Theorem: $\mathcal{Z} \subseteq Y^X$, where X is a set & Y a top. space.

For $\mathcal{Z}$ to be compact relative to the top. of pointwise convergence, it is sufficient that ① $\mathcal{Z}$ be pointwise closed in Y^X and ② $\mathcal{Z}[x]$ has compact closure $\forall x \in X$.

If Y is Hausdorff these conditions are also necessary.

Proof: use Tychonoff.

Remark: D = finite subsets of X ordered by inclusion. For a finite subset A, let χ_A be its char. fn. Then $\{\chi_A \mid A \overset{\text{finite}}{\subseteq} X\} \to$ constant fn 1. This is clearly not such a good notion of convergence for many purposes. ~~Mo~~ The more interesting topologies are those larger than that of pointwise convergence.

~~We~~ Remark: Suppose $(\mathcal{Z}, \mathcal{T})$ is a larger top than ~~and~~ $(\mathcal{Z}, \mathcal{P})$ (=top of pointwise convergence). If $\mathcal{T}$ is compact & $\mathcal{P}$ is Hdf then they must be the same. The standard method of showing $(\mathcal{Z}, \mathcal{T})$ is compact is this: show $(\mathcal{Z}, \mathcal{P})$ is compact & then show convergence in $\mathcal{P}$ implies convergence in $\mathcal{T}$.

Suppose $A \subseteq X$. We have natural map $Y^X \xrightarrow{R} Y^A$ (R = restriction). R is 1-1 (when restricted to $\mathcal{Z}$) iff for $f \neq g$ in $\mathcal{Z}$ $\exists\, a \in A$ s/t $f(a) \neq g(a)$. In this case, A is said to distinguish elts of $\mathcal{Z}$.

The initial top ~~for~~ under $\mathcal{Z} \xrightarrow{R|\mathcal{Z}} Y^A$ is the ~~pointw~~ topology of pointwise convergence on A.

Propn: $\mathcal{Z} \subseteq Y^X$ (with Y Hdf). The topology $\mathcal{P}_A$ on $\mathcal{Z}$ of ptwise convergence on A is Hdf iff A distinguishes points of $\mathcal{Z}$. If $\mathcal{P}_X$ is compact (where $\mathcal{P}_X$ denotes the top. of pointwise convergence on X) and ~~A~~ A distinguishes points of $\mathcal{Z}$, then $\mathcal{P}$ and $\mathcal{P}_A$ are identical.

Proof: Easy / immediate.