METRIZATION: d pseudo-metric induces a uniformity.

(pseudo)-metrizability: when the uniformity is induced by a (pseudo) metric.

Characterization of a ~~pseudo~~ uniformity given by a pseudometric d:
Coarsest one for which d is uniformly cont w.r.t. product uniformity.
→ previous theorem.

Observe: top induced by a p-metric d is the uniform top. of the uniformity induced by d.

Metrization ~~lemma~~ Theorem: A uniform space is pseudo-metrizable if and only if it has a countable base.

COR: A uniform space is metrizable iff it is Hausdorff & has a countable base.

History: Alexandroff-Urysohn (1925) - ... - Weil-Bourbaki (1937)

Metrization Lemma: Let $\{U_n \mid n = 0, 1, 2, \ldots\}$ be a sequence of subsets of $X \times X$ s/t $U_0 = X \times X$, each $U_n \supseteq \Delta$, $U_{n+1} \circ U_{n+1} \circ U_{n+1} \subseteq U_n$ $\forall n$.
Then $\exists\, d : X \times X \to \mathbb{R}_{\geq 0}$ s/t (a) $d(x,y) + d(y,z) \geq d(x,z)$ $\forall\, x,y,z \in X$
and (b) $U_n \subseteq \{(x,y) \mid d(x,y) < 2^{-n}\} \subseteq U_{n-1}$ $\forall\, n \geq 1$

If each U_n is symmetric, then there exists a pseudo-metric d satisfying (b).

Proof of the Theorem modulo the lemma: Given a countable base $V_1, V_2, \ldots$ of a uniformity, we may find a symmetric base U_n for the uniformity satisfying the hypothesis of the lemma. The conclusion shows pseudo-metrizability of $\mathcal{U}$.