Reduction to the theorem of the above statement:

ETS the following: Choose a maximal collection $\mathcal{U}'$ of open sets of X s/t ① $A \cap U$ is meager $\forall\, U \in \mathcal{U}'$ and
② elts of $\mathcal{U}'$ are pairwise disjoint.

Then $A \cap (\bigcup_{U \in \mathcal{U}} U) = A \cap \overline{(\bigcup_{U \in \mathcal{U}'} U)}$. [Note: Maximal collection $\mathcal{U}'$ as above exists by Zorn.]

Proof: By contradiction. Suppose $x \in A \cap \left[(\bigcup_{U \in \mathcal{U}} U) \setminus \overline{(\bigcup_{U \in \mathcal{U}'} U)} \right]$

Put $V = X \setminus \overline{(\bigcup_{U \in \mathcal{U}'} U)}$. ~~This is not empty~~ Now $x \in A \cap \bigcup_{U \in \mathcal{U}} U = \bigcup_{U \in \mathcal{U}} A \cap U$

~~$\subseteq \overline{A \cap \bigcup_{U \in \mathcal{U}} U}$~~ ~~Consider~~

~~$[B \cap \overline{W} \subseteq \overline{B \cap W}$ for W open$]$~~ ~~Consider~~ So x belongs to ~~some~~ $A \cap U$ for some $U \in \mathcal{U}$. Consider $V \cap U$. Since $V \cap U \cap A \subseteq U \cap A$, we conclude that $V \cap U \cap A$ is meagre. ~~Thus we may ad~~

By the maximality of $\mathcal{U}'$, we get $V \in \mathcal{U}'$. But then $x \in A \cap V \cap U$ a contradiction.
because $x \notin \overline{(\bigcup_{U \in \mathcal{U}'} U)}$. QED

Statement without proof of an important theorem

$(X, \mathcal{U}) \xrightarrow{f} (Y, \mathcal{V})$ map of uniform spaces is uniformly open if $\forall\, U \in \mathcal{U}$ $\exists\, V \in \mathcal{V}$ s/t $\forall\, x \in X$ $f(U[x]) \supseteq V[fx]$.

Theorem $X^{\text{complete p.m}} \xrightarrow[\text{Continuous \& Uni. open}]{f} Y^{\substack{\text{Hdf} \\ \text{uniform space}}}$

Then $f(X)$ is complete.