# I §9. No.3. Relative Compactness (Continued)

__Propn:__ $X^{\text{Hausdorff}} \supseteq A$. Then $A$ is $\overset{\text{relatively}}{\text{compact}} \iff \bar{A}$ is compact.

__Pf:__ $\Leftarrow$ $\because A \subseteq \bar{A}$; $\Rightarrow$ $A \subseteq C^{\text{compact}}$; but $C$ is closed since $X$ is Hdf.
So $A \subseteq \bar{A} \subseteq C^{\text{compact}}$. Now $\bar{A}$ is closed in $C$, so it is compact. ▨

__Propn:__ $A^{\text{relatively } QC \text{ } \cancel{\text{compact}}} \subseteq X$. Then every filter base in $A$ has a cluster point in $X$.

__Proof:__ $A \subseteq C^{QC}$. Then every filter base in $A$ has a cluster point in $C$. ▨

__Example 1:__ $=X= [-1,1]/\sim$ where $\sim$ has equivalence classes $\{-1\}$, $\{1\}$, ~~[illegible]~~ $\{0\}$, and $\{-x,x\}_{0<x<1}$. The image of $[-1,0]$ is compact in $X$ but not closed.
The intersection of the images of $[-1,0]$ & $[0,1]$ (both of which are compact) is not quasi-compact. Their union is not compact ($\because$ it is not Hausdorff).

__Example 2__ $X = \mathbb{N} \amalg A$ with $A$ infinite. Define topology on $X$ by specifying the nbhd filters for every point of $X$. If $x \in \mathbb{N}$, then $\{\{x\}\}$ is taken to be a filter base of the filter $\mathcal{N}_x$. If $x \in A$, then $\{\{x, m, m+1, \dots\} \mid m \in \mathbb{N}\}$ is taken to be a filter base of $\mathcal{N}_x$. Show that this choice defines a topology on $X$, which is accessible but not QC, and not Hdf. $X$ however admits dense compact subsets.

__Remark:__ Converse of the $\overset{\text{last}}{\text{}}$ Propn $\Xi$ is not true. Example?