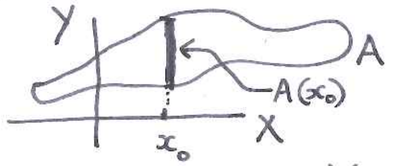
<u>No. 2</u> Sections of an open/closed sets; Projection of an open set; Partial continuity.

Let A be open/closed subset of $X \times Y$. Then for $x_0 \in X$, the set $A(x_0) :=$ $\{a \in A \mid p_X(a) = x_0\}$ is open/closed in Y.

<u>Proof:</u> $Y \to X \times Y$ given by $y \mapsto (y, x_0)$ is a homeomorphism onto its image. $A(x_0)$ is the preimage in Y of the intersection of A with the image.



<u>COR</u> (of proof) Suppose we give X the final top w.r.t. $X \times Y \xrightarrow{p_X} X$ where $Y \neq \emptyset$. This topology is the same as the original top. ($\because$ for any $y \in Y$, $X \to X \times Y$ given by $x \mapsto (x, y)$ is a continuous section of p_X.)

<u>COR:</u> projections from $\prod X_i$ to X_i are open. (but not always closed; e.g. $\{(x,y) \mid xy = 1\}$ ~~projects to~~ $\subseteq \mathbb{R} \times \mathbb{R}$ projects to $\mathbb{R} \setminus \{0\}$)

<u>Separate Continuity:</u> Suppose $X \times Y \to Z$ is continuous at (x_0, y_0). Then $X \to Z$ given by $x \mapsto f(x, y_0)$ is continuous at x_0 ($\because$ it is the composition with f of ~~map~~ $X \to X \times Y$ given by $x \to (x, y_0)$ which is a homeo.)

<u>Caution:</u> It is possible for $X \times Y \xrightarrow{f} Z$ to be discontinuous although $X \to X \times Y \xrightarrow{f} Z$ given by $x \mapsto (x, y_0) \mapsto f(x, y_0)$ and $Y \to X \times Y \xrightarrow{f} Z$ given by $y \mapsto (x_0, y) \mapsto f(x_0, y)$ are continuous $\forall\, x_0 \in X$ & $y_0 \in Y$.

E.g. define $\mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by $f(0,0) = 0$ & $f(x,y) = xy / x^2 + y^2$ for $(x,y) \neq (0,0)$.