<u>Remark:</u> X/R is Hausdorff $\Rightarrow$ Each R-equivalence class in X is closed. ($\because X/R$ accessible)

<u>Propn:</u> (generalizes the "diagonal is closed" definition of Hausdorffness)
~~Iffn~~ For a relation R on a top space to be Hausdorff (this means, $\frac{X}{R}$ is Hdf) it is necessary that the graph of R in $X \times X$ is closed (this graph by defn is the pull-back of the diagonal in $X/R \times X/R$). If ~~[illegible]~~ R is open, then it is also sufficient.

<u>Proof:</u> Necessity: Obvious since ~~[illegible]~~ (the graph C) is the pullback of the diagonal in $\frac{X}{R} \times \frac{X}{R}$.

! Sufficiency: Since R is open, (the bijection) $\frac{X \times X}{R \times R} \simeq \frac{X}{R} \times \frac{X}{R}$ is a ~~bijection~~ homeo.
~~But~~ On the other hand, since C is the pull-back of the diagonal from $X \times X / R \times R$, it being closed in $X \times X$ is equivalent to the diagonal being closed in $\frac{X \times X}{R \times R}$ (i.e., diagonal of $X/R \times X/R$ thought of as a subset of $X \times X / R \times R$). QED

<u>Remarks:</u>
- If $f \overset{\text{cont}}{:} X \to Y$ with Y Hdf, let R be the equiv. reln $x \sim x'$ iff $fx = fx'$. Then X/R is Hdf. (For $X/R \xrightarrow{\bar{f}} Y$ is continuous)
- Let s be a continuous section of a quotient map $X \to X/R$ with X Hdf. Then (a) X/R is Hdf (b) $s(X/R)$ is a closed subsp. of X (~~[illegible]~~) (Proof: $\frac{X}{R} \xrightarrow{s} X$ is a continuous map into a Hdf space which separates points.

! Thus (a) follows. For (b) Note that $s(X/R)$ is the locus where the continuous maps $id: X \to X$, $s \circ \pi : X \xrightarrow{\pi} X/R \xrightarrow{s} X$ agree. Since X is Hdf $s(X/R)$ is closed.)