Symmetric Functions: Problem Set 9

1. Show that the definitions of the induced representation $\mathrm{Ind}_H^G(W)$ (given in lecture) make it a representation of G.

2. Prove Frobenius reciprocity; given a H-map $f : W \to U$, there is a unique G-map $\tilde{f}$ such that $\tilde{f} \circ i = f$ where i is the natural inclusion from $W \to \mathrm{Ind}_H^G(W)$.

3. For each irrep W of S_3, compute the decomposition of $\mathrm{Ind}_{S_3}^{S_4} W$ into irreps of S_4, using the inner product version of Frobenius reciprocity.

4. If X is a transitive G-set and H is the stabilizer of an element of X, prove that $\mathrm{Ind}_H^G(W)$ is isomorphic to $\mathbb{C}[X]$, where W is the one dimensional trivial representation of H.

5. Let W be an irrep of H. Prove that $\langle \mathrm{Ind}_H^G(W), \mathrm{Ind}_H^G(W) \rangle \leq [G : H]$.