

Reduction to the theorem of the above statement:

ETS The following: Choose a maximal collection \mathcal{U}' of open sets of X s/t ① $A \cap U$ is meager $\forall U \in \mathcal{U}'$ and ② elts of \mathcal{U}' are pairwise disjoint.

Then $A \cap \overline{\bigcup_{U \in \mathcal{U}'} U} = A \cap \overline{\bigcup_{U \in \mathcal{U}} U}$. [Note: Maximal collection \mathcal{U}' as above exists by Zorn.]

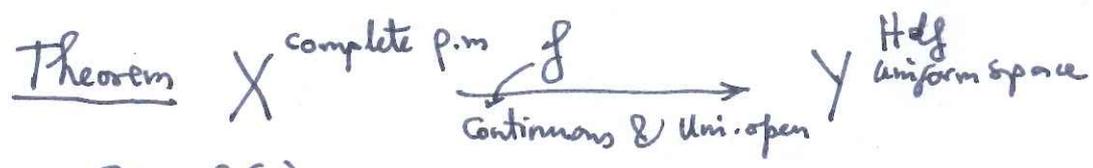
Proof: By contradiction. Suppose $x \in A \cap \left[\overline{\bigcup_{U \in \mathcal{U}} U} \setminus \overline{\bigcup_{U \in \mathcal{U}'} U} \right]$

Put $V = X \setminus \overline{\bigcup_{U \in \mathcal{U}'} U}$. This is not empty. Now $x \in A \cap \overline{\bigcup_{U \in \mathcal{U}} U}$

Consider $V \cap U$. Since $V \cap U \cap A \subseteq U \cap A$, we conclude that $V \cap U \cap A$ is meager. Thus we may ad By the maximality of \mathcal{U}' , we get $V \in \mathcal{U}'$. But then $x \in A \cap V \cap U$ a contradiction. because $x \notin \overline{\bigcup_{U \in \mathcal{U}'} U}$. QED

Statement without proof of an important theorem

$(X, \mathcal{U}) \xrightarrow{f} (Y, \mathcal{V})$ map of uniform spaces is uniformly open if $\forall U \in \mathcal{U} \exists V \in \mathcal{V}$ s/t $\forall x \in X \ f(U[x]) \supseteq V[x]$.



Then $f(X)$ is complete.