

I §8 Hdf spaces & Regular spaces No. 4 Regular spaces

(35)

Propn: TFAE for a top space X (a) (denoted (O_{III})) The set of closed nbhds form a fundamental system of nbhds (of any point of X)

(b) (denoted (O'_{III})) $x \notin C^{\text{closed}} \Rightarrow \exists$ nbhds A of x & B of C s/t $A \cap B = \emptyset$.

Def: Regular if Hdf & O_{III} (equivalently ~~O_{III}~~ Hdf & O'_{III}).

Remark: Subspaces of regular spaces are regular. (H & O_{III} go down to subspaces)

- If every point of a space has a closed regular nbhd, then the space is regular.

! (Proof: Use H^1 & O_{III} .)

Counter-examples:

Product of regular spaces is regular.