FUNCTION SPACES†

$\mathcal{F}$ be a subset of Y^X = functions from X^{set} to $Y^{\text{top sp}}$.

Suppose Y^X is given the product top & $\mathcal{F}$ the subspace top.

- Then a net $\{f_n, n \in D\} \to g$ iff $\{f_n(x), n \in D\} \to g(x)$ $\forall x \in X$.
- Subbase: $\{f \mid f(x) \in U\}$ as x varies over X and U over open subsets of Y.
- $\mathcal{F} \xrightarrow{e_x} Y$: evaluation maps exist for each x in X. These are continuous (and open if $\mathcal{F} = Y^X$). The top. on $\mathcal{F}$ is the smallest s/t each e_x is continuous. A map g from a top space Z to $\mathcal{F}$ is continuous iff $e_x \circ g$ is continuous $\forall x \in X$.
- The top. of X (if any) is immaterial.
- Y Hdf/Regular $\Rightarrow$ so is $\mathcal{F}$. But local compactness, first/second countability of Y do not pass to $\mathcal{F}$.

Terminology: $\mathcal{F}$ is <u>pointwise closed</u> if $\mathcal{F}$ is closed in Y^X (product top).

For $A \subseteq X$: $\mathcal{F}[A] = \{f(a) \mid a \in A, f \in \mathcal{F}\}$

For $x \in X$: $\mathcal{F}[x] = \{f(x) \mid f \in \mathcal{F}\}$ Note $\mathcal{F}[x] = e_x(\mathcal{F})$.

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† Function space means a ~~set~~ family of functions from a set X to a top. space ~~Y~~ or uniform space Y. We will mostly take X to be a top space and families of functions consisting of continuous maps. Briefly the purpose is to define topologies & uniformities for families of continuous fns and prove compactness, completeness, and continuity properties for the resulting spaces.