No. 1 Product spaces **Elementary set**: $\prod U_i$ with U_i open in X_i & $U_i = X_i$ for almost all i.

A Subbasis $\bigcup p_i^{-1}(\mathcal{G}_i)$ where $\mathcal{G}_i$ is a subbase for X_i. In particular, elementary sets in which at most one U_i is not equal to X_i.

A Base: Elementary sets $\prod U_i$ with $U_i \in \mathcal{B}_i$ where $\mathcal{B}_i$ is a base for X_i. In particular, the elementary sets themselves.

Special case: finite products. Topology on $\mathbb{R}^n$. Base: open boxes

Recall universal property of ~~init~~ product top: $Y^{\text{top space}}$, $f_i^{\text{set maps}}: Y \to X_i$, $y \in Y$

Then $\prod f_i : Y \to \prod X_i$ is continuous at y iff every f_i is continuous at y.

Remarks:
- Let X_i, Y_i be two families of top spaces indexed by I. Let $f_i : X_i \to Y_i$ be set maps. Then $\prod X_i \to \prod Y_i$ is continuous at $(a_i) \in \prod X_i$ iff f_i is cont. at a_i $\forall i$.
- Let $X \xrightarrow{f} Y$ be a set map of top. spaces. Then f is continuous iff the graph of f $\Gamma(f): X \to X \times Y$ given by $x \mapsto (x, f(x))$ is a homeomorphism of X onto its image (as a sub of $X \times Y$). (Proof: $\Leftarrow$ $f = p_Y \circ \Gamma$. $\Rightarrow$ Γ is continuous; p_X is continuous inverse of Γ.)
- Associativity of top. products. $X_i, i \in I$, be top spaces. Let $J_\varkappa$, $\varkappa \in K$, be a partition of I. For $\varkappa \in K$, let $X'_\varkappa = \prod_{i \in J_\varkappa} X_i$. Then the canonical mapping $\prod_{i \in I} X_i \to \prod_{\varkappa \in K} X'_\varkappa$ is a homeomorphism. (Proof: Transitivity of initial topologies.)
- CoR: If σ is a permutation of the set I, then $(x_i) \mapsto (x_{\sigma i})$ is a homeo of $\prod_{i \in I} X_i \to \prod_{i \in I} X_{\sigma(i)}$. (Proof: $K = I$ and $J_i = \{\sigma i\}$.).
- Let $X \xrightarrow{f_i} Y_i$ be a family of set maps into top spaces Y_i. The initial top on X w.r.t. f_i coincides with the inverse image top on X w.r.t. $X \xrightarrow{\prod f_i} \prod Y_i$. (Proof: transitivity of initial topologies)
- Product of subspaces is subspace of the product.

~~Note~~ $\Rightarrow$ Composing with $\prod Y_i \xrightarrow{p_i} Y_i$, we see that $\prod X_i \xrightarrow{p_i \circ \prod f_i} Y_i$ is continuous at (a_i) $\forall i$. Now consider $X_i \xrightarrow{g_i} \prod X_j$ given by $x \mapsto (x_j)$ where $x_j = a_j$ for $j \neq i$ and $x_i = x$. Observe that ~~f_i~~ $p_i \circ \prod_j f_j \circ g_i = f_i$ is continuous. ▨