COMPLETIONS

Theorem: Each ~~pseudo-metric~~ metric space can be mapped by a one-to-one isometry onto a dense subspace of a complete ~~pseudo-metric~~ metric space.

Theorem: Each uniform space is a dense subspace of a complete uniform space. Each Hdf uniform space is a dense subspace of a Hdf complete uniform space.

Defn of a completion. Completion of a Hdf uniform space is unique up to a unique isomorphism.

Exercises:

COMPACT UNIFORM Spaces As we've seen a completely regular top is the uniform topology of some uniformity (∵ the space is a subspace of a product of p.metrizable spaces) but the uniformity is not unique. If however the space is compact & regular, the uniformity is determined, and topological invariants ~~are~~ uniform invariants.

Propn: Suppose X is a compact uniform space. Then every nbhd of the diagonal is an entourage. Every p.metric that is continuous is uniformly continuous.

CORR: If X is ~~the~~ a compact regular top space then the family of all nbhds of the diagonal is a uniformity for X and the corr. uniform top is just the top on X.

CORR: Each cont. fn ~~from~~ on a compact uniform space with values in a uniform space is uniformly continuous.

~~Totally bounded uniformities ∀ each p.metric d in the gage and ∀ ε>0, X is the union of finitely many sets of d-diameter less than ε.~~