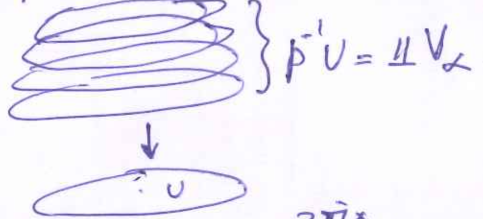
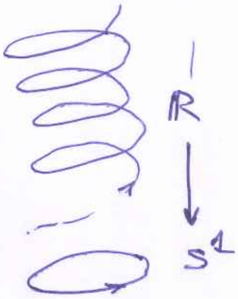
COVERING SPACES

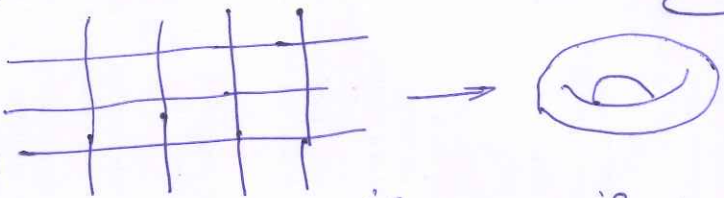
Def: $E \xrightarrow{p} B$ covering if continuous, surjective, and every point of B has an open set that is ~~evenly~~ evenly covered. $U^{open} \subseteq B$ is evenly covered if the inverse image $p^{-1}U$ is a disjoint union of open sets V_α of E each of which maps homeomorphically onto U by p. Each V_α is a slice of $p^{-1}(U)$.





Example: $\mathbb{R} \to S^1$ by $x \mapsto e^{2\pi i x}$ is a covering map.

$\mathbb{R} \times \mathbb{R} \to S^1 \times S^1$ is a covering map



$(x, t) \mapsto (\exp(2\pi i x), t)$

$\mathbb{R} \times \mathbb{R}_+ \to S^1 \times \mathbb{R}_+ \cong \mathbb{R}^2 \setminus \{0\}$ given by $(e^{i\theta}, r) \mapsto re^{i\theta}$ is a covering map

$S^1 \to S^1$ by $e^{2\pi i \theta} \mapsto e^{2\pi i n \theta}$ $n \in \mathbb{Z}$ is also a covering

Remark: ~~Every coveri~~ Covering $\Rightarrow$ local homeomorphism, i.e., $\exists$ an open cover of E each elt of which maps homeomorphically onto its image by p, which is open.
Not every local homeomorphism is a covering, e.g., $\mathbb{R}_+ \to S$ by $x \mapsto e^{2\pi i x}$.