No. 2. Continuity with respect to a subspace

Let $A \subseteq X^{\text{top sp.}}$ Let A be given the subspace top. Then (by the properties of the initial topology) the inclusion map is continuous and a map $g: Z^{\text{top}} \to X$ with image in A is continuous at $z \in Z$ iff it is continuous at z as a map into A (more precisely, if $g: Z^{\text{top}} \to A^{\text{subspace}}$ is continuous at z). The restriction to A of a continuous fn $X \to Y$ (or one which is only continuous at a point a belonging to A) is continuous (respectively, continuous at a), for composing with the inclusion $A \subseteq X$ (which is continuous) preserves continuity.

The restriction to A of a function f from X to $Y^{\text{top. sp}}$ could however be continuous without f being continuous at any point of X: Suppose that both A and CA are dense in X. Let $X \to \{0,1\}^{\text{discrete}}$ be the characteristic function of A (that is, points of A map to 1, those not in A to 0).

Local character of continuity: if $X \xrightarrow{f} Y$ be a map of top spaces and A be a nbhd of x in X, then f is continuous at x iff $f|_A$ is continuous at x. (For each nbhd of x in A is a nbhd of x in X.)

Propn: Let $\{A_i\}$ be a collection of subsets of X s/t either ① the interiors of the A_i cover X or ② the A_i form a LF closed cover of X. Then a map $f: X \to Y$ to a top. space Y is continuous iff the restrictions $f|_{A_i}$ are continuous ($\forall i$).

No. 3. Locally closed Subspaces A subset L of a top space X is locally closed at a point $x \in L$ if $\exists$ nbhd V_x of x s/t $V_x \cap L$ is closed in V_x. L is locally closed if it is locally closed at each of its points.

Remark: Suppose that $\forall x \in X$ (instead of only for x in L) $\exists$ a nbhd V_x of x s/t $V_x \cap L$ is closed in V_x. Then (since the interiors of V_x cover X) L is closed.

Propn: TFAE for $L^{\text{subset}} \subseteq X^{\text{top sp}}$ ① L is locally closed ② L is the intersection of a closed set and an open set ③ L is open in its closure.

Proof: ① ⇒ ② $\forall x \in L$ $\exists$ V_x^{open} nbhd of x s/t $V_x \cap L$ is closed in V_x. Put $U = \bigcup_{x \in L} V_x$. Then U is open and L is closed in U, So $L = U \cap C$ for some closed C.

② ⇒ ③ If $L = U \cap C$, with U^{open} & C^{closed}, then $L = U \cap \bar{L}$

③ ⇒ ① If $L = U \cap \bar{L}$ with U^{open}, we may take U to be the nbhd with the desired property $\forall x \in L$. QED

COR: The inverse image under a continuous map of a locally closed subset is a locally closed subset.