_0) \to (X,x_0)$ ~~are~~ induce the zero hom on fund. gps, Then $\pi_1(X,x_0)=0$.

→ Special van Kampen Theorem

CoR: For $n \geq 2$, the n-sphere S^n is simply connected.

CoR: $\mathbb{R}^n \setminus \{0\}$ is simply connected for $n \geq 3$

CoR: $\mathbb{R}^n$ and $\mathbb{R}^2$ are not homeomorphic for $n > 2$

Remark: $\underset{\text{non-homeo}}{\mathbb{R}^n \not\cong \mathbb{R}^m}$ for $m \neq n$ (invariance of domain) ~~but proof~~
without proof