Symmetric Functions: Problem Set 8

1. Use multiribbon tableaux to compute the character table of S_n for $n = 2, 3, 4, 5$.

2. Consider the partition $\lambda = (n, 1)$. Prove that $\chi^\lambda(w) = f - 1$ where f is the number of fixed points of the permutation $w \in S_{n+1}$.

3. Let $\delta = (n-1, n-2, \cdots, 0)$. Show that s_δ is a polynomial in the odd power sums $p_1, p_3, p_5, \cdots$.

4. Prove that the inner product on Λ_n has the following description:

$$\langle f, g\rangle = \frac{1}{n!}(\text{ the constant term of the product } (a_\delta f)(a_\delta g)^*)$$

where for a polynomial $f(x_1, x_2, \cdots, x_n)$, we let f^* denote $f(x_1^{-1}, x_2^{-1}, \cdots, x_n^{-1})$.