Subspaces of Hausdorff spaces are Hausdorff.

! Use H^1 for proof

<u>Propn</u>: If every point of a top space has a closed nbhd that is a Hausdorff space then the space is Hausdorff.

? <u>Example</u>: $\exists$ ~~$\nexists$~~ non-Hausdorff spaces in which every elt of the space has a nbhd that is Hausdorff. E.g. $[-1,1]/\sim$, where $x \sim -x$ if $x \neq 1, -1$.

<u>Propn</u>: Product of Hausdorff spaces is Hausdorff. Conversely, If a product of <u>non-empty</u> spaces is Hausdorff, then so is each factor.

! <u>Proof</u>: $(x_i) \neq (x_i')$ implies $x_{i_0} \neq x_{i_0}'$ for some i_0. Now p_{i_0} is a map into a Hausdorff space ~~that~~ (viz. X_{i_0}) and $p_{i_0}(x_i) \neq p_{i_0}(x_i')$. [See propn in No. 1] Conversely, if X_i are non-empty, each X_{i_0} is realized as a subspace of $\prod X_i$ QED

<u>Propn</u>: ~~When is an initial t~~ (Criterion for Hausdorffness of initial topology) Let X be a set given the initial topology w.r.t. $f_\alpha : X \to Y_\alpha$ top. space. Suppose that the Y_α are Hausdorff. Then X is Hausdorff iff the collection (f_α) "separates points of X".

<u>CoR</u>: ~~lim~~ The inverse limit of Hausdorff spaces (over an inverse system, where the underlying set is only a poset, not necessarily directed set) is Hausdorff and closed in the product.

<u>Proof</u>: $\varprojlim$ being a sub of the product, it is Hausdorff. $\forall$ pair α, β of indices s/t $\alpha \geq \beta$, consider $\prod X_\gamma \to$ ~~$X_\alpha \times X_\beta$~~ $X_\beta \times X_\beta$ given by $(x_\gamma) \mapsto ({}_{\alpha\beta}p\, x_\alpha, x_\beta)$. The pull-back of the diagonal under this map is closed. The $\varprojlim$ is the intersection of these closed sets over all such pairs (α, β). QED.

<u>Remark</u>: A direct sum of Hausdorff spaces is evidently Hausdorff.