Symmetric Functions: Problem Set 10

1. Let G be the dihedral group with $2n$ elements and H its subgroup isomorphic to the cyclic group of order n. What are the irreducible representations of H ? What are the dimensions of the irreducible representations of G ? For which irreps W of H is $\mathrm{Ind}_H^G W$ an irrep of G ?

2. Let G be the cyclic group C_n and let H be its cyclic subgroup isomorphic to C_d (where $d|n$). For each irrep W of H, describe the decomposition into G-irreps of $\mathrm{Ind}_H^G W$.

3. Let $H \subset K \subset G$, and let W be a representation of H. Prove :

$$\mathrm{Ind}_K^G \left(\mathrm{Ind}_H^K(W)\right) \cong \mathrm{Ind}_H^G(W)$$

(use the universal property of the induced representation)

4. Prove that the multiplication defined on the ring R (the span of class functions of S_d for all d) is commutative and associative.

5. Let V be a finite dimensional representation of S_d, and let χ denote its character. Prove that $\mathrm{ch}(\chi)$ is *Schur positive*, i.e., it can be written as a $\mathbb{Z}_+$ linear combination of Schur functions.

6. Prove that the product of any two Schur functions is Schur positive. The non-negative integers $c_{\lambda\mu}^{\nu}$ in:

$$s_\lambda\, s_\mu = \sum_\nu c_{\lambda\mu}^{\nu}\, s_\nu$$

are called Littlewood-Richardson coefficients.