Exercise Set 5:

1. How many different similarity classes are there of 10×10 nilpotent matrices?

2. How many distinct similarity classes are there of 4×4 complex matrices A satisfying $A^5 = A^3$?

3. Prove: If A & B are $n \times n$ matrices having the same char. poly., the same minimal polynomial, and with no eigenvalue having multiplicity more than 3, then A & B are similar.

4. Find the Jordan form of
$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

5. Classify up to similarity all 3×3 complex matrices A with $A^3 = I$; all 4×4 complex matrices A with $A^4 = I$; etc.

6. Let n be a positive integer, $n \geq 2$. Let N be an $n \times n$ matrix such that $N^n = 0$ but $N^{n-1} \neq 0$. Show that N has no square root. Show that N is similar to its transpose.

7. True or false: every complex $n \times n$ matrix is similar to its transpose.

8. Use the binomial series for $(1+t)^{1/2}$ to ~~show~~ obtain a formula for a square root of $I + N$, where I is the identity $n \times n$ matrix and N is a nilpotent $n \times n$ matrix (over $\mathbb{C}$).

If c is any non-zero complex number, then $cI + N$ also has a square root.